Math. Struct. in Comp. Science (2017), *vol.* 27, *pp.* 1285–1286.
doi:10.1017/S0960129516000086 First published online 9 June 2016

Preface to the special issue: Continuity, computability, constructivity: from logic to algorithms 2013

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Received 18 February 2016; revised 22 February 2016

This issue of *Mathematical Structures in Computer Science* is composed mainly of papers submitted by participants of the Workshop 'Continuity, Computability, Constructivity: From Logic to Algorithms,' held in Gregynog, a conference centre of the University of Wales located in the beautiful nature of Mid Wales, in the last week of June 2013. In addition, several colleagues accepted our invitation to contribute to this volume.

The workshop was partially funded by the British Logic Colloquium, the College of Science of Swansea University, the European Union and the London Mathematical Society. It was the third in a series of workshops aiming to bring together researchers from real analysis, computability theory, constructive mathematics and related fields. The overall objective was to apply logical methods in these disciplines to provide a sound foundation for obtaining exact and correct algorithms. At the same time, the conference was the second annual meeting of the COMPUTAL project, which is a research network between Europe, Russia, South Africa and Japan funded by the European Union under the FP7-IRSES programme scheme.

The prevailing approach to computability with infinite data is Weihrauch's Theory of Type-Two Effectivity building upon earlier work by Grzegorczyk in the late 1950s. It is essentially based on the insight that data like the real numbers can be represented by infinite strings of finite objects. Strings of this kind are then processed by Turing machines that use these infinite strings as oracles.

In case such a string can be effectively generated, it can be finitely described by the generating programme, and instead of processing infinite strings one can transform programmes. This is Markov's approach and the basis of Russian Constructivism. The relation between both approaches is still an object of intensive studies.

In applications in which one is particularly interested in complexity considerations, other models are used as well. The BSS model, e.g., is obtained from the classical random access machine model by assuming that registers can not only store bits but real numbers, or elements of an algebraic structure as well.

Many important problems are not only hard to compute, but are even not computable at all. To measure their difficulty, one can classify them according to their descriptive complexity, leading to hierarchies such as the Borel hierarchy or the arithmetical hierarchy. Another approach is to relate their difficulty to the difficulty of certain well-known master problems using reducibility notions such as Turing reducibility of Weihrauch reducibility.

Investigations of this type are in the centre of Descriptive Set Theory. Traditionally, results in this theory were developed for Polish spaces. With the development of Theoretical Computer Science, spaces became important which are no longer Hausdorff. Prominent examples are domains in the area of Program Semantics, Noetherian spaces in Verification and qcb_0 spaces in Computable Analysis. Recent years have seen large efforts to extend the classical theory to such kind of spaces. This is, however, not an easy task as many of these spaces are no longer second countable.

Just as in computing with finite objects, the formal verification of written programmes is a tedious task. A well-known feature of constructive logic is that from the proof of a formula $(\forall x)(\exists y)A(x, y)$, i.e., a specification, one can extract a term (programme) t such that $(\forall x)A(x, t(x))$ is provable. Realizability interpretations can be used here. In order to apply this approach to computing with infinite objects, co-inductive characterizations of classical objects have turned out very useful.

The present special issue contains contributions to all these areas.

As usual, there are many people to be thanked. This is in particular true for the organizing committee of the Gregynog meeting. They did a wonderful job. Moreover, we want to thank the referees for having taken the burden of carefully reading and commenting the submissions. Last, but not least, we are very grateful to the Editor-in-Chief of *Mathematical Structures in Computer Science* for the opportunity to publish in this special issue of the journal.

The papers have undergone a rigorous reviewing process in accordance with the standards set by *Mathematical Structures in Computer Science*. Submissions by special issue editors were handled by others so as to protect the anonymity of the reviewers and to avoid conflicts of interest.

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