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Jovian encounter manifolds

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Abstract. Recent numerical studies have shown that the entire solar system is permeated with arch-like structures originating from all planets. Particles placed on such arches experience planetary close encounters after only one or few orbital revolutions. In this work, we are interested how thece arches, which we associate to encounter manifolds of Jupiter, appear in three dimensions for higher inclinations.

Our results show that about 0.5% of the observed domain $[a, e, i] = [2\ AU, 11.5\ AU] \times [0, 0.7] \times [0^\circ, 90^\circ]$ is covered by the manifolds. For inclinations up to $\sim 5^\circ$, the arch-like structures are almost unchanged compared to those initially observed in the orbital plane of Jupiter. At higher inclinations, the number of encounter orbits rapidly decreases to narrow domains where the manifolds stretch up to inclinations of 90° (and above) in a very steep manner.

Keywords. manifolds, Jupiter, encounters, chaos

1. Introduction

Chaos in the solar system has been investigated for more than a century from a variety of perspectives (Poincaré 1892; Sussman and Wisdom 1992; Laskar 1994; Batygin et al. 2015). It is generally accepted that chaos is present almost everywhere, but contrary to popular belief it is not completely disordered. Recent numerical studies have shown that the entire solar system is permeated with arch-like structures originating from all planets (Todorović et al. 2020). Particles placed on such arches experience planetary encounters after only one or few orbital revolutions, which is why they have been identified with the traces of stable invariant manifolds related to Lagrangian points L1 and L2 in the Sun-planet system (Koon *et al.* 2000; Gómez *et al.* 2004).

Invariant manifolds are crucial in our understanding of chaos, but also in the construction of natural space transfers (see e.g. Scantamburlo et al. (2022); Davis *et al.* (2010); Vaquero and Howell (2014); Conley (1968); Fitzgerald and Ross (2022); Daquin *et al.* (2016); Topputo et al. (2005)). However, their calculation is not trivial and is possible only in a narrow domain of three body energies in simplified dynamical models. The arches are numerically observed in the realm of high energies, they still do not have a proper mathematical interpretation and their general structure in phase space remains unknown.

The idea of this work is to investigate how the Jovian arc-like structures, initially obtained in the orbital plane of Jupiter (in the semi-major axis eccentricity (a, e) plane), appear in three dimensions at higher inclinations. In our discretized numerical treatment, we leave out the mathematical definitions and make a simple assumption that *encounter manifolds* (or just manifolds further in the text) are a collection of initial conditions leading to planetary close approaches in rapid times. Following this presumption we start our examination.

2. Methodology

The experiment was carried out following the analogy of dynamical mapping: we take a dense grid of equidistant initial conditions (ICs) in a given $a \times e \times i$ domain, track each particle for a fixed amount of time (for 100 years), and record their eventual entries into the close neighborhood of Jupiter. More precisely, ICs are selected inside a box defined with $a \times e \times i = [2AU, 11.5AU] \times [0, 0.7] \times [0^\circ, 90^\circ]$, with resolution of 0.001 AU in semi-major axis, 0.0014 in eccentricity and 1° in inclination, which makes a total number of 432 250 000 orbits.

The initial epoch was set to 7th January 2022 (JD 2459586.5) and the remaining orbital elements, the argument of pericenter ω, and the longitude of the node Ω were equal to the corresponding elements of Jupiter for the same epoch. The mean anomaly M was shifted for 60 degrees in front of the planet, because we construct the result around Jupiter's Lagrangian point L4, but also for the consistency with Todorović et al. (2020).

We have developed a Python program using the REBOUND N-body code (Rein and Liu 2012), where all parameter settings relevant to the computations are simplified and automatized (more details will be provided in future publications). The orbits were integrated using the WHfast scheme (Wisdom & Holman 1991; Rein and Tamayo 2015), with Jupiter as the only planet in the model. Besides observing close encounters, we also calculated FLIs Froeschlé et al. (1997); Froeschlé and Lega (2000); Froeschlé et al. (2000); Todorović (2017); Daquin *et al.* (2016); Guzzo and Lega (2015) and MEGNOs (Cincotta and Simó 2000; Hinse *et al.* 2010; Rein and Tamayo 2016; Kováčová et al. 2022) for each IC, but in this work, we limit our attention only to Jovian approaches smaller than 0.1 AU.

The computation lasted for about two months on the ARIS supercomputer† located in Athens (Greece).

