computation of opacities for twenty mixtures (X varying from 0·9996 to 0·30) in the ranges ($3000 \leqslant T \leqslant 10^9$°K; $10^{-12} \leqslant \rho \leqslant 10^{10}$ gm/cm^3) as well as on monochromatic absorption and scattering coefficients at and below 10^5°K, the results to be published in *Los Alamos Reports* and in the *Astrophysical Journal Supplements*.

T. D. Kusnetsova and D. A. Frank-Kamenetski (**3**) have evaluated the opacity for completely ionized hydrogen according to the exact formulae of the 'Bremsstrahlung' theory and have given a convenient interpolation formula, while Hayashi, Hôshi and Sugimoto have discussed the corrections to the opacity due to free-free transitions in degenerate matter (**4**).

One may also mention here the work of D. H. Samson (**5**) on the effects of the equilibrium concentration of electron-positron pairs on the properties of stars at very high temperatures and the series of papers by C. A. Rouse (**6**) on ionization equilibrium and equation of state for a large range of stellar conditions.

BIBLIOGRAPHY

1. Arking, A., Herring, J. *Astr. J.*, **67**, 110, 1962; *Report of the Institute for Space Studies*, New-York.
2. Cox, A. N. 'Stellar Absorption coefficients and opacities' to appear as chap. II of Vol. **8** (*Stellar Structure*) of *Stars and Stellar Systems*, Chicago Press (to be published soon).
3. Kusnetsova, T. D., Frank-Kamenetski, D. A. *Astr. Zu.*, **39**, 247, 1962.
4. Hayashi, C., Hôshi, R., Sugimoto, D. *Progr. theor. Phys.*, Suppl. no. 22, 3A, 5, 1962.
5. Sampson, D. H. *Astrophys. J.*, **135**, 261, 1962.
6. Rouse, C. A. *Astrophys. J.*, **134**, 435, 1961; *ibid.*, **135**, 599, 1962; *ibid.*, **136**, 636 and 665, 1962; *ibid.*, **137**, 1286, 1963.

III. CONVECTION, EXTERNAL CONVECTION ZONES, SURFACE BOUNDARY CONDITIONS.

Thermal convection either laminar or turbulent is of interest for many fields (hydrodynamics, meteorology, oceanography, geophysics) beside our own. Unfortunately, it is impossible to present here a complete survey of all the work accomplished in these different disciplines although much of it may be of interest for our particular problem.

As far as laminar convection is concerned, Chandrasekhar's book (Chap. II-VI) (**1**) provides an exhaustive and synthetic survey of the associated linear problem including the effects of rotation and magnetic fields when compressibility is negligible. A very short summary of significant results in the same general field together with some remarks on non-linear effects is presented in a paper by Y. Nakagawa (**2**).

Applications to stellar conditions [cf. general accounts by M. Schwarzschild (**3**) and E. Böhm-Vitense (**4**)] requires extensions to take into account:

1. compressibility both in the structure of the layer as well as during the motion of its elements,
2. more realistic boundary conditions,
3. the effects of non-linear terms.

The discussion of the first two points has mainly been approached by the study of convection (usually laminar modes) in superposed polytropic layers (one thermally stable, the other unstable—or of different degrees of instability). In this line, one may report a paper by S. Kato and W. Unno (**5**) discussing especially the effect of compressibility on the scale of motion at marginal stability, one by S. Kato (**6**) concerning the influence of the variation of the superadiabatic temperature gradient on the critical Rayleigh number and the flow pattern, one by P. Souffrin (**7**) (an unstable polytropic layer bounded on both sides by stable isothermal layers) attention being paid essentially to the penetration of the currents in the stable zone, and one by E. Spiegel and W. Unno (**8**) on the same general type of questions. One may perhaps consider that this type of approach culminates in a paper by K. H. Böhm (**9**) in which the same kind of

https://doi.org/10.1017/S0251107X00015108 Published online by Cambridge University Press

linear analysis of the unstable laminar modes is carried out for a realistic model of the external layers of a star.

