PAPER 27

ON MERIDIONAL CIRCULATIONS IN STELLAR CONVECTIVE ZONES

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ABSTRACT

It is shown in outline by a discussion of the equation of motion in all its components and by taking regard of the geometrical properties of turbulent exchange, that in general the state of pure rotation (without meridional circulations) is not a possible stationary state of motion for convective zones of stars.

It would not be possible, in the time available to me to give a complete survey of existing sunspot theories, to which so much effort has been devoted in recent years by a number of competent workers. Instead I will rather aim at grouping and discussing these theories from a point of view, suggested by the theory of stellar structure which will be developed below. In doing so, I shall make use of certain results concerning the structure of thermal convection and on meridional circulations in convective zones in stars. Only part of these results have been published; more recent ones are outlined in a preliminary note, which has been distributed to this symposium.

I start then by indicating which conceptions regarding the internal structure of the sun will be used as a basis for our discussion.

1. The hydrogen convection zone is regarded as deep, that is to say, of a depth of the order of 0·1 or 0·2 $R_{\odot}$, and of nearly adiabatic structure, such that

$$\Delta\frac{d\ln T}{d\ln P}\equiv\frac{d\ln T}{d\ln P}-\left(\frac{d\ln T}{d\ln P}\right)_{ad}\ll 1 \qquad \begin{cases} R_{\odot}=\text{solar radius}\\ T=\text{temperature}\\ P=\text{pressure,}\end{cases}$$

(but >0) everywhere, except near the photosphere, by several powers of 10. The convective transport of energy is assumed to dominate; for the detailed picture reference is made to the work of Mrs Boehm-Vitense [1] and to my own work [2, 3].

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2. For the main part of the interior it is assumed that the radiative transport of energy leads to a sub-diabatic temperature gradient.

3. The existence of a central convective zone, due to the contribution of the carbon cycle to the energy production, has become doubtful; if it exists at all, its instability cannot be pronounced, most of the energy sources being outside.

A crucial point for the hydromagnetics of sunspots and of the solar cycle, in particular for the interpretation of the migration of the activity zones on the surface, is the state of rotation and of large-scale circulation. On the surface we observe pronounced differential rotation such that

$$\frac{\partial \omega}{\partial \phi} < 0 \qquad \begin{cases} \omega = \text{angular velocity of rotation} \\ \phi = \text{latitude,} \end{cases}$$

and there are at least observational indications for $\partial\omega/\partial r$ being < 0, where r is the distance from the centre (U. Becker [4]).

From the theory of internal structure, one has, for the radiative deep interior, the theorem of von Zeipel–Vogt–Eddington, according to which in general the state of large-scale motion of a star with non-vanishing angular momentum of rotation, must be one of differential rotation with superposed meridional circulations (of velocity v_m), such that the isobaric and the isothermic surface will generally not coincide. Their angle as well as the relative temperature difference between axis and equator on isobaric surfaces, which I denote by

$$\Delta^* \ln T,$$

will in general be of the order of (g acceleration of gravity, ϵ energy production in erg. $g^{-1}\,sec^{-1}$)

$$\frac{\omega^2 r}{g} \quad (\approx 2\cdot 10^{-5} \text{ on the solar surface})$$

and v_m of the order of $(\omega^2 r/g)\,(\epsilon/g)$, that is to say exceedingly slow. Also the initial conditions may still be reflected in the actual law of rotation of the sun. If magnetic fields are present, one has furthermore the familar law of isorotation, of Ferraro [5] and Alfvén [6].

In a convective zone one would, at first sight, expect $\Delta^* \ln T = 0$ owing to the efficiency of the turbulent exchange ('Austausch'), and likewise $\nabla\omega \approx 0$ for the same reason. On the other hand we observe at the surface $\nabla\omega \neq 0$, from which Bjerknes [7] and Randers [8] have concluded that a meridional circulation of the order of 1 m/sec must be present in order to keep stationary this state of rotation in spite of the observed turbulence of the surface layers. The meridional circulation, in the scheme of Bjerknes and Randers, had to be supposed to be maintained by a given $\Delta^* \ln T > 0$ (the sign being fixed by the direction of the required circulation, towards the equator near the surface).