3. Results and discussion

According to our results, about 0.5 per cent of the observed space is occupied by the encounter manifolds i.e. 2080251 (out of 432250000) particles approach Jupiter to a distance smaller than 0.1 AU in 100 years. Let us remark that this result refers only to the above-described selection of parameters since the number of encounter orbits constantly grows with the passage of time and changes for different approaching distances from the planet.

These 2 million particles are placed along certain surfaces (which we identified with encounter manifolds) presented in Figs. 1 and 2. For better visibility, we show the same data from two perspectives (on the two figures) and colour them based on inclination. ICs on lower inclinations are grey, on higher inclinations pink, and the ones in between are white. The given interval in $a \in [3.5, 11.5]$ AU is somewhat smaller than the observed domain, since the number of approaching orbits between 2 AU and 3.5 AU was negligible. Manifolds are not observed in the main belt.

Although we are in unknown territory, a certain geography is discernible. The arch-like structures initially observed in the orbital plane of Jupiter remain relatively unchanged up to inclinations of about $i \sim 5^\circ$. For $i > 5^\circ$ the number of ICs leading to Jovain approaches sharply decreases, but the manifolds survive up to $i = 90^\circ$ (although in limited parts). At the top of the interval, the structures seem cut off, which suggests that they extend on even higher inclinations. The authors hypothesize that they could wrap around the Sun similar to how they wrap around the planet (see e.g. Lega and Guzzo (2016); Paez and Guzzo (2020)).

† https://doc.aris.grnet.gr/system/hardware/

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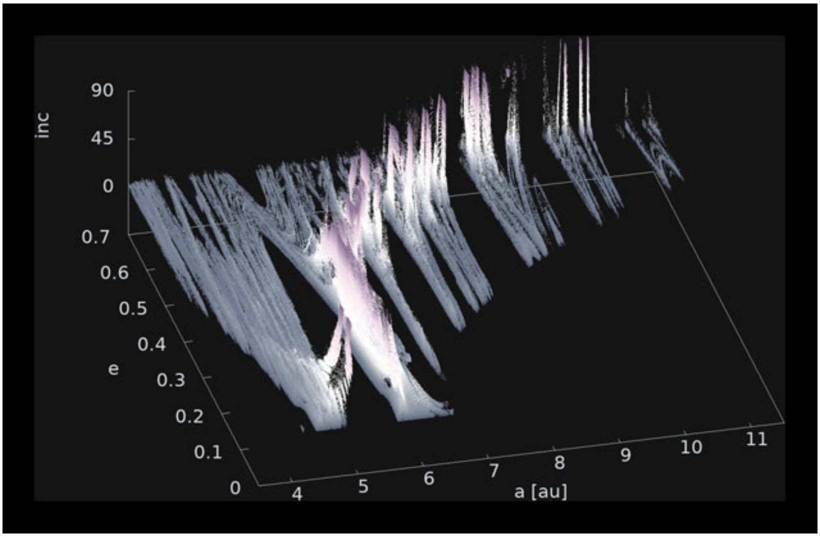

Figure 1. Encounter manifolds composed of initial conditions reaching Jupiter distance smaller than 0.1 AU in 100 years, in the region $a \times e \times i = [3.5, 11.5]$ $AU \times [0, 0.7] \times [0, 90]$ deg. The lowest inclination are coloured grey, the highest are pink, and those in between are white. The arch-like structures initially observed in the orbital plane of Jupiter, remain almost unchanged for inclination below 5 degrees. Manifolds reaching higher inclinations are limited to small areas around the Trojan island (the gap at $a \in 5.2$ AU) and along a narrow direction spreading diagonally through the whole domain. They extend up to 90 degrees (and above) in a steep manner almost vertically.

