This department welcomes short notes and problems believed to be new. Contributors should include solutions where known, or background to the problem in case the problem is unsolved. Send all communications concerning this department to Leo Moser, University of Alberta, Edmonton, Alberta.

PROBLEMS FOR SOLUTION

P 11. (Conjecture) Given n points at the vertices of a strictly convex polygon. Is it true that no distance can be determined more than $5(n-1)/3$ times? (If $n \equiv 1 \pmod 3$) there is a configuration in which one distance is determined $5(n-1)/3$ times.)

P. Erdős and L. Moser

P 12. If $x_1, x_2, \ldots, x_p$ are p points in affine n-space, their convex hull P consists of all the points $\lambda_1 x_1 + \lambda_2 x_2 + \ldots + \lambda_p x_p$ where $\lambda_1 + \lambda_2 + \ldots + \lambda_p = 1$, $\lambda_i \geqslant 0$. The point y is an extremal point of P if $y \in P$ and if y does not lie on a segment connecting two other points of P. Let D be a convex region in n-space. The function $w = f(x_1, x_2, \ldots, x_p)$ associates continuously with any p-tuple of points of D another point w in their convex hull P which is not an extremal point of P. Choose $a_1, a_2, \ldots, a_p$ arbitrarily in D and define $a_{n+p+1} = f(a_{n+1}, a_{n+2}, \ldots, a_{n+p})$, $n = 0, 1, 2, \ldots$. Does the sequence $a_1, a_2, \ldots$ necessarily converge? Cf. Azpeitia, Proc. Amer. Math. Soc. 9(1958), 428-432.

P. Scherk

P 13. Angular measure in a Minkowski plane is sometimes defined to be proportional to the area of the corresponding sector of the unit circle U, and sometimes proportional to the arc length of the sector of U. Determine all Minkowski metrics in which the two measures are proportional.

H. Helfenstein

P 14. Prove that

$$\sum_{d=1}^{n} \left[\frac{n}{d}\right] \frac{1}{d} \prod_{p|d} (1 - p) = 1 + 1/2 + 1/3 + \ldots + 1/n$$

https://doi.org/10.1017/S0008439500024899 Published online by Cambridge University Press

where the product runs over all prime divisors of d and an empty product is taken to be 1.

L. Moser

P 15. (Conjecture) Every point on the perimeter of an ellipse is a vertex of an inscribed triangle of maximum area. There are other closed convex curves with this property, e.g. parallelograms. Is it true that the ellipse is the only closed strictly convex (no proper segments) plane curve with this property [cf. R.P. Bambah, Proc. Nat. Inst. Sci. India, Part A 23 (1957), 540-543]?

H. Helfenstein

SOLUTIONS

P 2. Put $S_k(n) = \Sigma_{a \leq n} a^k$, $T_k(n) = \Sigma a^k$, the second sum extending over all $a \leq n$ such that $(a,n) \neq 1$ and a does not divide n. Let $\mu(n)$ denote Moebius' function. Prove that

(i) $\quad T_k(n) = -\Sigma_{d|n, d>1} (1 + \mu(d) S_k(n/d))$,

(ii) n divides $S_1(n)$ (that is, n is multiply perfect) if and only if

$$T_1(n) \equiv 1 (\text{mod } n) \text{ if n is odd}, (1+n/2)(\text{mod } n) \text{ if n is even.}$$

J.C. Hayes and P. Scherk

Solution by the proposers. Let $\varphi_k(n) = \Sigma_{a \leq n, (a,n)=1} a^k$. Then $S_k(n) = \Sigma_{d|n} \Sigma_{a \leq n, (a,n)=d} a^k = \Sigma_{d|n} d^k \varphi_k(n/d)$ and by Moebius inversion $\varphi_k(n) = \Sigma_{d|n} \mu(d) d^k S_k(n/d)$. Now put $\sigma_k(n) = \Sigma_{d|n} d^k$ Then

(1) $\quad T_k(n) = S_k(n) - \varphi_k(n) - \sigma_k(n) + 1$

$$= S_k(n) - \Sigma_{d|n} \mu(d) d^k S_k(n/d) - \Sigma_{d|n} d^k + 1.$$

This yields the first assertion $T_k(n) = -\Sigma_{d|n, d>1} d^k (1 + \mu(d) S_k(n/d))$.

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