I am now going to discuss an analogue to the von Zeipel–Vogt–Eddington theorem, which, in contrast, applies to convective zones in rotating stars. The starting point is a property of turbulent exchange due to thermal instability, which as such has been discussed already long ago, but has received little attention, that is to say its nonisotropy [9]. Since the primary acceleration in thermal convection is anti-parallel to the direction of gravity, whereas the motions in the plane perpendicular to that direction are only necessitated by the conservation of mass, it is obvious that the turbulence of largest scale under such conditions cannot be isotropic. This follows also from a consideration of the respective properties of large-scale motion and of turbulent exchange in the earth's atmosphere, where the conservation of angular momentum in large-scale motions is the basis for some well-known laws of the winds. It follows, that the turbulent exchange, which strictly speaking is a tensor, for the transport of angular momentum of rotation, cannot be reasonably approximated, by one scalar coefficient. The simplest approximation expressing the non-isotropic character of large-scale turbulence, is a combination of the two scalar coefficients, one of which (A_1) may be said to denote the isotropic part, whereas the other one (A_2) expresses a monotropic transport of angular momentum, in the direction of gravity. The flux of angular momentum of rotation, in this approximation, is thus thought of as being made up by two parts, one of which is analogous to that due to ordinary viscosity proportional to A_1 and to the gradient of the angular velocity of rotation, the other one being essentially the product of the monotropic exchange coefficient A_2 and of the radial derivative of the angular momentum of rotation per unit mass. In the absence of meridional circulations, it would follow, that ω can depend only on the distance r from the centre, its rate of change with r being given by

$$\frac{d \ln \omega}{d \ln r} \approx \frac{-2 A_2}{A_1 + A_2}.$$

For the discussion which follows we assume, as is true for the sun, $\omega^2 r/g \ll 1$ and disregard the contribution of the pressure of radiation and of turbulence to the total pressure gradient. We fix our attention to the momentum balance in a meridional plane. Writing the equations in question in such a form that all terms have the dimension of accelerations, we see by taking the curl that in a stationary state (cf. Randers [8])

$$\text{curl } [\boldsymbol{\omega}[\boldsymbol{\omega r}]] = \text{curl } \left(\frac{1}{\rho} \nabla P\right) + \text{terms only due to } v_m \text{ (e.g. } A_1 \Delta^2 \mathbf{v}_m).$$

Using only the equation of state (by an expansion effectively in spherical harmonics) it may be shown that

$$\left|\text{curl } \frac{1}{\rho} \nabla P\right| \approx \left(\frac{g}{r} \Delta^* \ln T\right) \sin 2\phi$$

$$+ \text{terms of higher order} \equiv \omega^2 \left(\frac{g}{r\omega^2} \Delta^* \ln T\right) \sin 2\phi + \ldots$$

The expression in brackets is either of order unity, in the general case of baroclinic rotation, or $\ll 1$, the special case of almost barotropic rotation ($\Delta^* \ln T \ll \omega^2 r/g$). The ratio g/r varies along r approximately as $1/r^3$.

Independent information regarding $\Delta^* \ln T$ may be gained from the equation of energy transfer by turbulence

$$H_k = \text{const. } P v_t \, \Delta \frac{d \ln T}{d \ln P} \text{ erg. cm}^{-2} \text{ sec}^{-1} \quad (v_t = \text{velocity of large-scale turbulence}).$$

The non-dimensional constant is of the order unity, if c_p is not increased by partial ionization, or if the effectivity of the transport is not diminished in some way (see below).

Apart from the alternative last mentioned, $\Delta^* \ln (H_k/Pv_t)$ would again be of order $\omega^2 r/g$, and hence ($(d \ln T/d \ln P)_{ad}$ being the same on axis and equator)

$$\Delta^* \Delta \frac{d \ln T}{d \ln P} = \Delta^* \frac{d \ln T}{d \ln P} \text{ of order } \frac{\omega^2 r}{g} \Delta \frac{d \ln T}{d \ln P},$$

that is to say, small compared with unity to a higher order, owing to the smallness of $\Delta \frac{d \ln T}{d \ln P}$ itself. Integrating over $d \ln P$, the change of $\Delta^* \ln T$ along r is obtained, which is still $\ll \omega^2 r/g$.

