CORRESPONDENCE.

OPINION OF THE LATE DR. WILLIAM WHEWELL AS TO THE GRAPHIC METHOD.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Having regard to the nature of the objections that Mr. Woolhouse has made against the Graphic Method of adjusting Mortality Tables, and, in particular, his expression of opinion that it is unscientific, I think it will be useful to lay before your readers the opinion regarding the method given by the late Dr. William Whewell, who must be admitted to have been an eminently scientific man. This opinion is contained in a passage in his *Novum Organon Renovatum*, which has lately come under my notice; although, as I have had the book for many years, I may possibly have read the passage previously and forgotten all about it. In the chapter entitled *Special Methods of Induction applicable to Quantity*, he has a section devoted to what he calls the "Method of Curves"; and it is from this section that the following passages have been extracted.

It will be noticed that, although Dr. Whewell had not in view the application of the Method of Curves to the adjustment of mortality observations, many of his remarks are quite as appropriate as if they had been written with special reference to this subject. In this connection I would draw attention especially to his statements, *First*, that though the observed facts may appear irregular, the correct facts which they represent are really regular; and, *Secondly*, that the irregular and abrupt deviations, which appear in the course of the original observations as plotted down in a curve, are in most cases small in extent when compared with those bendings which denote the effects of regular law. This is equivalent to saying, in the case of Mortality observations, (1) that although the observed probabilities of death appear irregular as we pass from one age to the next, yet the true probabilities must be really regular; and (2) that the irregularities in the observed series of probabilities are, in most cases, small in extent as compared with the general sweep of the curve considered as a whole. These are the fundamental principles on which we rely in graduating a mortality table by the Graphic Method; and the extract not only admirably describes the process of adjustment, but is even more useful as explaining the principles which establish its claim to be a thoroughly scientific process and not a mere empirical contrivance.

I am, Sir,

Your obedient Servant,

T. B. SPRAGUE.

Edinbro,
12 *March* 1892.

Extract from WHEWELL'S "*Novum Organon Renovatum*", Book III, ch. vii, p. 204. (3rd Ed. 1858.)

The Method or Curves proceeds upon this basis: that when one quantity undergoes a series of changes depending on the progress of another quantity, * * * * this dependence may be expressed by means of a *curve*. In the language of mathematicians, the variable quantity, whose changes we would consider, is made the *ordinate* of the curve, and the quantity on which the changes depend is made the *abscissa*. In this manner, the curve will exhibit in its form a series of undulations, rising and falling so as to correspond with the alternate Increase and Diminution of the quantity represented, at intervals of Space which correspond to the intervals of Time, or other quantity, by which the changes are regulated. Thus, if we set up, at equal intervals, a series of ordinates, representing the Height of all the successive High Waters brought by the tides at a given place, for a year, the curve which connects the summits of all these ordinates will exhibit a series of undulations, ascending and descending once in about each Fortnight; since, in that interval, we have, in succession. the high spring tides and the low neap tides. The curve thus drawn offers to the eye a picture of the order and magnitude of the changes to which the quantity under contemplation, (the height of high water), is subject.

Now the peculiar facility and efficacy of the Method of Curves depends upon this circumstance;—that order and regularity are more readily and clearly recognized, when thus exhibited to the eye in a picture, than they are when presented to the mind in any other manner. To detect the relations of Number, considered directly as Number, is not easy: and we might contemplate for a long time a Table of recorded Numbers, without perceiving the order of their increase and diminution, even if the law were moderately simple. * * * *

