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INVARIANTS IN ABSTRACT MAPPING PAIRS

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Abstract

In a topological vector space, duality invariant is a very important property, some famous theorems, such as the Mackey-Arens theorem, the Mackey theorem, the Mazur theorem and the Orlicz-Pettis theorem, all show some duality invariants.

In this paper we would like to show an important improvement of the invariant results, which are related to sequential evaluation convergence of function series. Especially, a very general invariant result is established for an abstract mapping pair $(\Omega, B(\Omega, X))$ consisting of a nonempty set Ω and $B(\Omega, X) = \{f \in X^{\Omega} : f(\Omega) \text{ is bounded}\}$, where X is a locally convex space.

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1. Introduction

Let X be a locally convex space with the dual space X'. Various admissible polar topologies lie between the weak topology $\sigma(X, X')$ and the strong topology $\beta(X, X')$, for example, the Mackey topology $\tau(X, X')$. If a property P of X is shared by all admissible polar topologies lying between $\sigma(X, X')$ and $\tau(X, X')$, then P is called a *duality invariant*.

The Mackey-Arens theorem and the Mackey theorem show that the continuity of linear functionals on X and the boundedness of subsets of X are duality invariants. If A is a convex subset of X, then the closure of A is a duality invariant by the Mazur theorem, and the Orlicz-Pettis-McArthur theorem says that for $\{x_j\} \subset X$ the subseries convergence of $\sum x_j$ is also a duality invariant.

A few results have expanded the invariant ranges of boundedness and subseries convergence [9, 5, 2]. Moreover, in 1998, Li Ronglu [4] has found two invariants which are invariable over all admissible polar topologies lying between $\sigma(X, X')$ and $\beta(X, X')$ as follows (see also [12, 15, 14]).

THEOREM A. *Let $\lambda = c_0$ or l^p, $0 < p < +\infty$. Then for every $\{x_j\} \subset X$ the following conditions are equivalent*:

$$(\sigma_0) \qquad \forall (t_j) \in \lambda, \quad \sum_{j=1}^{\infty} t_j x_j \text{ is } \sigma(X, X')\text{-convergent.}$$

$$(\beta_0) \qquad \forall (t_j) \in \lambda, \quad \sum_{j=1}^{\infty} t_j x_j \text{ is } \beta(X, X')\text{-convergent.}$$

Recently, Theorem A was improved by the following generalization of linear functions see [7].

Let $MC(0) = \{\varphi \in \mathbb{C}^{\mathbb{C}} : \lim_{t\to 0} \varphi(t) = \varphi(0) = 0, \ \varphi(ts) = \varphi(t)\varphi(s)\}$. Then for a vector space X and $\varphi \in MC(0)$, let

$$QH_{\varphi}(X, \mathbb{C}) = \{f \in \mathbb{C}^X : f(tx) = \varphi(t) f(x), \ t \in \mathbb{C}, \ x \in X\}.$$

The identity function $\varphi_0(t) = t$ belongs to $MC(0)$ and $QH_{\varphi_0}(X, \mathbb{C})$ includes all linear functionals and many nonlinear functionals whenever $\dim X > 1$. Moreover, if $\varphi \in MC(0)$ but $0 \neq \varphi \neq \varphi_0$, then each nonzero $f \in QH_{\varphi}(X, \mathbb{C})$ is not linear. Then we have

THEOREM B ([7, Corollary 3]). *If $\varphi_i \in MC(0)$, $i = 0, 1, \dots, n$, $\lambda = c_0$ or l^p, $0 < p < +\infty$, and $X^{\#} \subset \bigcup_{i=0}^{n} QH_{\varphi_i}(X, \mathbb{C})$, then for every $\{x_j\} \subset X$ the following conditions are equivalent*:

$$(\sigma_1) \qquad \forall (t_j) \in \lambda, \quad \sum_{j=1}^{\infty} t_j x_j \text{ is } \sigma(X, X^{\#})\text{-convergent,}$$

that is, there is $x \in X$ such that $\sum_{j=1}^{\infty} x'(t_j x_j) = x'(x)$, for all $x' \in X^{\#}$.

$$(\beta_1) \qquad \forall (t_j) \in \lambda, \quad \sum_{j=1}^{\infty} t_j x_j \text{ is } \beta(X, X^{\#})\text{-convergent,}$$

that is, there is $x \in X$ such that if $A \subset X^{\#}$ and A is pointwise bounded on X, then $\lim_n \sum_{j=1}^{n} x'(t_j x_j) = x'(x)$ uniformly for $x' \in A$.

In this paper we prove an important improvement of the invariant results, which are related to sequential evaluation convergence of function series. Especially, a very general invariant result is established for an abstract mapping pair $(\Omega, B(\Omega, X))$ consisting of a nonempty set Ω and $B(\Omega, X) = \{f \in X^{\Omega} : f(\Omega) \text{ is bounded}\}$, where X is a locally convex space.

2. A function family and a sequence family

Let $C(0) = \{\varphi \in \mathbb{C}^{\mathbb{C}} : \lim_{t\to 0}\varphi(t) = \varphi(0) = 0\}$.

For $\varphi \in C(0)$ and vector spaces X and Y, a function $T : X \to Y$ is said to be *φ-radiative* if for every $x \in X$ and $t \in [0, 1]$ there is an $s \in [0, |\varphi(t)|]$ such that $T(tx) = sT(x)$. Let $R_\varphi(X, Y)$ be the family of φ-radiative functions.

If X is a topological vector space, then let $\mathcal{N}(X)$ be the family of neighborhoods of $0 \in X$ and, for $\varphi \in C(0)$ and $U \in \mathcal{N}(X)$, a function $T : X \to Y$ is *(φ, U)-radiative* if for every $x \in U$ and $t \in [0, 1]$ there is $s \in [0, |\varphi(t)|]$ for which $T(tx) = sT(x)$. Let $R_{\varphi,U}(X, Y)$ be the family of (φ, U)-radiative functions.

