VARIABILITY AND INSTABILITIES OF ACCRETION DISKS

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ABSTRACT. In the Seyfert galaxy NGC 1566 and quasars 3C 345, 3C 446 long term ($\sim 10^3$ days) variations have been observed. We propose to explain them as a limit cycle behaviour, similar to that well studied in the case of dwarf novae outbursts. The key element of our analysis is the relationship between the accretion rate $\dot{M}$ and the surface density Σ.

1. OBSERVATIONAL BASIS AND FUNDAMENTAL THEORETICAL IDEAS

This lecture gives a review of only some of the properties of accretion disks which are relevant to quasars, Seyfert galaxies and other active galactic nuclei. Most of the facts presented here belong to the standard theory of accretion (see e.g. an excellent review of Pringle 1981). The new theoretical discovery is the possibility of a limit cycle behaviour of the accretion flow.

In several active galactic nuclei quasi periodic bursts have been observed. They have a steep rise time of the order of 20 days and a period of about 1500 days. Alloin, Pelat, Phillips, Fosbury and Freeman (1985) have published data on the Seyfert galaxy NGC 1566. The data is also presented at this Symposium (Alloin, poster session, published in this book).

A schematic light curve of NGC 1655 is shown in Fig. 1. The observed variability of the broad emission lines H_α and H_β is interpreted as a variability of the accretion rate in the accretion disk orbiting the black hole in the centre of NGC 1655. The shaded areas are equal. This defines both the characteristic accretion rate $\dot{M}_0=3\times10^{-1}\dot{M}_E$ and the two timescales of 900 and 400 days. Two regimes occur along the rising branch: first

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a mild increase on a timescale of 310 days followed by a steep rise on a timescale less than 20 days. The decreasing part of the bursts corresponds to an exponential decay between 390 and 475 days. The three successive sub-bursts observed in the last cycle are separated by 310 days.

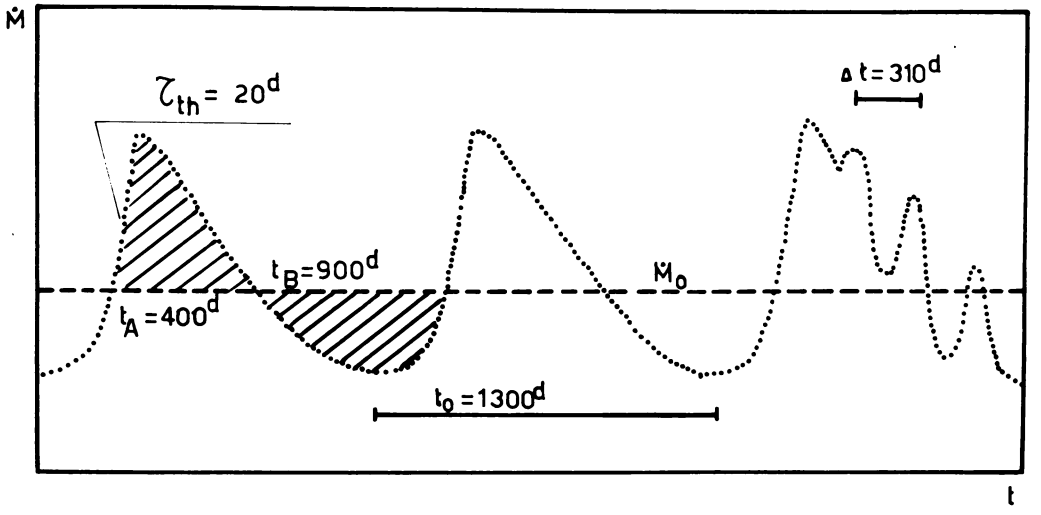

FIGURE 1. The light curve of NGC 1655 (schematic).