For the case of pure rotation ($v_m = 0$) the argument given above ($\omega \equiv \omega(r)$ only), leads to

$$|\text{ curl } [\boldsymbol{\omega} \; [\boldsymbol{\omega r}]] \,| \approx -\boldsymbol{\omega}^2 \frac{d \ln \omega}{d \ln r} \sin 2\phi + \text{terms of higher order with}$$

$d (\ln \omega)/d (\ln r)$ of order -1.

The boundary condition at the inner boundary of the convective zone, which is least favourable for meridional circulations in the zone, is obviously $\Delta^* \ln T = 0$. Since $\Delta \ln P$ along r in the convective zone is only of order of 10 it would follow that

$$\Delta^* \ln T \text{ of order } 10 \frac{\omega^2 r}{g} \Delta \frac{d \ln T}{d \ln P},$$

that is

$$\ll \omega^2 r/g$$

throughout the whole convective zone. On the other hand, assuming for the argument the absence of such circulations it would follow that the curl of $[\boldsymbol{\omega}[\boldsymbol{\omega r}]]$ is of the order ω^2 itself.

Hence the curl of $\frac{1}{\rho} \nabla P$ would be small compared with ω^2, but that of $[\boldsymbol{\omega}[\boldsymbol{\omega r}]]$ of order ω^2. It follows, from the contradiction thus reached, that $v_m = 0$ is no possible stationary solution of the momentum equations. It may be concluded, then, that the normal state of motion of the convective zone in a rotating star is such that meridional circulations regulate together with the turbulent exchange the flow of momentum in such a way that the law of rotation is stationary, which in this case must be expected to lead to a variation of ω, both with latitude and with depth. Since the depth is not very small compared with the solar radius, these relative variations would not be expected to be relatively small, as is actually observed to be the case for the variation of ω with latitude and is indicated by the aforementioned observations for the variation with depth.

Further discussion, which cannot be presented here, shows that the effect of partial ionization and/or the local diminishing of the effectivity of transport—by the stabilizing action of rotation (Cowling [10]; Biermann [11]) or by magnetic fields (Biermann [12]; Chandrasekhar [13, 14]) must not be disregarded for the actual law of rotation and for the geometry of the circulations, which therefore might be not quite simple. From the analogy of the earth's atmosphere one might expect in the equatorial regions considerable circulations with likewise considerable variations of ω, whereas in the polar regions almost solid body rotation without meridional circulations would be expected. As in the earlier reasoning of Bjerknes and Randers, the observed state of differential rotation together with the isotropic part of turbulent exchange would lead to a meridional velocity of the order of 1 m/sec.

The magnetic observations, as discussed by Babcock at this symposium—which I choose to take at their face value—indicate that in the polar regions, down to a distance of about 45°, an apparently permanent poloidal magnetic field is present, with an intensity of the order of 1 gauss. This field may well be a relict of a primordial magnetic field of the sun. On the other hand, in the middle and low latitudes the observations strongly indicate the presence only of a local field of essentially toroidal shape as indicated by the BM regions and the sunspots. Both seem to be similar in their general properties and with respect to the total magnetic flux, the lifetime of the magnetic fields brought to the surface being apparently considerably longer than that of visible sunspots. The main difference between a BM region and a bipolar spot appears to be the concentration of the magnetic flux in the photosphere, as to which references is made to the paper of Schlüter and Temesváry to be discussed later. Owing to the long lifetime of magnetic fields even in the surface regions of the sun, the magnetic fields connected with BM regions and spots must be supposed to be brought to the surface by convection, as was pointed out long ago by Cowling [15]. Hence the theoretical picture of differential rotation and meridional circulations as discussed before, and the magnetic observations, seem to fit quite well, in the sense, that the near axis regions seem to be the seat of the permanent poloidal field, the law of rotation conforming to the law of Ferraro and Alfvén, whereas in the low latitudes the meridional circulations do not seem to permit any permanent poloidal component, but strongly favour magnetic fields of toroidal character as long-lived features.