It is easy to see that $T(0) = 0$ for every $T \in R_{\varphi,U}(X, Y)$ and

$$R_\varphi(X, Y) \subset \bigcap_{U\in\mathcal{N}(X)} R_{\varphi,U}(X, Y), \qquad \bigcup_{\substack{|\varphi(\cdot)|\le|\psi(\cdot)| \\ U\subset V}} R_{\varphi,V}(X, Y) \subset R_{\psi,U}(X, Y).$$

EXAMPLE 2.1. (1) $\varphi_0(t) = t$, $t \in \mathbb{C}$. If $T : X \to Y$ is homogeneous, then $T \in R_{\varphi_0}(X, Y)$ so $R_{\varphi_0}(X, Y)$ includes all linear operators. Moreover, if $\varphi \in MC(0)$ and $\varphi(t) \ge 0$ for $t \ge 0$, then $QH_\varphi(X, Y) \subset R_\varphi(X, Y)$, for example, for an associative algebra X over $\mathbb{R}$ and $T(x) = \sqrt{2}x^3$, for every $x \in X$, $T \in QH_\varphi(X, X) \subset R_\varphi(X, X)$, where $\varphi(t) = t^3$.

(2) Let $\varphi(t) = \pi t/2$, $t \in \mathbb{C}$. If $0 < t < 1$ and $0 < |x| \le \pi/2$, then

$$\sin tx = \frac{\sin tx}{tx}\,\frac{x}{\sin x}\,t\sin x$$

and

$$0 < \frac{\sin tx}{tx}\,\frac{x}{\sin x}\,t \le \frac{x}{\sin x}\,t \le \frac{\pi}{2}t = \varphi(t)$$

so $\sin \in R_{\varphi,(-\pi/2,\pi/2)}(\mathbb{R}, \mathbb{R})$.

(3) Let $(X, \|\cdot\|)$ be a normed space and define $T : X \to \mathbb{R}$ by

$$T(x) = \begin{cases} e^{\sqrt{\|x\|}} - 1, & \|x\| \le 1, \\ \sqrt{\|x\|}, & \|x\| > 1. \end{cases}$$

For $0 < t < 1$ and $0 < \|x\| \le 1$, $\|tx\| < 1$ and

$$T(tx) = e^{\sqrt{t\|x\|}} - 1 = \frac{e^{\sqrt{t\|x\|}} - 1}{e^{\sqrt{\|x\|}} - 1}\left(e^{\sqrt{\|x\|}} - 1\right) = \frac{e^{\xi}\sqrt{t}\sqrt{\|x\|}}{e^{\eta}\sqrt{\|x\|}}T(x) = e^{\xi-\eta}\sqrt{t},$$

where $0 < \xi < \sqrt{t}\sqrt{\|x\|} < \sqrt{\|x\|}$, $0 < \eta < \sqrt{\|x\|}$ and $\xi < \eta$ so

$$0 < e^{\xi-\eta}\sqrt{t} < \sqrt{t} < e\sqrt{t}.$$

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Let $0 < t < 1$ and $\|x\| > 1$. If $t\|x\| \le 1$, then

$$T(tx) = e^{\sqrt{t\|x\|}} - 1 = \frac{e^{\sqrt{t\|x\|}} - 1}{\sqrt{\|x\|}} \sqrt{\|x\|} = \frac{e^{\xi}\sqrt{t}\sqrt{\|x\|}}{\sqrt{\|x\|}} T(x) = e^{\xi}\sqrt{t}\,T(x),$$

where $0 < \xi < \sqrt{t\|x\|} \le 1$ so $0 < e^{\xi}\sqrt{t} < e\sqrt{t}$. If $t\|x\| > 1$, then

$$T(tx) = \sqrt{t\|x\|} = \sqrt{t}\,T(x)$$

and $0 < \sqrt{t} < e\sqrt{t}$. Hence $T \in R_{\varphi}(X, \mathbb{R})$, where $\varphi(t) = e\sqrt{|t|}$.

(4) Let $c_{00} = \{(a_j) \in \mathbb{C}^{\mathbb{N}} : a_j = 0 \text{ eventually}\}$ and $0 < p < +\infty$. Define $T : l^p \to l^p$ by

$$T((a_j)_{j=1}^{\infty}) = \begin{cases} 0 = (0, 0, \ldots), & (a_j) \in c_{00}, \\ \left(a_j \Big/ \left(\sum_{i=j}^{\infty} |a_i|^p\right)^{1/2p}\right)_{j=1}^{\infty}, & (a_j) \in l^p \backslash c_{00}. \end{cases}$$

If $(a_j) \in c_{00}$, then $t(a_j) = (ta_j) \in c_{00}$ and $T(t(a_j)_{j=1}^{\infty}) = 0 = 0T((a_j)_{j=1}^{\infty})$. For $t > 0$ and $(a_j) \in l^p \backslash c_{00}$,

$$\frac{ta_j}{\left(\sum_{i=j}^{\infty} |ta_i|^p\right)^{1/2p}} = \sqrt{t}\frac{a_j}{\left(\sum_{i=j}^{\infty} |a_i|^p\right)^{1/2p}}$$

so $T(t(a_j)_{j=1}^{\infty}) = \sqrt{t}\,T((a_j)_{j=1}^{\infty})$. Thus $T \in R_{\varphi}(l^p, l^p)$, where $\varphi(t) = \sqrt{|t|}$.

Let X be a vector space and $\lambda(X) \subset X^{\mathbb{N}}$; $\lambda(X)$ is said to be c_0-*decomposable* if for every $(x_j) \in \lambda(X)$ there exist $(t_j) \in c_0$ and $(z_j) \in \lambda(X)$ such that $(x_j) = (t_j z_j)$, that is, $x_j = t_j z_j$ for all j; $\lambda(X)$ is said to be c_0-*composite* (respectively, l^{∞}-*composite*) if $(t_j z_j) \in \lambda(X)$ for every $(t_j) \in c_0$ (respectively, l^{∞}) and $(x_j) \in \lambda(X)$. Clearly, a c_0-decomposable family is l^{∞}-composite if and only if it is c_0-composite.