Abramowicz, Alloin, Lasota and Pelat (1985) have suggested an explanation for this variability: a limit cycle which works due to thermal and viscous instabilities present in the innermost region of an accretion disk orbiting a black hole. This explanation uses the following properties of accretion disks models:

a) There is a relation between the shape of the $\dot{M}(\Sigma)$ curve, describing the dependence of the accretion rate M on surface density Σ, and the stability of the disk: a negative slope corresponds to unstable disks. An S-shaped $\dot{M}(\Sigma)$ curve indicates the possibility of a cyclic outburst behaviour in accretion disks (Bath and Pringle 1981).

b) The equilibrium states of the innermost regions of an accretion disk around a black hole can be described by an S-shaped $\dot{M}(\Sigma)$ curve: for small accretion rates the disks are stable (positive slope), while for higher accretion rates well-known thermal and viscous instabilities develop (negative slope), and for still higher accretion rates they are stabilized by efficient cooling by advection (positive slope).

The rest of the paper discusses these two points. Because definitions and equations relevant to the thin accretion disk theory are basic for this discussion, we collect them in Tables 1 and 2.

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TABLE 1.: The structure of thin accretion disks

M=black hole mass	Q^+=viscous heat production rate
$\dot{M}$=accretion rate	Q^-=radiation loss rate
R= distance from the hole	q=horizontal heat flux
α=viscosity coefficient	γ=heat balance coefficient
P=pressure	R_{in}=inner edge of the disk
ρ=density	H=half thickness
T=temperature	f=radial function
V_S=sound velocity	v=accretion velocity
Σ=surface density	$v_K=(GM/R)^{1/2}$=Keplerian velocity
ν=kinematic viscosity	$\ell_K=(GMR)^{1/2}$=Keplerian momentum
κ=opacity coefficient	ℓ=specific angular momentum

(Gravitational radius) = $R_G=3\times10^{11}(M/10^6 M_\odot)$ [cm]

(Eddington accretion rate) = $\dot{M}_E=1\times10^{23}(M/10^6 M_\odot)$ [g/sec]

R_G and $\dot{M}_E$ give convenient scaling factors for the distance and accretion rate. The heat fluxes Q^+,Q^- and q are defined per unit surface area.

TABLE 2.: The structure equations of thin accretion disks

(1)	$\Sigma=2H\rho$	definition of surface density
(2)	$H/R=V_S/V_K$	vertical hydrostatic equilibrium
(3)	$V_S^2=P/\rho$	sound velocity
(4)	$\dot{M}=2\pi R\Sigma V$	mass conservation
(5)	$Q^+=f(3GM\dot{M})/(4\pi R^3)$	viscous heat generation rate
(6)	$Q^-=(4acT^4)/(3\rho H\kappa)$	radiation heat loss
(7)	$Q^+=Q^-+q$	heat balance
(8)	$q=(1-\gamma)Q^+$	horizontal heat flux
(9)	$3\pi\nu\Sigma=\dot{M}f$	angular momentum balance
(10)	$\nu=\alpha V_S H$	kinematic viscosity
(11)	$P=(\mathcal{R}/\bar{u})\rho T+(a/3)T^4$	equation of state
(12)	$f=[\ell(R)-\ell_K(R_{in})]/\ell(R)$	radial function
(13)	$\ell_K^2=GMR$	specific Keplerian angular momentum
(14)	$V_K^2=GM/R$	Keplerian velocity
(15)	$R_{in}=3R_G$	inner edge
(16)	$\ell=\ell_K$	angular momentum distribution
(17)	$\kappa=\kappa(\rho,T)$	opacity coefficient
(18)	$\alpha=\alpha(\rho,T)$	viscosity coefficient
(19)	$\gamma=\gamma(\rho,T)$	heat balance coefficient

Additional definition: $\beta \equiv P_{gas}/P = (\mathcal{R}/\bar{u})\rho T/P$
The physical constants a,c,G,$\bar{u}$,$\mathcal{R}$ have the standard meanings (e.g. Tassoul 1978, Appendix A).