Turning now to the several schemes proposed for understanding the cycle of solar activity and the properties of sunspots, I will start by indicating, as a central problem, the question of whether the change of location of the activity belts with the cycle is due to mass motion or due to a change of the physical state, that is to say to a wave-type phenomenon.

The picture proposed by Alfvén [16, 17, 18, 19] and Walén [20] about ten years ago, and similarly some other schemes proposed by more recent workers, agree in assuming a poloidal magnetic field also in low latitudes, along which disturbances may be propagated giving rise to travelling or to standing waves. Now I do not want to repeat the arguments advanced already earlier, e.g. by Cowling against these schemes. What I want to emphasize is, that, apart from the low value of the magnetic intensity now observed, the theoretical results concerning circulations presented here, again seem to disfavour strongly any possibility that stationary poloidal magnetic fields exist in the regions in question. One might argue, of course, that the influence of magnetic fields on the deceleration of meridional circulations has not been taken account of in the analysis above; but it is seen without much difficulty, that although magnetic fields being able to restore the balance of momentum can easily be constructed mathematically, their properties are such, that they should not be expected theoretically, and there is no observational indication whatsoever of their presence. We infer that on the sun in these regions the magnetic fields should adjust themselves to the circulations maintained by the conditions discussed above (and not vice versa).

We next turn to the scheme advanced already long ago from hydrodynamic considerations by Bjerknes [7, 21]. Replacing Bjerknes's vortex girdles around the axis by a toroidal magnetic field, it is seen at once that the general pattern is compatible with the state of rotation and of meridional circulations derived above. This does not mean, of course, that the theory has been proved to be correct. There is, e.g. lacking a proof that an arrangement of two girdles of opposite direction in each hemisphere is a stable one, and that it is continuously reinforced in some way. What I would like to indicate is only that the general background of this scheme is in better harmony with what one should expect from the theory of internal structure, than one probably used to think.

There are two points which I would like to note in passing. Magnetic girdles of purely toroidal structure would be dynamically unstable, since in a sense their specific weight is smaller than that of the layers in which they are embedded. This is probably one main cause for the appearance of BM regions and spots on the surface as suggested by Elsasser and Parker [22]; but to secure dynamical stability over long periods of time, the magnetic fields must be of such a structure that their specific weight is effectively equal to that of the regions to which they belong. Hence fields of the type studied by Lundquist [23], Lüst and Schlüter [24], called force-free-fields, are suggested.

The second remark is this. There has been a tendency to set aside the theoretical possibility that impressed electro-motive forces, play a role in creating and maintaining toroidal magnetic fields around the axis of rotation. Now it may easily be shown, from my work in 1949 [25] that the magnetic flux, created by such forces in the inner parts of the hydrogen convection zone, has roughly the order of magnitude, observed in big sunspot groups and BM regions. Furthermore this mechanism seems to be a very natural one to create complete magnetic girdles around the axis. But obviously more questions remain open here than have been answered.

Lastly I would like to discuss briefly the scheme recently proposed by Parker [26]. This scheme has several features in common with the general pictures of the solar magnetic fields we had discussed earlier. In particular, according to Parker, toroidal fields are the main feature of the low latitude regions, poloidal fields dominating in the polar regions. Parker assumes that the toroidal field is continuously reinforced from the poloidal field by differential rotation, and the poloidal field vice versa from the toroidal field by cyclonic motion. In this connexion Parker notices the theoretical possibility of migratory dynamo waves in the convective zone, which he identifies with the migration of the activity belts. Now from his expression for the velocity of this migration, it is found that the latter depends only on the velocity of differential rotation and on that of the cyclonic motion (not on the magnetic field intensity) and with the most probable choice of these velocities there is a discrepancy in the velocity of migration of roughly three powers of 10. On the other hand, no account has been taken in Parker's picture as published, of any meridional circulation; a possible line of further development of this work is to be desired. From what has been said earlier, it may be concluded, that the observed migration of the activity belts should be at least partially connected with the circulations postulated by the theoretical considerations presented above.