EXAMPLE 2.2. (1) A topological vector space X is said to be *braked* if for every $(x_j) \in c_0(X) = \{(z_j) \in X^{\mathbb{N}} : z_j \to 0\}$ there is a scalar sequence $\lambda_j \to \infty$ for which $\lambda_j x_j \to 0$ [3, page 43]. Thus, X is braked if and only if $c_0(X)$ is c_0-decomposable. Every metrizable topological vector space and the nonmetrizable $(l^1,$ weak) are braked and, especially, every (LF) space (for example, the space $\mathfrak{D}$ of test functions) is not metrizable but braked [7].

(2) $g : X \to [0, +\infty)$ is called a *gauge on* X if $g(0) = 0$, $g(tx) \le g(x)$ for $|t| \le 1$, $x \in X$ and there is a $M > 0$ such that $g(tx) \le |t|g(x)$ whenever $|t| \ge M$, $x \in X$. For $0 < \beta \le 1$, every β-norm $\|\cdot\| : X \to [0, +\infty)$ ($\|0\| = 0$, $\|tx\| = |t|^{\beta}\|x\|$, $\|x_1 + x_2\| \le \|x_1\| + \|x_2\|$) is a gauge on X. For a gauge $g : X \to [0, +\infty)$ and $0 < p < +\infty$, let

$$l^p(X; g) = \left\{(x_j) \in X^{\mathbb{N}} : \sum_{j=1}^{\infty} [g(x_j)]^p < +\infty\right\},$$

then $l^p(X; g)$ is both c_0-decomposable [7, Lemma 1] and l^{∞}-composite.

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(3) Let $(X, \|\cdot\|)$ be a normed space and $\lambda(X) = \{(x_j) \in X^{\mathbb{N}} : \exists \delta \in (0,1)$ such that $\|x_j\| = j^{\delta},\ \forall j \in \mathbb{N}\}$. Then $\lambda(X)$ is c_0-decomposable but not c_0-composite.
(4) Let X be a topological vector space. If $\lambda(X) \subset l^{\infty}(X)$ such that $\lambda(X) \backslash c_0(X) \neq \phi$, then $\lambda(X)$ is not c_0-decomposable.

3. Sequential evaluation convergence

LEMMA 3.1 ([1], [11, page 12]). *Let G be an abelian topological group and let $x_{ij} \in G$ for $i, j \in \mathbb{N}$. If each subsequence $\{m_i\}$ of $\{i\}$ has a further subsequence $\{n_i\}$ such that*

(1) $\lim_{i\to\infty} x_{n_i n_j} = 0$ *for* $j \in \mathbb{N}$ *and*
(2) $\lim_{i\to\infty} \sum_{j=1}^{\infty} x_{n_i n_j} = 0$,

then $x_{ii} \to 0$.

This is a special case of the Antosik-Mikusiński basic matrix theorem ([11, page 10] and [16]).

LEMMA 3.2. *Let X be a vector space and V a convex subset of X such that $0 \in V$. If $x_1, x_2, \ldots, x_n \in X$ and $M > 0$ such that*

$$M \sum_{j\in\Delta} x_j \in V, \quad \textit{for every nonempty } \Delta \subseteq \{1, 2, \ldots, n\},$$

then $\sum_{j=1}^{n} s_j x_j \in V$, *for every* $0 \le s_j \le M$, $j = 1, 2, \ldots, n$.

PROOF. If $Mx \in V$ and $0 \le s \le M$, then $sx = s(Mx)/M + (1 - s/M)0 \in V$. Assume that the conclusion holds for $n = k$. Let $M > 0$ and $x_1, x_2, \ldots, x_k, x_{k+1} \in X$ such that

(i) $M \sum_{j\in\Delta} x_j \in V$, for every $\phi \neq \Delta \subseteq \{1, 2, \ldots, k, k+1\}$.

Let $0 \le s_j \le M$, $1 \le j \le k+1$. Without loss of generality, assume that $s_1 = \max_{1\le j\le k+1} s_j$. Then (i) implies that

$$s_1 \sum_{j\in\Delta} x_j = \frac{s_1}{M}\left(M \sum_{j\in\Delta} x_j\right) + \left(1 - \frac{s_1}{M}\right) 0 \in V, \quad \forall \phi \neq \Delta \subseteq \{1, \ldots, k+1\}$$

and, in particular,

(ii) $s_1(y_1 + \cdots + y_m) \in V$, for all $\{y_1, \ldots, y_m\} \subseteq \{x_1 + x_{k+1}, x_2, \ldots, x_k\}$, and
(iii) $s_1(x_1 + x_{k+1}) + \sum_{j=2}^{k} s_j x_j \in V$

by the inductive assumption and (ii). Observing that $\sum_{j=1}^{k} s_j x_j \in V$, by (i) and the inductive assumption, (iii) implies that

$$\sum_{j=1}^{k+1} s_j x_j = \left(1 - \frac{s_{k+1}}{s_1}\right) \sum_{j=1}^{k} s_j x_j + \frac{s_{k+1}}{s_1} \left[s_1(x_1 + x_{k+1}) + \sum_{j=2}^{k} s_j x_j \right] \in V. \qquad \square$$

Let S be a nonempty set and $\{s_j\} \subset S$, $S^{\#} \subset \mathbb{C}^S$. Referring to the weak convergence in linear analysis, we say that $\sum_{j=1}^{\infty} s_j$ is $\sigma(S, S^{\#})$*-convergent* if there is $s \in S$ such that $\sum_{j=1}^{\infty} s'(s_j) = s'(s)$ for each $s' \in S^{\#}$. Similarly, $\sum_{j=1}^{\infty} s_j$ is $\beta(S, S^{\#})$*-convergent* if there is $s \in S$ such that $\lim_n \sum_{j=1}^{n} s'(s_j) = s'(s)$ uniformly with respect to $s' \in A \subset S^{\#}$ whenever A is pointwise bounded on S, that is, $\sup_{s' \in A} |s'(t)| < \infty$, for all $t \in S$.