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2. THE $\dot{M}(\Sigma)$ CURVE

Consider a sequence of accretion disks for which the mass of the central black hole M, the equation of state $P=P(\rho,T)$, the opacity law $\kappa=\kappa(\rho,T)$, and the viscosity law $\nu=\nu(\rho,T)$ are fixed. The distance from the centre, R is also fixed. We put $R=5R_G$ as most of the radiation produced by the disk originates there.

In Fig. 2 we present (after Bath and Pringle, 1981) an intuitive proof that

$$\frac{d\ln\dot{M}}{d\ln\Sigma} > 0 \Rightarrow \text{stability}, \quad \frac{d\ln\dot{M}}{d\ln\Sigma} < 0 \Rightarrow \text{instability} \tag{2.1}$$

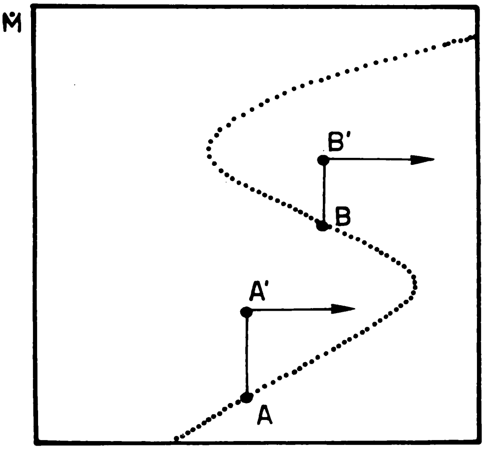

FIGURE 2. Stability of the accretion disks sequence (Bath and Pringle criterion).

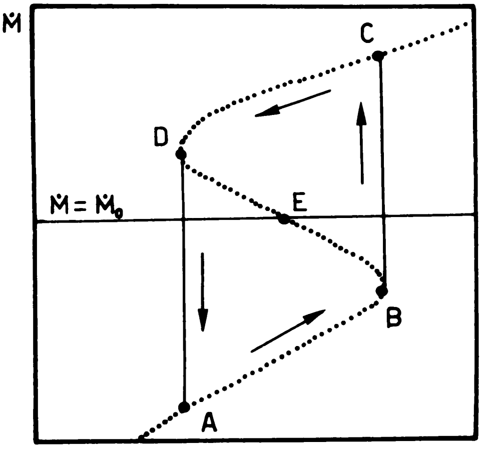

FIGURE 3. The limit cycle behaviour connected with an S-shaped sequence.

The point A' lies off the equilibrium sequence and can be considered as the perturbed state of the model A. In the perturbed state A' the accretion rate is higher than in the equilibrium state A. Thus, the model A' is over-supplied with matter and its surface density will increase. This brings the model A' back to the equilibrium curve - the model A is stable. A similar argument shows that model B is unstable.

Linear stability of accretion disks has been formally studied only in the case of thin disks ($\dot{M} << \dot{M}_E$). Two types of instabilities are present: thermal instabilities (Pringle et al. 1973, Shakura and Sunyaev 1976) and viscous instabilities (Lightman and Eardley 1974, Shakura and Sunyaev 1976).

Using the phenomenological coefficients defined by Piran (1978):

$$\mathcal{K} \equiv \left(\frac{\partial\ln Q^-}{\partial\ln H}\right)_\Sigma, \ \mathcal{L} \equiv \left(\frac{\partial\ln Q^-}{\partial\ln\Sigma}\right)_H, \ \mathfrak{M} \equiv \left(\frac{\partial\ln Q^+}{\partial\ln H}\right)_\Sigma, \ \mathfrak{N} \equiv \left(\frac{\partial\ln Q^+}{\partial\ln\Sigma}\right)_{\dot{H}} \tag{2.2}$$