But the Method of Curves not only enables us to obtain laws of nature from *good* Observations, but also in a great degree, from observations which are very *imperfect*. For the imperfection of observations may in part be corrected by this consideration;—that though they may appear irregular, the correct facts which they imperfectly represent, are really regular. And the Method of Curves enables us to remedy this apparent irregularity, at least in part. For when observations thus imperfect are laid down as Ordinates, and their extremities connected by a line, we obtain, not a smooth and flowing curve, such as we should have if the observations contained only the rigorous results of regular laws; but a broken and irregular line, full of sudden and capricious twistings, and bearing on its face marks of irregularities dependent, not upon law, but upon chance. Yet these irregular and abrupt deviations in the curve are, in most cases, but small in extent, when compared with those bendings which denote the effects of regular law. And this circumstance is one of the great grounds of advantage in the Method of Curves. For when the observations thus laid down present to the eye such a broken and irregular line, we can still see, often with great ease and certainty, what twistings of the line are probably due to the irregular errors of observation; and can at once reject these, by drawing a more regular curve, cutting off all such small and irregular sinuosities, leaving some to the right and some to the left; and then proceeding as if this regular curve, and not the irregular one, expressed the observations. In this manner, we suppose the errors of observation to balance each other; some of our corrected measures being too great and others too small, but with no great preponderance either way. We draw our main regular curve, not *through* the points given by our observations, but *among* them: drawing it, as has been said by one of the philosophers (Sir J. Herschel,*

* Mr. Sorley, in his "Observations on the Graduation of Mortality Tables" (*J.I.A.*, xxii, 309) has mentioned Sir J. Herschel's connection with the Graphic Method, and has made some lengthy quotations from the paper here referred to.—T. B. S.

Ast. Soc. Trans. vol. v, p. 1) who first systematically used this method, "with a bold but careful hand". The regular curve which we thus obtain, thus freed from the casual errors of observation, is that in which we endeavour to discover the laws of change and succession.

By this method, thus getting rid at once, in a great measure, of errors of observation, we obtain data which are *more true than the* individual *facts themselves*. The philosopher's business is to compare his hypotheses with facts. But if we make the comparison with separate special facts, we are liable to be perplexed or misled, to an unknown amount, by the errors of observation; which may cause the hypothetical and the observed result to agree, or to disagree, when otherwise they would not do so. If, however, we thus take the *whole mass of the facts*, and remove the errors of actual observation, (*Ibid.*, vol. v, p. 4), by making the curve which expresses the supposed observation regular and smooth, we have the separate facts corrected by their general tendency. We are put in possession, as we have said, of something more true than any fact by itself is.

One of the most admirable examples of the use of this Method of Curves is found in Sir John Herschel's *Investigation of the Orbits of Double Stars*. The author there shows how far inferior the direct observations of the angle of position are, to the observations corrected by a curve in the manner above stated. "This curve once drawn", he says, "must represent, it is evident, the law of variation of the angle of position, "with the time, not only for instants intermediate between the dates of "observations, but even at the moments of observation themselves, much "better than the individual *raw* observations can possibly (on an average) "do. It is only requisite to try a case or two, to be satisfied that by "substituting the curve for the points, we have made a nearer approach to "nature, and in a great measure eliminated errors of observation." "In "following the graphical process", he adds, "we have a conviction, almost "approaching to moral certainty, that we cannot be greatly misled."

A LONG-LIVED FAMILY.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The vital history, as annexed, of a family of 11 children is so remarkable that it would seem worthy of insertion in the *Journal:* the father died in 1845, aged 85, and the mother in 1811, aged about 45. I would add that the father, one of the founders of a great life office, was the head of a well-known house now rivalling, in the vigour of its official existence, the longevity of certain of its members.

	Sex	Born	Died	Age at Death
1	F.	7 Dec. 1789	24 Jan. 1892	102·1
2	F.	23 Feb. 1791	10 Nov. 1886	95·7
3	M.	28 March 1792	10 Jan. 1881	88·8
4	F.	9 April 1793	10 Aug. 1875	82·3
5	F.	16 Feb. 1795	27 March 1891	96·1
6	M.	4 Sept. 1796	1796	(say) ·1
7	M.	20 Nov. 1798	28 Sept. 1841	42·9
8	M.	15 Dec. 1799	8 June 1869	69·5
9	M.	23 Jan. 1801	29 Dec. 1888	87·9
10	M.	21 April 1802	3 Feb. 1892	89·8
11	F.	27 June 1806	8 May 1807	·9