THEOREM 3.3. *Let X, Y be vector spaces and φ, $\psi \in C(0)$, $\lambda(X) \subset X^{\mathbb{N}}$. If $\lambda(X)$ is both c_0-decomposable and c_0-composite, then for every $\{T_j\} \subset R_\varphi(X, Y)$ and $Y^{\#} \subset R_\psi(Y, \mathbb{C})$ the following conditions are equivalent*:

$$(\sigma_2) \qquad \forall (x_j) \in \lambda(X), \quad \sum_{j=1}^{\infty} T_j(x_j) \ \textit{is}\ \sigma(Y, Y^{\#})\textit{-convergent}.$$

$$(\beta_2) \qquad \forall (x_j) \in \lambda(X), \quad \sum_{j=1}^{\infty} T_j(x_j) \ \textit{is}\ \beta(Y, Y^{\#})\textit{-convergent}.$$

PROOF. Assume that (σ_2) holds, that is, for every $(x_j) \in \lambda(X)$ there is $y \in Y$ such that $\sum_{j=1}^{\infty} (y' \circ T_j)(x_j) = y'(y)$, for every $y' \in Y^{\#}$.

Let $(x_j) \in \lambda(X)$. If $A \subset Y^{\#}$ such that A is pointwise bounded on Y but the convergence of $\sum_{j=1}^{\infty} (y' \circ T_j)(x_j)$ is not uniform with respect to $y' \in A$, then there exist $\varepsilon > 0$, $\{y'_k\} \subset A$ and an integer sequence $m_1 < n_1 < m_2 < n_2 < \cdots$ such that

$$\left| \sum_{j=m_k}^{n_k} (y'_k \circ T_j)(x_j) \right| \geq \varepsilon, \quad k = 1, 2, \ldots. \tag{3.1}$$

There exist $(t_j) \in c_0$ and $(z_j) \in \lambda(X)$ for which $(x_j) = (t_j z_j)$. Then $\delta_k = \max_{m_k \leq j \leq n_k} |t_j| \to 0$ and, observing that $T_j(0) = 0$ and $y'(0) = 0$ for $y' \in Y^{\#}$, each $\delta_k > 0$ by (3.1). Since $\delta_k \to 0$ and $\varphi(\delta_k) \to 0$, without loss of generality, we assume that $\delta_k < 1$ and $|\varphi(\delta_k)| < 1$ for all $k \in \mathbb{N}$. Then for $m_k \leq j \leq n_k$ there exist $0 \leq \theta_j \leq |\varphi(\delta_k)| < 1$ and $0 \leq s_j \leq |\psi(\theta_j)|$ such that

$$(y'_k \circ T_j)(x_j) = y'_k\left(T_j\left(\delta_k \frac{t_j}{\delta_k} z_j\right)\right) = y'_k\left(\theta_j T_j\left(\frac{t_j}{\delta_k} z_j\right)\right) = s_j (y'_k \circ T_j)\left(\frac{t_j}{\delta_k} z_j\right).$$

Let $r_k = \max_{m_k \leq j \leq n_k} |\psi(\theta_j)|$ and $\alpha > 0$. Since $\lim_{t \to 0} \psi(t) = \psi(0) = 0$, there is $\eta > 0$ such that $|\psi(t)| < \alpha$ whenever $|t| < \eta$. Moreover, $\varphi(\delta_k) \to 0$ so there is

$k_0 \in \mathbb{N}$ for which $|\varphi(\delta_k)| < \eta$ whenever $k > k_0$. Hence, if $k > k_0$ and $m_k \le j \le n_k$, then $0 \le \theta_j \le |\varphi(\delta_k)| < \eta$ and $0 \le r_k = \max_{m_k \le j \le n_k} |\psi(\theta_j)| < \alpha$. Thus, $r_k \to 0$ and (3.1) becomes

$$(3.2)\quad (\forall m_k \le j \le n_k)\,(\exists s_j \in [0, r_k]) \text{ such that } \left|\sum_{j=m_k}^{n_k} s_j\,(y'_k \circ T_j)\left(\frac{t_j}{\delta_k} z_j\right)\right| \ge \varepsilon,\ k \in \mathbb{N}.$$

Let $k \in \mathbb{N}$. If $r_k\left|\sum_{j\in\Delta}(y'_k \circ T_j)(t_j z_j/\delta_k)\right| < \varepsilon$ for each nonempty $\Delta \subset \{m_k, m_k+1, \dots, n_k\}$, then $\left|\sum_{j=m_k}^{n_k} s_j\,(y'_k \circ T_j)(t_j z_j/\delta_k)\right| < \varepsilon$ by Lemma 3.2 for the case $X = \mathbb{C}$ and $V = \{t \in \mathbb{C} : |t| < \varepsilon\}$. This contradicts (3.2) and, hence, there is $\Delta_k \subset \{m_k, m_k+1, \dots, n_k\}$ for which $r_k\left|\sum_{j\in\Delta_k}(y'_k \circ T_j)(t_j z_j/\delta_k)\right| \ge \varepsilon$. Thus, we have a sequence $\{\Delta_k\}$ of finite subsets of $\mathbb{N}$ such that

$$(3.3)\qquad \max \Delta_k < \min \Delta_{k+1}, \qquad \left|r_k \sum_{j\in\Delta_k}(y'_k \circ T_j)\left(\frac{t_j}{\delta_k} z_j\right)\right| \ge \varepsilon, \quad k \in \mathbb{N}.$$

We claim that the matrix

$$\left\{r_i \sum_{j\in\Delta_k}(y'_i \circ T_j)\left(\frac{t_j}{\delta_k} z_j\right)\right\}_{i,k\in\mathbb{N}}$$

satisfies conditions of Lemma 3.1. In fact, for a subsequence $\{m_i\}$ of $\{i\}$ let $\{n_i\} = \{m_i\}$ and $\alpha_j = t_j/\delta_{n_k}$ if $j \in \Delta_{n_k}$ $(k = 1, 2, \dots)$ and $\alpha_j = 0$ otherwise. Now let $u_j = \alpha_j z_j$ for each $j \in \mathbb{N}$.