Although their main object is somewhat different, one may add to this series two papers by P. O. Vandervoort (**10**) devoted to the effects of compressibility, through sound waves and convection, on the Rayleigh instability at the surface of discontinuity separating two compressible media.

As to the difficult problem of the non-linear terms, one may perhaps distinguish the effect of finite amplitudes on purely laminar convection and their effects when turbulence arises, which should be the rule in cosmical bodies. Typical of the first group is the paper of L. L. Segel and J. J. Stuart (**11**) discussing the influence of non-linear terms on the critical Rayleigh number R_a and on the preferred mode and its shape in cellular convection. In a similar vein, G. Veronis (**12**) has shown that, at least in a particular case, sub-critical convection ($R_a < (R_a)_c$, linear) can occur for finite perturbations and he has studied the penetration of the currents outside the primitively unstable region. The discussion of the solution of the non-linear equations of cellular convection and heat transfer has also been tackled by H. L. Kuo (**13**). Although quite different in spirit, we may also include in this group a paper by J. R. Herring (**14**) in which numerical integrations, retaining only those non-linear terms which affects the mean temperature distribution, allow one to follow the establishment of a steady state which qualitatively agrees fairly well with experimental results and in which dominant modes correspond closely to those maximizing the total heat transport. In this respect, see also the recent paper by L. Howard (**15**).

As far as properly turbulent convection is concerned, an attempt was made by P. Ledoux, M. Schwarzschild and E. Spiegel (**16**) to derive the stationary spectrum of turbulent convection by balancing the energy input due to buoyancy evaluated from the linear theory of laminar convection in the Boussinesq approximation against the dissipation into modes of smaller scales due to the non-linear interaction terms evaluated by Heisenberg's heuristic formula. In the second part of Kato's paper (**6**) one finds an extension of this type of approach to the case of a variable temperature gradient.

Recently other models and hypothesis for the turbulent-energy transfer associated with the non-linear terms have been studied by R. H. Kraichnan and E. A. Spiegel (**17**) and by I. Dungstad (**18**) and useful references on this type of question may be found in (**19**). Let us note also that a new spectral equation has been recently proposed by S. A. Kaplan (**20**) for magneto-hydrodynamic convection.

There has appeared also a tendency to generalize the mixing-length treatment so as to cover some of these non-linear effects without recourse to a detailed description of the turbulence spectrum and Kraichnan (**21**) was able, in this way, to obtain expressions for the heat transport, mean temperature, r.m.s. values of the velocity and temperature fluctuations at any arbitrary Prandtl number σ and which, for σ small, reduce to those found in (**16**). E. Spiegel (**22**) has given an expression of the convective heat flux which is not restricted to the case of small mixing-length and which permits to evaluate the penetration of currents in convectively stable regions. On the other hand, W. Unno (**23**) has shown that the linear theory can be used and is essentially equivalent to the mixing-length theory if one introduces an eddy viscosity reducing the Reynold number to about 30.

As far as the internal structure of the stars is concerned, refinements of the theory of turbulent convection are mainly needed for the external convective zones which affect the surface boundary conditions which have to be imposed on the interior solution. In this respect, and in the frame of the mixing-length theory, a semi-empirical approach has often been used trying to determine the mixing-length l in terms of some significant scale-height from a comparison of theoretical models and observational data. Such an attempt by M. Shimoda (**24**) for the

https://doi.org/10.1017/S0251107X00015108 Published online by Cambridge University Press

giant sequence of globular clusters concludes that correct boundary conditions can only be formulated within a theory allowing a complete derivation of the overall structure of the convective flow, but C. Hayashi and R. Hôshi (**25**), considering the same problem, show that the models lie anyway in a fairly narrow region in the H-R diagram in agreement with the observations. For very small masses (**26**), they find that the surface boundary conditions become independent of the efficiency parameter in the convective energy transport. K. S. Krishna Swamy and R. S. Kushwaha (**27**) have discussed, on the basis of atmospheric models, the determination of the constant E fixing the interior convective solution at different points in the H-R diagram. On the other hand, Faulkner, Griffiths and Hoyle (**28**) have developed a method allowing integration downwards from the photosphere through any convective zone that may exist without using explicitly the notion of mixing-length l although, apparently, it is equivalent to adopt a continuously variable l proportional to the local temperature scale-height. They intend to give the pressure and the depth at which $T = 10^{5}$°K is reached as functions of M, L, T_{eff} and the metal abundance A.