In discussing the theory of turbulence in stellar atmosphere last year in Dublin (cf. *I.A.U. Transactions*, vol. 9), I emphasized the conclusion, that the various aspects of solar activity are probably all based on the turbulence of the hydrogen convection zone. This is also in harmony with what has been discussed today; but the essential role of the law of rotation in determining, e.g., the pattern of the spot activity has perhaps become somewhat clearer by the present discussion.

In closing, I would like to emphasize that this discussion was mainly based on one theoretical argument which is believed to be of some bearing, but it is obvious that in a complex problem like this some other aspects, to which earlier writers have given closer attention, have to be reconsidered also (cf. Temesváry [27]) with further references).

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Discussion

Alfvén: The main difference between my opinion and Professor Biermann's is that the source of energy of the sunspots, according to my point of view, is the centre of the sun and the energy goes out as waves to the solar surface where they create solar activity, sunspots and perhaps also prominences at higher latitudes. There could, of course, be given many arguments against this view and at least the same number in favour of it; I think that in this, as in many other cases, the final decision could only come from observations. This theory results in a correlation between the sunspots of the northern hemisphere and the sunspots of the southern hemisphere. The correlation has been checked independently by Galvenius and Wold by statistical methods and by Whittle and they have found a correlation which supports the theory. I have myself made another correlation analysis which also seems to support it. It would be very interesting to discuss the objections against it because, again, the decision of problems like this cannot be given by theoretical arguments alone because we know too little about the interior of the sun.

Tuominen: The convective zone is thin as compared to the solar radius. Is it therefore possible that the velocity of the meridional circulation in it does not change its sign between the equator and the pole? If the circulation takes place in different directions at different latitudes, it will be difficult to see the connexion between the motion of the sunspot belts and the meridional circulation in the convective zone.

Biermann: I am not aware of any necessary contradiction in this case, since the sunspot belts are confined to latitudes $<40°$; the direction of the meridional circulation might be different at higher latitudes, as suggested, e.g. by the displacement of the zones of prominences, and of the coronal isophotes, with phase in the cycle of solar activity. In the inner parts of the convective zone in the spot latitudes the velocity v_m will depend on the detailed structure of the velocity field, which we do not know. I am doubtful whether the simplified picture which I have discussed today contains as yet the whole truth, and I indicated earlier that certainly wave-type phenomena also have to be thought of. But the structure of the circulation itself might be rather complex, on account of the complex character of the physical effects which should be relevant.

Tuominen: In the terrestrial atmosphere the circulation is in different directions at different latitudes and it may be so also in the sun.

Biermann: Yes, that is what I would expect. But I think that the system of circulation might be more complex than the corresponding system in the earth's atmosphere.

Tuominen: But then it should have nothing to do with the motion of sunspots in a belt?

Biermann: O, yes, it should. The motion of the sunspot zone and that of an individual spot are not necessarily identical. I would like to emphasize that in my picture the meridional circulation is an essential element of the migration of the spot belts in the cycle of solar activity, but that contributary causes might be important for the motion of individual spots.

Schatzman: With reference to the existence of the inner convective zones, referred to by Professor Alfvén, the important factor is the effective power of temperature. This power is usually supposed to be 4, but if one takes into account the effect of electron screening in thermonuclear energy generation rate one finds that the effective power is slightly smaller, may be 3, which reduces considerably the likelihood of the existence of a central convective zone.

Tayler: The existence or non-existence of a small convective core has very small effect on the observed solar properties and does not seem to effect the mechanism proposed by Professor Biermann. It would, however, be critical to the Alfvén theory.

Biermann: I mentioned the central convective zone exclusively on account of its (postulated) relation to some sunspot theories; I perfectly agree that its existence or otherwise does not essentially affect the internal structure in its overall features.