Since $\lambda(X)$ is both c_0-decomposable and c_0-composite, $\lambda(X)$ is also l^∞-composite. So $(\alpha_j) \in l^\infty$ shows that $(u_j) \in \lambda(X)$ and, by (σ_2), there is $y \in Y$ such that

$$\sum_{j=1}^{\infty}(y' \circ T_j)(u_j) = y'(y), \quad y' \in Y^{\#}.$$

Observing that $T_j(0) = 0$ for $j \in \mathbb{N}$ and $y'(0) = 0$ for $y' \in Y^{\#}$, we have

$$y'(y) = \sum_{j=1}^{\infty}(y' \circ T_j)(u_j) = \sum_{k=1}^{\infty}\sum_{j\in\Delta_{n_k}}(y' \circ T_j)\left(\frac{t_j}{\delta_{n_k}} z_j\right), \quad y' \in Y^{\#}.$$

Since $\{y'_{n_i}\} \subset A$, where A is pointwise bounded on Y and $r_{n_i} \to 0$, $r_{n_i} y'_{n_i}(y) \to 0$ as $i \to \infty$. Moreover,

$$\lim_{i\to\infty} r_{n_i} \sum_{j\in\Delta_k}(y'_{n_i} \circ T_j)\left(\frac{t_j}{\delta_k} z_j\right) = \sum_{j\in\Delta_k}\lim_{i\to\infty} r_{n_i}(y'_{n_i} \circ T_j)\left(\frac{t_j}{\delta_k} z_j\right) = 0.$$

Hence, by Lemma 3.1, $r_i \sum_{j\in\Delta_i}(y'_i \circ T_j)(t_j z_j/\delta_i) \to 0$ as $i \to \infty$. This contradicts (3.3) and, hence, $(\sigma_2) \Rightarrow (\beta_2)$ holds. □

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The c_0-decomposability of $\lambda(X)$ cannot be omitted in Theorem 3.3.

EXAMPLE 3.1. If $\lambda \subset l^\infty$ but $\lambda\backslash c_0 \neq \phi$, then λ is not c_0-decomposable and there exist a locally convex space X with the dual X' and a sequence $\{T_j\} \subset L(\mathbb{C}, X) \subset R_{\varphi_0}(\mathbb{C}, X)$ such that $\sum_{j=1}^\infty T_j(s_j)$ is $\sigma(X, X')$-convergent for each $(s_j) \in \lambda$ but $\sum_{j=1}^\infty T_j(t_j)$ cannot be $\beta(X, X')$-convergent whenever $(t_j) \in \lambda\backslash c_0$.

In fact, let $X = (l^\infty, \sigma(l^\infty, l^1))$ and $T_j : \mathbb{C} \to X$, $T_j(t) = te_j$, where

$$e_j = (0, \ldots, 0, \overset{(j)}{1}, 0, 0, \ldots).$$

For $(s_j) \in \lambda \subset l^\infty$ and $(\alpha_j) \in l^1 = X'$,

$$\left|\left\langle (s_j) - \sum_{j=1}^n T_j(s_j), (\alpha_j)\right\rangle\right| = \left|\sum_{j=n+1}^\infty s_j\alpha_j\right| \leq \sup_k |s_k| \sum_{j=n+1}^\infty |\alpha_j| \to 0$$

as $n \to +\infty$, so $\sigma(X, X') - \sum_{j=1}^\infty T_j(s_j) = (s_j)$, for every $(s_j) \in \lambda$.

Let $B = \left\{(\alpha_j) \in l^1 : \sum_{j=1}^\infty |\alpha_j| \leq 1\right\}$ and $(t_j) \in \lambda\backslash c_0$. There exists an increasing $\{j_k\} \subset \mathbb{N}$ and a $\delta > 0$ such that $|t_{j_k}| \geq \delta$ for all $k \in \mathbb{N}$ and

$$\left|\left\langle (t_j) - \sum_{j=1}^n T_j(t_j), e_{j_k}\right\rangle\right| = |t_{j_k}| \geq \delta, \quad \forall j_k > n.$$

Observing that $\{e_j\} \subset B$ and $(X, \beta(X, X')) = (l^\infty, \beta(l^\infty, l^1)) = (l^\infty, \|\cdot\|_\infty)$, if $\sum_{j=1}^\infty T_j(t_j)$ is $\beta(X, X')$-convergent, then

$$\beta(X, X') - \sum_{j=1}^\infty T_j(t_j) \neq (t_j) = \sigma(X, X') - \sum_{j=1}^\infty T_j(t_j).$$

However, this is impossible since both $(X, \beta(X, X'))$ and $(X, \sigma(X, X'))$ are Hausdorff and $\beta(X, X')$ is stronger than $\sigma(X, X')$. This shows that $\sum_{j=1}^\infty T_j(t_j)$ cannot be $\beta(X, X')$-convergent.

THEOREM 3.4. *Let X be a topological vector space, $U \in \mathcal{N}(X)$, $\lambda(X) \subset l^\infty(X)$, and let Y be a vector space. If $\lambda(X)$ is both c_0-decomposable and c_0-composite, then for every $\varphi, \psi \in C(0)$, $Y^\# \subset R_\psi(Y, \mathbb{C})$ and $\{T_j\} \subset R_{\varphi,U}(X, Y)$, the conditions (σ_2) and (β_2) are equivalent.*

PROOF. As stated in Example 2.2 (4), $\lambda(X) \subset c_0(X)$ and $\lambda(X)$ is l^∞-composite. Then, for $(\alpha_j) \in l^\infty$ and $(z_j) \in \lambda(X)$, $(\alpha_j z_j) \in \lambda(X)$ and $\alpha_j z_j \to 0$ so $\alpha_j z_j \in U$ eventually. Now the desired equivalence follows from the arguments similar to those given in the proof of Theorem 3.3. □