Finally let us mention a discussion by Ruben (**29**) which suggests a new version for boundary conditions allowing a direct link from the interior to the atmosphere through the transition region in which $\bar{\mu}$ and κ change continuously.

BIBLIOGRAPHY

1. Chandrasekhar, S. *Hydrodynamic and Hydromagnetic Stability*, Clarendon Press, Oxford, 1961.
2. Nakagawa, Y. *Bull. Soc. R. Sci. Liège*, **31**, 685, 1962.
3. Schwarzschild, M. *Astrophys. J.*, **134**, 1, 1961.
4. Böhm-Vitense E. *IAU Symposium no.* 12: *'Aerodynamic Phenomena in Stella Atmospheres', Nuov. Cim. Suppl.*, **22**, 330, 1961; Part IV, A and B of this symposium are devoted entirely to the problem of convection and contain many interesting comments both from the point of view of hydrodynamics and astrophysics.
5. Kato, S., Unno, W. *Publ. astr. Soc. Japan*, **12**, 427, 1960.
6. Kato, S. *Publ. astr. Soc. Japan*, **13**, 410, 1962.
7. Souffrin, P. *C.R. Acad. Sci. Paris*, **252**, 2997, 1961. cf. also *ibid.* **252**, 2073, 1961 and *Ann. Astrophys.*, **26**, 170, 1963, in which transient aspects in limited regions of a field in turbulent convection, stationary in the large, are studied on the basis of the characteristic instability of the different modes in the model developed in the first reference.
8. Spiegel, E. L., Unno, W. *Publ. astr. Soc. Japan*, **14**, 28, 1962.
9. Böhm, K. H. *Astrophys. J.*, **137**, 881, 1963; cf. also *Astrophys. J.*, **138**, 297, 1963.
10. Vandervoort, P. O., *Astrophys. J.*, **134**, 699, 1961; *Astrophys. J.*, **136**, 1115, 1962.
11. Segel, L. A., Stuart, J. J. *J. Fluid Mech.*, **13**, 289, 1962.
12. Veronis, G. *Astrophys. J.*, **137**, 641, 1963.
13. Kuo, H. L. *J. Fluid Mech.*, **10**, 611, 1961.
14. Herring, J. R. *J. atmos. Sci.*, **20**, 325, 1963.
15. Howard, L. *J. Fluid Mech.*, **17**, 405, 1963.
16. Ledoux, P., Schwarzschild, M., Spiegel, E. *Astrophys. J.*, **133**, 184, 1961
17. Kraichnan, R. H., Spiegel, E. A. *Research Report no. HSN*-4, Courant Institute of Mathematical Sciences, N.Y. University, 1962.
18. Dugstad, I. *Meteor. Ann.* (Oslo), **4**, no. 7, 1962.
19. *Mécanique de la Turbulence*, Paris, C.N.R.S., 1962.
20. Kaplan, S. A. *Astr. Zu.*, **40**, 1047, 1963.
21. Kraichnan, R. H. *Phys. Fluids*, **5**, 1374, 1962.
22. Spiegel, E. A. *Astrophys. J.*, **138**, 216, 1963; for an application, cf. Cameron, A. G. W., and D. Ezer, *Astr. J.*, **68**, 278, 1963.
23. Unno, W. *Publ. astr. Soc. Japan*, **13**, 276, 1961.
24. Shimoda, M. *Publ. astr. Soc. Japan*, **13**, 424, 1961.
25. Hayashi, C., Hôshi, R. *Publ. astr. Soc. Japan*, **13**, 442, 1961.

https://doi.org/10.1017/S0251107X00015108 Published online by Cambridge University Press

26. Hayashi, C., Hôshi, R., Sugimoto, D. *Progr. theor. Phys.*, Suppl. no. 22, 1962.
27. Swamy Krishna, K. S., Kushwaha, R. S. *Astrophys. J.*, **135**, 802, 1962.
28. Faulkner, J., Griffiths, K., Hoyle, F. (to be published in *M.N. RAS*).
29. Ruben, G. V. *Astr. Zu.*, **40**, 855, 1963.

