INVITED DISCOURSES

ASTRONOMY IN ANCIENT GREECE

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Abstract: Science has spread from western Europe where it developed into recognisably-modern form in the seventeenth century, stimulated by Copernicus's claim that the Earth is a planet. Copernicus however was an astronomer in the Greek tradition, whose task was to reproduce the planetary paths by geometrical constructions using uniform circular motions. Eudoxus's attempt to do this with nests of concentric spheres had been superseded by the use of the more flexible techniques necessary to meet the observational standards of the Hellenistic era. Ptolemy's Almagest synthesised the Greek achievement but its shortcomings led Copernicus to make the Earth a planet.

Our purpose this evening is to travel back in time for two thousand years and more, and to try to enter the minds of those men of the period who worked to understand the heavens and to explain and predict the motions of the stars and planets, and who communicated their conclusions to their fellow-men in the Greek language. And I shall outline reasons why their achievement was of decisive importance, not only for the history of astronomy but for the development of modern science as we know it.

First, some general comments by way of orientation. Those like myself who spent most of their school years studying Latin and Greek were encouraged to think that Greek culture came to an end with the death of Alexander the Great in 323 BC and that of Aristotle the following year. But by then Greek astronomy was still in its infancy, and Claudius Ptolemy, the dominating figure in our story, died around 170 AD, some five centuries later. That is to say, major astronomers wrote in Greek for at least five hundred years after the decline of Athenian culture, and for our purpose the years after Christ are as important as those before.

Nor, for the most part, was astronomy written on the main land of what is now Greece. Of where the great astronomers lived and worked we know extraordinarily little, but the places mentioned are mostly in

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the eastern Mediterranean: the islands such as Rhodes and Samos, the towns of Asia Minor, and especially the great centre of learning at Alexandria. In short, the great astronomers lived in a different age and in different regions from the great dramatists whose works were performed in this theatre.

The next point is a little more controversial. Almost every possible thought, however absurd, occurred to some Greek philosopher at some time; and occasionally we find suggestions that the Earth is spinning, or even that it is in some sort of orbit. Archimedes and Plutarch both tell us expressly that Aristarchus of Samos in the third century BC proposed that the Earth not only spins on its axis but that it moves in a circular orbit around the Sun. They do not explain why Aristarchus proposed this, though the motion of the Earth about the Sun must surely have involved considerations of the apparent motions of the planets. Nor do they make it clear whether Aristarchus believed the Earth to be in truth moving about the Sun, or whether he merely argued that this would be one way of accounting for the observed facts. Aristarchus had inherited a speculative tradition, but by the third century BC astronomy was becoming more sober and serious and technical, and we know of only one very minor figure who took the suggestion of Aristarchus seriously. That is to say, we make the mistake of looking at Greek astronomy with our eyes rather than theirs, if we seize on Aristarchus as a precursor of Copernicus, and exaggerate the importance of an hypothesis whose impact in Antiquity was almost nil. Instead, we must understand their astronomy in terms of the questions they thought important, the evidence at their disposal, and the conceptual and mathematical tools they could deploy. To see the problems as they saw them is not easy; but it is more difficult still to rid ourselves of the ethical conventions of the science of today: the duty to say what is our work and what we have taken from the writings of others, to say what is a genuine observation and what is an observation adjusted to fit a theory and what is no observation at all but a straightforward calculation. But here too one can say that the historian who condemns past astronomers as fools or criminals is most probably condemning himself as an historian.

Next, a word about historical records. For any period, records survive of only a minute fraction of the events that actually take place -- will any record survive to tell your biographer what you had for breakfast today? The historian has no possibility of access to events of which no records survive, and he must accept that when he has made sense of the surviving records then he has done all that is possible. Even so, in the modern period the surviving records are often overwhelming in number, and the historian singles out an incident here and there to illustrate his general theory. But when we turn to the study of Antiquity, the situation is the very opposite. Very little has come down to us, and we must make the most of the fragmentary records that we have. There are several reasons for this scarcity of source material. With few exceptions, what we have of Greek astronomy is not original documents contemporary with the author, but copies of copies of copies, laboriously and expensively made, and perhaps involving one or even several translators. In most cases a work was never widely in demand, or if it was then the demand fell off with time, so that no copy has come down to us and the work is lost. This can apply even to works of the first importance, if their content is assimilated into a later and even more successful work. For then there is no longer any purpose in the potential reader's paying good money to recopy the earlier work, which therefore vanishes. This happened in Greek mathematics when Euclid's Elements synthesised the achievements of his mathematical predecessors: the writings of these predecessors thereupon vanished. But the historian of mathematics is lucky, in that Euclid wrote about 300 BC, when some of the greatest mathematicians of Antiquity were still to come. Ptolemy wrote his Almagest in the middle of the second century AD; he drew extensively on the work of his great predecessor Hipparchus, and so of the writings of Hipparchus we possess only one minor example. The very success of Ptolemy as a synthesiser, *coming at the end of the high period of Greek astronomy, brought about* the destruction of the works of even his greatest predecessors; and this apparent dearth of predecessors in turn enhanced Ptolemy's own stature in the eyes of later civilisations.