COROLLARY 3.5. (i) *Let $\varphi, \psi \in C(0)$ and Y be a vector space, $Y^\# \subset R_\psi(Y, \mathbb{C})$. If X is a braked space and $U \in \mathcal{N}(X)$, then for $\lambda(X) = c_0(X)$ and $\{T_j\} \subset R_{\varphi, U}(X, Y)$, and the conditions (σ_2) and (β_2) are equivalent.*

(ii) *Let X, Y be vector spaces and $g : X \to [0, +\infty)$ a gauge and $U_\varepsilon = \{x \in X : g(x) \le \varepsilon\}$, $Y^\# \subset R_\psi(Y, \mathbb{C})$. Then for $\lambda(X) = l^p(X; g)$ $(0 < p < +\infty)$ and $\{T_j\} \subset R_{\varphi, U_\varepsilon}(X, Y)$, the conditions (σ_2) and (β_2) are equivalent.*

COROLLARY 3.6. *Let X be a vector space and $\varphi, \psi \in C(0)$, $X^\# \subset R_\psi(X, \mathbb{C})$, $D_\varepsilon = \{z \in \mathbb{C} : |z| \le \varepsilon\}$. If $\lambda \subset l^\infty$ and λ is both c_0-decomposable and c_0-composite, then for every $\{F_j\} \subset R_{\varphi, D_\varepsilon}(\mathbb{C}, \mathbb{C})$ and $\{x_j\} \subset X$ the following conditions are equivalent*:

$$(\sigma_3) \qquad \forall (t_j) \in \lambda, \quad \sum_{j=1}^{\infty} F_j(t_j)x_j \text{ is } \sigma(X, X^\#)\text{-convergent.}$$

$$(\beta_3) \qquad \forall (t_j) \in \lambda, \quad \sum_{j=1}^{\infty} F_j(t_j)x_j \text{ is } \beta(X, X^\#)\text{-convergent.}$$

PROOF. Define $T_j : \mathbb{C} \to X$ by $T_j(z) = F_j(z)x_j$, $j \in \mathbb{N}$. If $0 \le t < 1$ and $z \in D_\varepsilon$, then $T_j(tz) = F_j(tz)x_j = sF_j(z)x_j = sT_j(z)$, where $0 \le s \le |\varphi(t)|$. Hence, $\{T_j\} \subset R_{\varphi, D_\varepsilon}(\mathbb{C}, X)$ and the desired equivalence follows from Theorem 3.4. □

Let $\lambda \subset \mathbb{C}^{\mathbb{N}}$. We say that a series $\sum x_j$ in a topological vector space X is λ-*multiplier convergent* or, simply, λ-mc if $\sum_{j=1}^\infty t_j x_j$ converges for every $(t_j) \in \lambda$ (see [11, 14]). It was shown ([6]) that a sequentially complete locally convex space X contains no copy of $(c_0, \|\cdot\|_\infty)$ if and only if c_0-mc, l^∞-mc and $\{0, 1\}^{\mathbb{N}}$-mc are equivalent for series in X and if and only if for every c_0-mc series $\sum x_j$ in X the series $\sum_{j=1}^\infty t_j x_j$ converges uniformly with respect to $(t_j) \in \{(\alpha_j) \in l^1 : \sum_{j=1}^\infty |\alpha_j| \le 1\}$ (see [6]; [11, page 143]). In fact, λ-mc was one of the key issues in analysis during the last century.

We say that $\lambda (\subset \mathbb{C}^{\mathbb{N}})$ is mc-invariable if for every vector space X and $X^\# \subset R_\varphi(X, \mathbb{C})$, where $\varphi \in C(0)$, each λ-multiplier $\sigma(X, X^\#)$-convergent series in X is λ-multiplier $\beta(X, X^\#)$-convergent. By Corollary 3.6, c_0 and $l^p (0 < p < +\infty)$ are mc-invariable and, especially, Corollary 3.6 gives a simple method for construction of mc-invariable families.

EXAMPLE 3.2. (1) Let $U = (-\pi/2, \pi/2)$ and for each $j \in \mathbb{N}$, define $F_j : \mathbb{R} \to \mathbb{R}$ by $F_j(x) = \sin(j^{-j}x)$. If $0 \le t < 1$ and $0 \le |x| \le \pi/2$, then

$$F_j(tx) = \sin(tj^{-j}x) = s\sin(j^{-j}x) = sF_j(x),$$

where $0 \le s \le \pi t/2 = \varphi(t)$ so $\{F_j\} \subset R_{\varphi,U}(\mathbb{R}, \mathbb{R})$ (see Example 2.1 (2)). For $(t_j) \in c_0$,

$$|F_j(t_j)| = |\sin(j^{-j}t_j)| \le j^{-j}|t_j| \le \sup_k |t_k| j^{-j}.$$

Then $\lambda_0 = \{(\sin(j^{-j}t_j))_{j=1}^\infty : (t_j) \in c_0\} \subset \bigcap_{p>0} l^p$, that is, λ_0 is a very small family and the λ_0-multiplier $\sigma(X, X^\#)$-convergence is a very weak condition. However, Corollary 3.6 shows that λ_0 is mc-invariable.

(2) Define $F_j : \mathbb{R} \to \mathbb{R}$ by $F_j(x) = e^{j|x|} - 1$, $j \in \mathbb{N}$. Then, for $0 < t < 1$ and $x \ne 0$,

$$F_j(tx) = \frac{e^{tj|x|} - 1}{e^{j|x|} - 1} F_j(x) = e^\alpha t F_j(x),$$

where $\alpha < 0$ so $\{F_j\} \subset R_{\varphi_0}(\mathbb{R}, \mathbb{R})$ (see Example 2.1 (3)) and $\lambda_\infty = \{(e^{j|t_j|} - 1)_{j=1}^\infty : (t_j) \in l^2\}$ is mc-invariable. Notice that λ_∞ includes unbounded sequences.