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and slightly modifying his original results, one can write the criteria for thermal and viscous instability as

$$\text{thermal:}\quad \left(\frac{\partial \ln Q^-/Q^+}{\partial \ln H}\right)_H = \mathcal{K} - \mathfrak{M} > 0\ , \tag{2.3a}$$

$$\text{viscous:}\quad \left(\frac{\partial \ln \Sigma\nu}{\partial \ln \Sigma}\right)_{Q^+=Q^-} = \frac{\mathcal{K}\mathfrak{N} - \mathfrak{M}\mathcal{L}}{\mathcal{K} - \mathfrak{M}} > 0\ . \tag{2.3b}$$

Criteria (2.3a) and (2.3b) are together a sufficient and necessary condition for stability. We have computed that

$$\left(\frac{d\ln \dot{M}}{d\ln \Sigma}\right) = \frac{\mathcal{K}\mathfrak{N} - \mathfrak{M}\mathcal{L}}{\mathcal{K} - \mathfrak{M}}\ . \tag{2.4}$$

This proves that (2.1) is sufficient and necessary for the viscouse stability, so $(d\ln\dot{M}/d\ln\Sigma) < 0$ means that the disk is unstable with respect at least to viscous perturbations.

In the case of the standard accretion disk model:

$$\mathcal{K} = 4\,\frac{1+\beta}{4-3\beta}\ ,\ \mathcal{L} = -\,\frac{\beta}{4-3\beta}\ ,\ \mathfrak{M} = 2,\ \mathfrak{N} = 1 \tag{2.5}$$

and the criterion (2.3) gives $\beta > 2/5$ for stability: radiation pressure supported think disks are unstable (Shakura and Sunyaev, 1976).

Bath and Pringle (1981) also noticed that an S-shaped $\dot{M}(\Sigma)$ curve indicates a possibility of cyclic variation in the accretion flow. Fig. 3 provides an elementary explanation of this. Let the basic accretion rate in the system be $\dot{M} = \dot{M}_0$. This corresponds to an unstable equilibrium model E. It cannot be realised by the flow. Consider point A. The equilibrium accretion rate $\dot{M}(A)$ is smaller than the actual accretion rate $\dot{M}_0$. The model A is oversupplied and will evolve, along the path AB, in the direction of increasing surface density, as indicated by the arrow. From the point B further evolution is only possible after a jump to C. Here, however, the equilibrium accretion rate is higher than the actual one. The surface density now decreases down to the point D, where another jump, to A, must take place. This closes the cycle.

We proceed further by noticing that the turning points of the $\dot{M}(\Sigma)$ curve are given by $\mathcal{K} - \mathfrak{M} = 0$.

If a particular $\dot{M}(\Sigma)$ has N turning points, then the equation $\mathcal{K} - \mathfrak{M} = 0$ has N different solutions, which means that $\mathcal{K} - \mathfrak{M}$ is an N-th degree polynomial in $\dot{M}$. Because both $\mathcal{K}$ and $\mathfrak{M}$ are defined through logarithmic derivatives of physics quantities, this means that there exists a physical quantity X with a very strong dependence on the accretion rate, $X \sim \exp(k\dot{M}^N)$, where k=const.

Suppose, that a particular $\dot{M}(\Sigma)$ curve has one stable and one unstable branch, as shown in Fig. 4. This means that one of the physical quantities varies according to $X \sim \exp(k\dot{M})$. To change this into $X \sim \exp(k\dot{M}^2)$, which is needed for the existence of an S-shaped $\dot{M}(\Sigma)$ curve, one can assume that a free parameter, e.g. the viscosity coefficient, varies like an exponential function of $\dot{M}$. (Fig. 4).