With these preliminaries, let us quickly travel back in time from the present, beginning with a word about science and scientists in the modern world. We have at this General Assembly astronomers from every nation on earth, sharing in a common enterprise. There is world-wide agreement about what it is to be a scientist -- what a scientist does, the questions he asks, the kind of answer that is acceptable, and the methods for arriving at such an answer and communicating it to other scientists. These agreements are inculcated in our education, and enforced by appointments committees to posts for teaching or research, by referees who control access to scientific journals, and by the electors to membership of scientific societies. Today this agreement is worldwide. But if we travel back in time to the year 1700, say, we find modern science only in western Europe. In England and France there are major scientific societies -- the Royal Society and the Académie des Sciences -- and scientific journals; scientific teaching posts in universities, and research institutions such as the Greenwich and Paris Observatories. Most important of all, there is the Principia of Isaac Newton which the eighteenth century would take as a model of a mature science. But the rest of the world has yet to learn from western Europe what science is.

If we go back another hundred years, to 1600, we find no major scientific societies, no scientific journals, and only here and there a post where an astronomer or mathematician might make a career. But two men are at work whose achievements would set physical science on the path to Newton's Principia. Galileo would soon use the telescope to extend the range of the human senses, something no Greek would have envisaged, and he would become the first human being to see mountains on the Moon, and phases of Venus, the satellites of Jupiter, and so forth. Armed with this evidence against the old world-picture of

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Aristotle and Ptolemy, he became a militant Copernican and set himself to create a physics in which it made sense for the Earth to be in motion about the Sun while we passengers on Earth are totally unaware of it. Kepler was a convinced Copernican from his youth, and he would soon use the accurate observations of Tycho Brahe to develope a profoundly new, dynamical astronomy within which each planet would orbit the Sun in a single curve, the ellipse studied by the mathematicians of Antiquity.

In the work of Galileo and Kepler we find powerful new elements marking a fundamental advance beyond anything present in Copernicus only a few decades earlier. Copernicus's De revolutionibus (1543) is thoroughly Greek in spirit and method; it is very obviously modelled in layout on Ptolemy's Almagest, addressing the same problems and offering the same kind of answer by means of similar techniques. Indeed, in some ways it was more faithful to the totality of Greek natural philosophy and astronomy than was Ptolemy's Almagest. Copernicus was not the first of the moderns but the last of the medievals, and last of the astronomers in the Greek tradition; yet his claim that the Earth is a planet convinced both Kepler and Galileo and helped spark off the Scientific Revolution that taught first western Europe and then the whole world what science is. This, in brief, is why Greek astronomy is fundamentally important in the whole history of science.

The basic aim of Copernicus's De revolutionibus is to use combinations of uniform circular motions to provide geometrical constructions that would accurately reproduce the observed motion of each planet across the sky. He was, in fact, answering the challenge said to have been thrown down by Plato in the first half of the fourth century BC: "By the assumption of what uniform and orderly motions can the apparent motions of the planets be accounted for?"