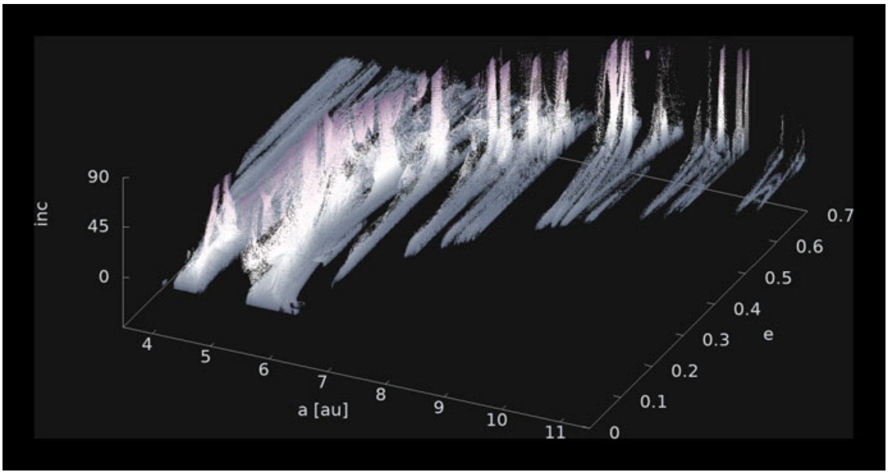

Figure 2. The same data set as in Fig.1 from another perspective. Manifolds occupy about 5 % of the observed space.

The limited parts from which manifolds stretch up to 90° inclinations are located around the stable Trojan island around ~ 5.2 AU (a closer look is given in Fig. 3), and along an unidentified diagonal form which extends in the entire (a, e) domain (more studies will be provided in future work).

In Fig.3 covering the interval between $a \in [4.5, 6.1]$, the layered complex manifold surfaces are better visible. As expected, encounter orbits are absent in the rounded gap of the Trojan island. But it is somewhat paradoxical that this region where

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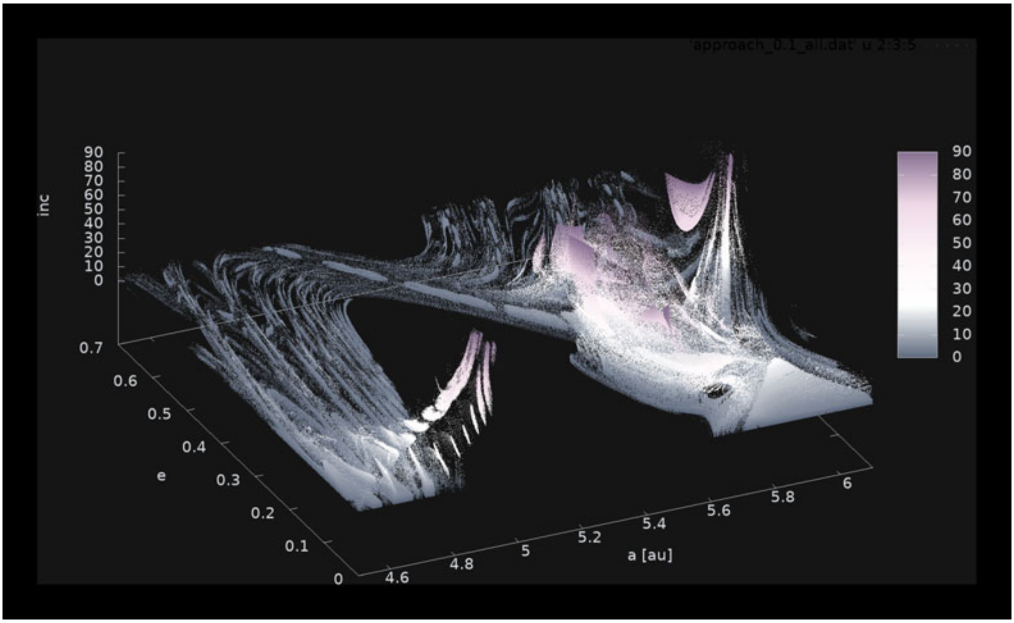

Figure 3. A closer look at the encounter manifolds for $a \in [4.5, 6.1]$ AU around the Jupiter Trojan island corresponding to the L4 point. ICs at low inclinations (coloured in grey) are vertically oriented and form the same structures as observed in Todorović et al. (2020). The white and pink particles lie on multi-layered complex surfaces. A somewhat larger representation of manifolds is notable on the right side of the island. Further research is needed to explain these results.

the stabilities are studied for times comparable to the lifetime of the solar system (Holt *et al.* 2020; Levison et al. 1997; Giorgilli and Skokos 1997; Di Sisto et al. 2014; Efthymiopoulos and Sándor 2005), is surrounded by highly unstable manifolds, whose orbits can hardly survive one century.

4. Conclusion

We conducted massive numerical investigations and detected Jovian encounter manifolds in three dimensions (a, e, i). Although the result has provided new insights into their global structure, such representation is still partial. Encounter manifolds are observed for only one planet, one set of remaining orbital elements, and one epoch. How the ensemble of all the manifolds would look and what is their theoretical explanation, is certainly a challenge for further investigations. It remains an open question whether we will ever see them in their entirety.

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