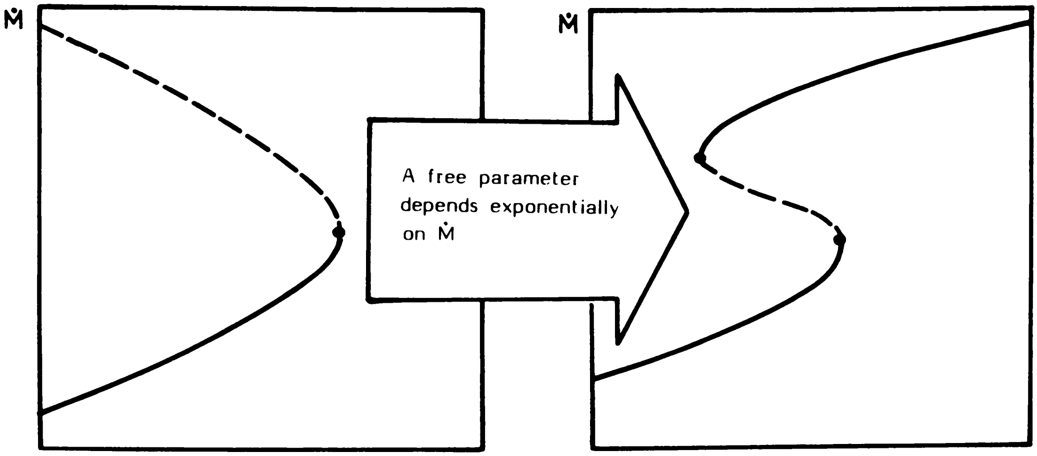


FIGURE 4. How to get an S-shaped $\dot{M}(\Sigma)$ curve.

3. THE INNERMOST REGION OF THE DISK

Abramowicz and Lasota (in preparation) noticed that for accretion rates $\dot{M} \leq 0.02\,\dot{M}_E$ the accretion is stable. This is reflected by the positive slope of the $\dot{M}(\Sigma)$ curve, which is well approximated by Shakura-Sunyaev (1973), formula valid for $\beta >> 2/5$:

$$\frac{\dot{M}}{\dot{M}_E} = 2.5\mathrm{x}10^{-4} \left(\frac{M}{10^6 M_\odot}\right)^{\frac{5}{3}} \alpha^{\frac{4}{3}} \left(\frac{R}{R_G}\right) f^{-1} \Sigma^{\frac{5}{3}} . \qquad (3.1)$$

At $\dot{M} = 0.02\dot{M}_E$ the $\dot{M}(\Sigma)$ curve bends and its slope becomes negative. This corresponds to the thermal and viscous instabilities found by Pringle, Rees and Pacholczyk (1974), Pringle (1976), Lightman and Eardley (1974), and Lightman (1974). The instabilities result because of the insufficient cooling. The analytic approximation of Shakura and Sunyaev, valid for $\beta << 2/5$, reads:

$$\frac{\dot{M}}{\dot{M}_E} = 1.3\times10^1 \alpha^{-1} (\frac{R}{R_G})^{\frac{3}{2}} f^{-1} \Sigma^{-1} . \tag{3.2}$$

At still higher accretion rates the $\dot{M}(\Sigma)$ curve bends again and accretion disks become stable. The stability is restored because of a very efficient cooling by advection (Abramowicz, 1981). This results from the fact that the accretion flow onto a black hole is transonic: its sonic point lies close to the inner edge (cusp) of the disk. The heat is thus removed by advection at a speed close to the speed of sound. This cooling mechanism works for both small ($\dot{M}/\dot{M}_E \ll 1$) and large ($\dot{M}/\dot{M}_E > 1$) accretion rates, but in the former case it is relevant only in the immediate vicinity of the inner radius. Cooling by advection can be efficient only if its characteristic time scale is not longer than the thermal timescale,

$$t_{th} = 1.6 \times 10^{-3}/\alpha (R/R_G)^{3/2} (M/10^6 M_\odot) \quad [\text{days}]$$

In a thin, gas pressure dominated disk the characteristic time for radial motion is the viscous time, $t_{vis}=t_{th}/(H/R)^2$, which is much longer than the thermal one (as $H/R \ll 1$). At higher accretion rates, close to $\dot{M}_E$, this is no longer true. The angular momentum distribution changes from being almost Keplerian to being almost constant (see e.g. Muchotrzeb and Paczyński, 1982). To move a matter element from place to place in a fluid with constant angular momentum no viscous torque is necessary. In addition black holes accrete matter with non-zero angular momentum. This is a relativistic effect, since in a Newtonian potential there is an infinite potential barrier for non-zero angular momentum. As a result when accretion rate is high, $\dot{M} > \dot{M}_E$ advection is not related to viscous processes and may proceed on a much faster timescale than t_{vis}. For such high accretion rates, however, the disk cannot be considered to be thin and the standard approximations are no longer valid.