The assumptions underlying Plato's question are very remarkable indeed. First of all, the universe is assumed to be a cosmos. Today we do not link the astronomical term 'cosmos' with the cosmetics that a lady uses to perfect the beauty and harmony and symmetry of her appearance, but the root word is the same. A cosmos is first of all a rational universe, one that the human intellect can penetrate. It is not the domain of the gods with their whims and fancies, their playthings and their enemies, gods whose behaviour a mortal man can neither control nor predict. There can be a science of the universe because the universe is a cosmos. But a cosmos is not merely rational; it has beauty and harmony and symmetry. Copernicus found Ptolemy's treatment of the planets one by one unsatisfactory because there was no perception of the unity of the universe; the result of assembling the various independent bits would be, he said, "monster rather than man". Kepler, in his first publication, believed he had penetrated the mind of the divine geometer who created the universe, when he found that the planetary orbits were spaced out on the pattern of a nest of regular solids and spheres. The cosmos had a long and important history, and the conviction on the part of the early Greeks that the universe is a cosmos set the scene for the science of astronomy.

At the outside of the cosmos was the sphere of the heavens. We are so used to working with spherical geometry that we think it self-evident that the heavens appear spherical to the observer, but a study of other early civilisations shows that this is far from being the case. Curiously, at the other extreme, many people today are convinced that the spherical shape of the Earth is an early modern discovery and that explorers like Christopher Columbus feared they would sail off the edge of a flat Earth. In fact, the spherical shape of the Earth was known to Pythagoreans long before Plato, and in the writings of Plato's pupil, Aristotle, which were so central to medieval education, we have several proofs of this: that the shadow of the Earth during an eclipse of the Moon is always circular in outline, and that the stars are seen differently as we travel north and south. Indeed Aristotle quotes a value for the circumference of the Earth that is too large, but only by a factor of less than two. The Greek sense of symmetry demanded that the spherical Earth be at the centre of the spherical heavens, and there was nothing about the appearances to contradict this. Plato's challenge, therefore, assumed a cosmos that was lawlike, and comprised a spherical Earth at the centre of a spherical heavens.

Now the stars goes round us every day, and since the Greeks were the boldest, not to say the most irresponsible of speculative thinkers, a number of early philosophers suggested that the Earth itself might be spinning or otherwise in motion. Since Aristotle's method of exposition is to give the views of others before giving his own, he sets out these theories, thereby ensuring that every medieval and Renaissance student of astronomy considered them too. But Aristotle, like other Greek writers, has one especially decisive argument against the motion of the Earth; and although it is not the custom in Invited Discourses to perform experiments, I would like to reproduce the Aristotelian test, and prove to you experimentally that the Earth is indeed at rest. I take this heavy stone, and I throw it straight up in the air. You will see that it falls back into my hand, and this proves that my hand did not move during the time that the stone was rising and falling. If the Earth had been in motion, and I and my hand with it, then my hand would have moved on in the second or two that it took the stone to rise and fall back again to the place from which it had been thrown. Instead, when it arrived back again, there was my hand, ready to catch it. To quote Aristotle: "heavy bodies forcibly thrown quite straight upward return to the point from which they started" (_De caelo_, 296b). (It goes without saying that post-Newtonian science provides a theoretical framework within which the experiment by no means proves that the Earth is at rest. Modern theory benefits from the heroic efforts of Galileo and others to make dynamical sense of Copernicus's claim. But as historians we have to see the experiment through Greek eyes, and interpret it within the context of _their_ theory, not ours. Within that context, the experiment decisively proves the Earth is at rest.)

Aristotle, even more than Plato, emphasises for us the contrast between Earth and heavens. On Earth there is coming to be and passing away, life and death, and this is reflected in the fact that natural movements (a stone in falling, smoke in rising) take place in a straight line, towards or away from the centre of the Earth, which is also the centre of the universe. And such movements are temporary: the stone stops falling when it has fallen as far as it can. By contrast, in the heavens everything is eternal, stars never appear or disappear, and this is expressed in their movements, regular uniform cyclic movements that of their nature continue endlessly without change.

Mainstream Greek astronomy, then, accepted a spherical Earth at rest in the centre of a spherical sky, with almost all the stars rotating daily about the Earth with a common and unchanging cyclic motion. Almost all the stars, but not quite all. Seven stars seemed irregular, whimsical, lawless in their movements: namely, the Sun, the Moon, Mercury, Venus, Mars, Jupiter and Saturn. These were the 'wandering stars', in Greek planetes, 'planet'. These seven planets seemed exceptions to the otherwise regular and lawlike cosmos. Plato's challenge was to show that the seven wanderers are not true exceptions but are equally lawlike and geometrical and regular in their motions - in other words, that the regularity is there, though it is more complex than with the fixed stars.