4. Series of abstract functions

Let Ω be a compact Hausdorff space and $C(\Omega, X)$ the space of continuous functions valued in a Banach space X. For $\{0,1\}^{\mathbb{N}}$-mc of $\sum f_j$, where $f_j \in C(\Omega, X)$, the Thomas theorem says that the following conditions are equivalent (see [13, 8]):

(1) $(\forall (t_j) \in \{0,1\}^{\mathbb{N}})$ $(\exists f \in C(\Omega, X))$ such that $\sum_{j=1}^\infty t_j f_j(\omega) = f(\omega)$, $\omega \in \Omega$.

(2) $(\forall (t_j) \in \{0,1\}^{\mathbb{N}})$ $(\exists f \in C(\Omega, X))$ such that $\lim_n \sum_{j=1}^n t_j f_j(\omega) = f(\omega)$ uniformly with respect to $\omega \in \Omega$.

Here $\{0,1\}^{\mathbb{N}}$ is not mc-invariable (see Example 3.1). It should also be pointed out that for mc-invariable $\lambda \subset \mathbb{C}^{\mathbb{N}}$ and λ-mc, a Thomas-type result holds in an even more abstract setting. In fact, we can consider the abstract mapping pair $(\Omega, B(\Omega, X))$ consisting of an abstract set Ω and $B(\Omega, X) = \{f \in X^\Omega : f(\Omega) \text{ is bounded}\}$, where X is a locally convex space. By a reasoning which is similar to the proof of Theorem 3.3, we have the following

THEOREM 4.1. *Let X be a locally convex space and $\Omega \ne \phi$, $\lambda \subset \mathbb{C}^{\mathbb{N}}$, $\{F_j\} \subset R_\varphi(\mathbb{C}, \mathbb{C})$, where $\varphi \in C(0)$. If λ is both c_0-decomposable and c_0-composite, then for every $\{f_j\} \subset B(\Omega, X)$ the following conditions are equivalent*:

(pwc) $(\forall (t_j) \in \lambda)$ $(\exists f \in B(\Omega, X))$ *such that weak*-$\sum_{j=1}^\infty F_j(t_j) f_j(\omega) = f(\omega)$, $\omega \in \Omega$.

(uc) $(\forall (t_j) \in \lambda)$ $(\exists f \in B(\Omega, X))$ *such that* $\lim_n \sum_{j=1}^n F_j(t_j) f_j(\omega) = f(\omega)$ *uniformly with respect to* $\omega \in \Omega$.

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PROOF. Fix $\omega \in \Omega$ and define $T_j : \mathbb{C} \to X$ by $T_j(z) = F_j(z)f_j(\omega)$, then $T_j \in R_\varphi(\mathbb{C}, X)$ and, by Theorem 3.3, the condition (pwc) is equivalent to the following

(pc) $(\forall (t_j) \in \lambda)\,(\exists f \in B(\Omega, X))$ such that $\sum_{j=1}^{\infty} F_j(t_j)f_j(\omega) = f(\omega)$, $\omega \in \Omega$.

Suppose that (pc) holds and $(t_j) \in \lambda$ but the convergence of $\sum_{j=1}^{\infty} F_j(t_j)f_j(\omega)$ is not uniform for $\omega \in \Omega$. Then there exist a convex $V \in \mathcal{N}(X)$, $\{\omega_k\} \subset \Omega$ and an integer sequence $m_1 < n_1 < m_2 < n_2 < \cdots$ such that

$$\sum_{j=m_k}^{n_k} F_j(t_j)f_j(\omega_k) \notin V, \quad k \in \mathbb{N}. \tag{4.1}$$

Let $(t_j) = (\eta_j \alpha_j)$, where $(\eta_j) \in c_0$ and $(\alpha_j) \in \lambda$. Then $\delta_k = \max_{m_k \le j \le n_k} |\eta_j| \to 0$ and each $\delta_k > 0$. Since $\{F_j\} \subset R_\varphi(\mathbb{C}, \mathbb{C})$, for sufficiently large k and $m_k \le j \le n_k$ there is $s_j \in [0, |\varphi(\delta_k)|]$ for which

$$F_j(t_j) = F_j(\eta_j \alpha_j) = F_j\left(\delta_k \frac{\eta_j}{\delta_k} \alpha_j\right) = s_j F_j\left(\frac{\eta_j}{\delta_k} \alpha_j\right)$$

and, without loss of generality, (4.1) becomes

$$0 \le s_j \le |\varphi(\delta_k)| \text{ for } m_k \le j \le n_k, \quad \sum_{j=m_k}^{n_k} s_j F_j\left(\frac{\eta_j}{\delta_k}\alpha_j\right) f_j(\omega_k) \notin V, \quad k \in \mathbb{N}.$$

Then, by Lemma 3.2, for each $k \in \mathbb{N}$ there is a $\Delta_k \subset \{m_k, m_k + 1, \dots, n_k\}$ such that

$$\max \Delta_k < \min \Delta_{k+1}, \quad |\varphi(\delta_k)| \sum_{j \in \Delta_k} F_j\left(\frac{\eta_j}{\delta_k}\alpha_j\right) f_j(\omega_k) \notin V.$$

Now consider the matrix

$$M = \left\{ |\varphi(\delta_i)| \sum_{j \in \Delta_k} F_j\left(\frac{\eta_j}{\delta_k}\alpha_j\right) f_j(\omega_i) \right\}_{i,k}.$$

Since each $f_j \in B(\Omega, X)$ so $\{f_j(\omega_i) : i \in \mathbb{N}\}$ is bounded, similarly to the proof of Theorem 3.3, the matrix M satisfies conditions of Lemma 3.1. Hence, Lemma 3.1 shows that $\lim_k |\varphi(\delta_k)| \sum_{j \in \Delta_k} F_j(\eta_j \alpha_j / \delta_k) f_j(\omega_k) = 0$. This is a contradiction so (pc) $\Rightarrow$ (uc) holds. □

It is also worthwhile observing that Theorem 4.1 has several interesting special cases.