Below we give a purely phenomenological description of the advective cooling.

The total amount of heat dQ carried in the horizontal direction by the advection form a volume $d\Upsilon = HdS = H2\pi RdR$ in a unit time is $dQ=(dV_S^2/dR)\rho V d\Upsilon$. Therefore the heat flux per unit surface, $q=dQ/dS$, equals:

$$q_{ad} = \frac{dV_S^2}{dR} H V \approx (\frac{H}{R}) V_S^2 V\rho . \tag{3.3}$$

The sign of the gradient dV_S^2/dR is fixed in (3.3) in such a way that q_{ad} describes cooling. Using the equations in Table 2 this may be expressed as:

$$q_{ad} = \frac{2}{3} f^{-1} (\frac{H}{R})^2 Q^+ \tag{3.4}$$

The horizontal heat flux carried by radiation (c.f. Maraschi et al. 1976; Muchotrzeb and Paczyński, 1982) can be estimate to be:

$$q_{th} = \left(\frac{H}{R}\right) Q^{-}. \tag{3.5}$$

Note, that for the equilibrium structure the horizontal radiation flux is more important than the advection flux in the case of thin disks, $H/R \ll 1$. However, for stability the advection flux is more important. This is because of the stability condition $\mathcal{K} - \mathfrak{M} > 0$. (See Table 3).

Note that because of the factor f^{-1} in equation (3.4) advection flux always dominates close to $R=R_{in}$ as $f(R_{in})=0$. This factor becomes dominant also when $\ell \not\approx$ const, because $f=[\ell(R)-\ell(R_{in})]/\ell(R)$. In precisely this situation the advection flux becomes efficient, because it acts on a timescale much shorter than the viscous one.

TABLE 3.: Stability properties of radiation pressure supported disks. In all three cases $\mathfrak{M} = 2$.

Q^{-}	$\mathcal{K}$	Stability condition $\mathcal{K} - \mathfrak{M} > 0$
vertical radiation	1	violated
horizontal radiation	2	marginally fulfilled
advection	4	fulfilled

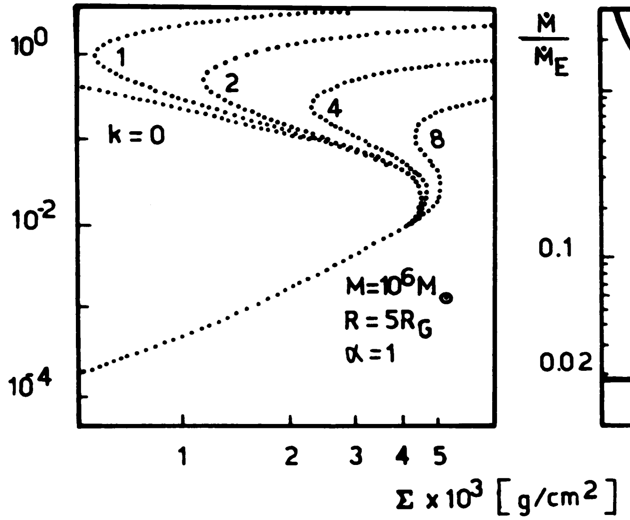
The key element is therefore the angular momentum distribution which determines the factor f. Computing $\ell=\ell(R)$ is not easy for $\dot{M} > \dot{M}_E$ (c.f. Muchotrzeb and Paczyński, 1982). The main difficulty is due to the transonic nature of the flow, which in turn implies the eigenvalue character of the mathematical model of it: the eigenvalue is the location of the inner edge, R_{in}. For small accretion rates $R_{in}=3R_G$ is a very good approximation for the eigenvalue, and this problem is not acute. The coefficient for the heat balance γ can, with the help of equations (8) and (3.4) be expressed as:

$$\gamma = \frac{2}{3} f^{-1} \left(\frac{H}{R}\right)^2 + 1. \tag{3.6}$$

Because there is no way today to compute f, the coefficient γ is unknown. However, as was explained before, for the S-shaped $\dot{M}(\Sigma)$ curve, it is sufficient

$$\gamma = \exp\left(-k \frac{\dot{M}}{\dot{M}_E}\right) , \tag{3.7}$$

with k=const. We have explained that this behaviour is very likely, because $\gamma \sim f^{-1}$ and the function f depends very strongly on $\dot{M}/M_E$, going to zero when $\dot{M}/\dot{M}_E >> 1$. Thus, we have assumed (3.7) and computed the resulting $\dot{M}(\Sigma)$ relations. They are shown in Fig. 5 and Fig. 6.



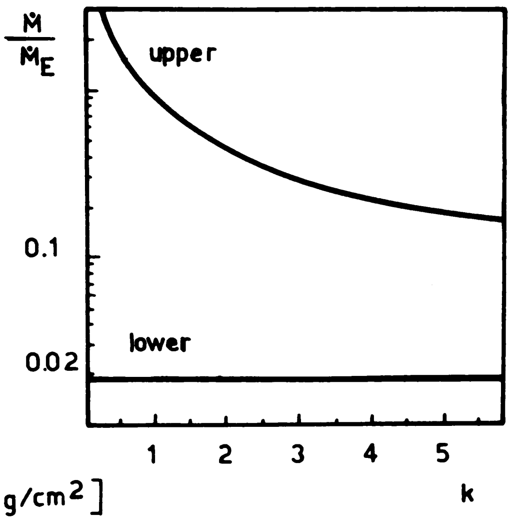


FIGURE 5. The sequences of equilibrium accretion disks models with advection heat flux (the strength of advection is measured by k).

FIGURE 6. The location of the lower turning point does not depend on the advection strength, but the location of the upper one does.

Results would be similar or identical if instead of (3.7) an exponential dependence on $\dot{M}$ of the α (viscosity) or κ (opacity) were assumed.

4. APPLICATION TO NGC 1566 AND CONCLUSIONS

Since the average accretion rate is $\dot{M}_0$=0.3$\dot{M}_E$, hence larger than at the lower turning point, $\dot{M}$=0.02$\dot{M}_E$, the inner regions of the accretion disk in NGC 1566 cannot settle to a steady state. The steep rising branch $\tau \gtrsim$ 20 days is the rising time for thermal instability (for this α must be greater than 10^{-2}). The three sub-bursts observed during the decay time may be connected with the fact that viscous instability breaks the disk into concentric rings. According to the interpretation of the variability observed in NGC 1566 as due to a limit cycle connected with the S-shaped $\dot{M}(\Sigma)$ curve one can estimate the accretion rate on the upper stable branch as

$$\dot{M}_{high} \approx \frac{t_A+t_B}{t_A} \dot{M}_0 \approx \dot{M}_E \quad , \tag{4.1}$$

by assuming that on the lower branch

$$\dot{M}_{low} << \dot{M}_E \quad . \tag{4.2}$$

This gives a good agreement with the theoretical S-curves in Fig. 5.

The hypothesis that a limit cycle instability operates at the centre of NGC 1566 requires more theoretical investigations. Of special importance are time dependent disk models for $\dot{M} > \dot{M}_E$. Observationally, statistical studies of long term variability would be relevant to the question whether this variability indeed occurs only in the range $10^{-2} < \dot{M}/\dot{M}_E \lesssim 1$ as our analysis predicts. More observational tests are described in the papers quoted in the introductory paragraph.

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DISCUSSION

Pranab Ghosh : 1) Comment : The fact that the turnover value of $\dot{M}$ for a Shakura-Sunyaev disc is practically independent of α and M is entirely as expected, since the physical variables in such a disc depend very weakly on α and fairly weakly on M.