IV. STELLAR STRUCTURE AND STELLAR EVOLUTION

(*a*) *General*

A general review of the problem of stellar evolution has been written by C. Hayashi, R. Hôshi and D. Sugimoto (**1**) which contains also original contributions or suggestions concerning various points which are reviewed in their respective sections. R. L. Sears and R. R. Brownlee have also prepared a general account (**2**) in which computational methods, evolutionary models and evolutionary tracks and their application to the determination of cluster ages are reviewed. To these we may add the lectures of M. H. Wrubel (**3**) which present a concise and very clear summary of the main physical and methodological aspects of the problem.

L. Henyey (**4**) has prepared a general account of the revised numerical methods used in his latest programme to compute evolutionary sequences complying as much as possible with the actual physics of the conditions encountered.

On the basis of a discussion of the ages of hot stars (under different assumptions for their evolution) and cold stars (still in the stage of gravitational contraction) in the Orion Nebula, A. G. Massevitch and E. V. Kotok (**5**) conclude that the duration of the process of star formation in this cluster is considerable of the order, of the age of the cluster itself.

(*b*) *Pre-main Sequence Contraction*

In a very important and now well-known paper, Hayashi (**6**) has shown that contracting stars in the low luminosity, low temperature part of the H-R diagram cannot be in equilibrium. Readjustment of the boundary conditions at the photosphere show that a gravitationally contracting star will be in convective equilibrium throughout its whole mass during a considerable part of this contraction and that the corresponding track in the H-R diagram will be very different from the usual radiative one and nearly vertical offering a possibility of explaining the H-R diagram of very young clusters like NGC 2264. This also reduces very considerably, especially for small masses, the duration of the contraction phases. The consequences for small mass stars are further discussed in (**1**, §10) ($M \simeq 2$ to $0{\cdot}6\ M_\odot$) and applied to the M-type stars in the Orion Nebula indicating masses of the order of $0{\cdot}1$ to $0{\cdot}2\ M_\odot$. It is also shown that for stars with $M < 1{\cdot}3\ M_\odot$, lithium but not beryllium may just burn at the bottom of the convection zone as they complete their evolution towards the main sequence. This discussion has been extended by Hayashi and Nakano (**7**) down to masses as small as $0{\cdot}05\ M_\odot$ taking also into account the dissociation of H_2 in the external layers. In following the contraction, they find that H-burning occurs for $M > 0{\cdot}08\ M_\odot$ but not in stars with $M < 0{\cdot}08\ M_\odot$ which contract directly to degenerate configurations. On reaching the main sequence, stars of the first group which may be compared with actual red dwarfs develop a radiative core if $M > 0{\cdot}26\ M_\odot$ while they remain wholly convective if $0{\cdot}25\ M_\odot \geqslant M \geqslant 0{\cdot}08\ M_\odot$. The Helmholtz-Kelvin time scale for these small masses is much reduced with respect to the previous radiative gravitational contraction, perhaps by as much as factor of 100 and, as shown by Kumar (**8**), cannot be much larger than 10^9 years.

The case of the Sun has been discussed by Weymann and Moore (**9**) who confirm Hayashi's result that such a star remains wholly convective as long as $T_{eff} < 4300°$ and show that the question of the depletion of lithium by the implicated mixing down to regions of fairly high temperatures is very delicate. This same problem has been discussed by Cameron and Ezer (**10**) who find a greater probability for lithium burning. General energy considerations on the dynamical instability arising in the early phases of contraction ($R > 57R_\odot$) due to ionization

https://doi.org/10.1017/S0251107X00015108 Published online by Cambridge University Press