EXAMPLE 4.1. (1) Let X, Y be Banach spaces, $\Omega = \{x \in X : \|x\| \le 1\}$, $\varphi \in C(0)$ and $\{F_j\} \subset R_\varphi(\mathbb{C}, \mathbb{C})$. Let λ be a c_0-decomposable and c_0-composite family of

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sequences in $\mathbb{C}$. If $\{T_j\} \subset L(X, Y)$ such that $\sum_{j=1}^{\infty} F_j(t_j)T_j(x)$ converges whenever $(t_j) \in \lambda$ and $x \in \Omega$, then the Banach-Steinhaus theorem shows that $\sum_{j=1}^{\infty} F_j(t_j)T_j(\cdot) \in L(X, Y)$ for each $(t_j) \in \lambda$. Fortunately, Theorem 4.1 gives a stronger conclusion as follows.

For each $(t_j) \in \lambda$, $\sum_{j=1}^{\infty} F_j(t_j)T_j$ converges in the operator norm, that is,

$$\lim_n \left\| \sum_{j=n}^{\infty} F_j(t_j)T_j \right\| = \lim_n \sup_{x\in\Omega} \left\| \sum_{j=n}^{\infty} F_j(t_j)T_j(x) \right\| = 0.$$

Note that even for the simplest case of $\lambda = c_0$ or l^p $(0 < p < +\infty)$ and $F_j(t) = t$, the Banach-Steinhaus theorem cannot assert that $\sum_{j=1}^{\infty} t_j T_j$ converges in the operator norm since the Banach-Steinhaus theorem only asserts that $\sum_{j=1}^{\infty} t_j T_j(x)$ is uniformly convergent on every relatively compact subset of X ([10, page 299]).

(2) A topological space Ω is said to be *pseudocompact* if every continuous $f : \Omega \to \mathbb{R}$ is bounded on Ω. A normal space is countably compact if and only if it is pseudocompact. Dini's lemma says that if Ω is pseudocompact and $\{f_n\}_{n=0}^{\infty}$ is a sequence in $C(\Omega, \mathbb{R})$ such that $f_n(\omega) \searrow f_0(\omega)$ at each $\omega \in \Omega$, then $\lim_n f_n(\omega) = f_0(\omega)$ uniformly for $\omega \in \Omega$. Since $\sum_{j=1}^{\infty}[f_j(\omega) - f_{j+1}(\omega)]$ is $\{0, 1\}^{\mathbb{N}}$-mc, the Dini lemma is also a Thomas type version. Now Theorem 4.1 implies a similar result as follows.

Let X be a locally convex space and λ a c_0-decomposable and c_0-composite family of sequences in $\mathbb{C}$, $\varphi \in C(0)$ and $\{F_j\} \subset R_\varphi(\mathbb{C}, \mathbb{C})$. If Ω is a pseudocompact space and $\{f_j\} \subset C(\Omega, X)$ such that $\sum_{j=1}^{\infty} F_j(t_j)f_j(\omega)$ converges whenever $(t_j) \in \lambda$ and $\omega \in \Omega$, then the following conditions are equivalent.

(i) $\sum_{j=1}^{\infty} F_j(t_j)f_j(\cdot) \in B(\Omega, X)$, for every $(t_j) \in \lambda$;

(ii) for every $(t_j) \in \lambda$, $\sum_{j=1}^{\infty} F_j(t_j)f_j(\omega)$ converges uniformly with respect to $\omega \in \Omega$;

(iii) $\sum_{j=1}^{\infty} F_j(t_j)f_j(\cdot) \in C(\Omega, X)$ for every $(t_j) \in \lambda$.

(3) Let Ω be a nonempty set and $\{f_j\} \subset B(\Omega, X)$ such that $\sum_{j=1}^{\infty} |f_j(\omega)| < +\infty$ at each $\omega \in \Omega$. If $\lambda = c_0$ or l^p $(0 < p < +\infty)$, then the following conditions are equivalent:

(iv) $\left\{ \sum_{j=1}^{\infty} \left[\exp\left(|t_j|/\sqrt[j]{j}\right) - 1\right] f_j(\omega) : \omega \in \Omega \right\}$ is bounded whenever $(t_j) \in \lambda$;

(v) for every $(t_j) \in \lambda$, $\sum_{j=1}^{\infty} \left[\exp\left(|t_j|/\sqrt[j]{j}\right) - 1\right] f_j(\omega)$ converges uniformly with respect to $\omega \in \Omega$.

In particular, if Σ is a σ-algebra of subsets of Ω and $\mu_j : \Sigma \to \mathbb{C}$ is a countably additive measure such that $\sum_{j=1}^{\infty} |\mu_j(A)| < +\infty$ at each $A \in \Sigma$, then for $\lambda \in \{c_0, l^\infty\ (0 < p < +\infty)\}$ the following conditions are equivalent:

(vi) for every $(t_j) \in \lambda$, $\sum_{j=1}^{\infty} \left[\exp\left(|t_j|/\sqrt[j]{j}\right) - 1\right] \mu_j(\cdot) : \Sigma \to \mathbb{C}$ is a bounded measure.

(vii) for every $(t_j) \in \lambda$, $\sum_{j=1}^{\infty}\left[\exp\left(|t_j|/\sqrt[j]{j}\right) - 1\right]\mu_j(\cdot) : \Sigma \to \mathbb{C}$ is a countably additive measure.

Observing that $\{(f_j(\omega))_{j=1}^{\infty} : \omega \in \Omega\} \subset l^1$ and using the resonance theorem instead of Theorem 4.1, and using linear analysis, we can also obtain the equivalence of the following conditions:

(vi′) $\left\{\sum_{j=1}^{\infty} t_j f_j(\omega) : \omega \in \Omega\right\}$ is bounded at each $(t_j) \in c_0$.
(vii′) For each $(t_j) \in c_0$, $\sum_{j=1}^{\infty} t_j f_j(\omega)$ converges uniformly for $\omega \in \Omega$.

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