(2) Question : All of us have been trying to understand the nature of the viscosity coefficient α for more than a decade now. Part of these efforts have come from the work of Bath and Pringle related to that which you just described, namely, their modelling of the time evolution of accretion discs and comparison with dwarf novae outbursts. Where, do you think, we stand today in our understanding ?

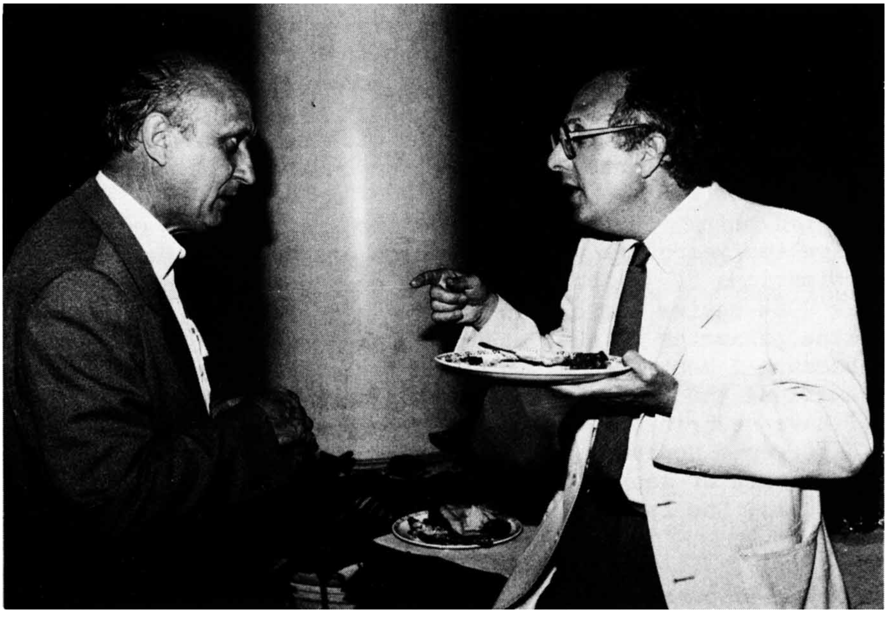
Abramowicz : I think, theoretical estimations of α are as bad today as they were ten years ago. However, there are very convincing observational estimations of α based on light curves of the dwarf novae. (e.g. Smak 1984 review article in PASP). In particular, it is now obvious that the parameter in the "low" and "high" state has different values. Because I mentioned again dwarf novae let me stress that although mathematical theory of limit cycle in dwarf novae case is similar to what I have described in my lecture, the physical nature of instability is different. However, dwarf nova mechanism may be relevant for outer, cool parts of accretion discs in AGN. Katy Clark in Oxford is working on this. She is getting periodicities of the order of a few years.

Burke : If the discs are stable, then they will tend to axisymmetry on the average. If the magnetic fields are generated by dynamo action, does not Cowling's theorem present difficulties ?

Abramowicz : Roger Blandford is perhaps the right person to answer that question.

Blandford : These objects are not exactly axially symmetric.

Sturrock : I wish to address the question of whether compact radio sources and jets are due to activity in the vicinity of the black hole or in the magnetosphere of the accretion disc. If the ejecta are produced near the black hole, it is hard to see why ejection is so frequently onesided. On the other hand, one might consider the possibility that activity is due to flare activity resulting from dynamo action in the accretion disc. If there is a probability P that this process will occur, at any time, on either surface of the disk, there is a probability P^2 of seeing two-sided ejection, 2P(1-P) of seeing one-sided ejection, and $(1-P)^2$ of seeing no ejection (and no compact central radio source). For quasars, $P \ll 1$ since most quasars are radio quiet, and this helps one understand why ejections from quasars are usually one-sided.



Leonid Matveyenko listens to Ken Kellermann

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