What kind of answer is Plato prepared to accept? The "uniform and orderly motions" he requires must in fact be combinations of uniform circular motions. Just as some recent cosmologists have insisted that a theory of the history of the universe must be without any special moment in time, because this is the way things must be, so Greek geometers and astronomers had a profound conviction that the heavens move with uniform circular motions: the cosmos simply is that kind of realm, and eternal, unchanging circular motions are the only motions so fundamental, so basic that of them no further questions can be asked.

The challenge posed by Plato shows a mature attitude to astronomy as an intellectual enquiry: an apparent anomaly in an otherwise satisfactory theory is to be resolved along agreed lines. The solution proposed by Plato's contemporary, the great mathematician Eudoxus, and preserved for us by Aristotle, is a brilliant mathematical tour de force; in that, although proceeding along what we know to be quite the wrong lines, Eudoxus generates the observed movements of each planet in qualitative outline by means of an elegant and simple geometrical construction. Each of the seven planets is thought of as attached to the equator of a sphere centred on the Earth and spinning about its poles with a uniform angular velocity. These poles are thought of as attached to a second sphere, outside the first and concentric with it, and spinning about different poles with a uniform angular velocity. The motion of the planet will of course be compounded from the motions of the two spheres.

Outside the seoond sphere is a third, again centred on the Earth. The poles round which the second sphere spins are attached to the third sphere, which spins about yet another axis, and again with a uniform angular velocity. This completes the geometrical model for the Sun and

Moon, but for the other five planets there is for each a fourth sphere, concentric with the other three and carrying the poles of the third sphere.

We have for each planet a beautifully simple and elegant geometrical model, consisting of either three of four spheres, all concentric with the centre of the universe which is the centre of the Earth, and all spinning with uniform angular velocities. In the case of the Moon, the outermost sphere spins once a day about the poles of the celestial equator; the middle sphere spins about the poles of the ecliptic once every 18.6 years, the Saros period within which eclipses usually recur; the third sphere is inclined slightly to the second, and carries the Moon round once every synodic month. This model worked surprisingly well, much more so than that of the Sun which follows a similar pattern. But the brilliance of Eudoxus in geometry, if not in astronomy, is shown in his models for the other planets. Again, the outermost of the spheres (of which now there are four for each planet) reproduces the daily spin of the entire sky, while the next sphere carries the planet along the ecliptic. The job of the other two spheres is to reproduce the retrograde loops that had earned the wandering stars their name. The mathematics underlying this was recovered less than a century ago. The axes about which the two spheres spin are inclined slightly to each other, and the two spheres rotate with equal and opposite angular velocities, completing one rotation in the synodic period of the planet. It turns out that the curve produced by these two motions is a kind of figure-of-eight, the intersection of a sphere and a cylinder tangent to the sphere at one point. As a result, the four spheres for Jupiter, say, reproduce the daily motion of the planet and its motion along the ecliptic compounded with a regular figure-of-eight that gives the planet a motion in latitude as well as causing it to retrograde from time to time. With three spheres for the Sun and Moon, four each for the other five planets, and one for the fixed stars, Eudoxus needed a total of only 27 spheres; and of these, no fewer than eight simply reflected the daily spin of the Earth.

Here we have, for the first time in history, a geometrical model which could in principle reproduce the motions of the planets for all time in the future -- and thereby, of course, confirm that the world is indeed a lawlike cosmos. Eudoxus the geometer expressed the mathematical laws governing the motions of the planets. In all probability he did not intend these spheres to be thought of as physically real, so that he was not theorising about the material structure of the heavens. Aristotle was soon to do this, making each nest of spheres physically real and introducing additional "cancelling-out" spheres between one nest and the next so that unwanted motions would not be transmitted down through the system. To the natural philosopher, these concentric spheres were profoundly attractive as a basis for the explanation of the cosmos: spheres spinning uniformly about the central Earth, as we clearly observe the sphere of the fixed stars to spin, hardly needed further explanation, so thoroughly were they in harmony with the rest of cosmology. Indeed, a last attempt to construct such a universe was made in the time of Copernicus, and in the intervening centuries the natural philosophers were never really at ease with anything else. Yet, as we can see, Eudoxus was on fundamentally the wrong track, and there was never any hope of the observed motions of the planets being accurately reproduced along these lines. Indeed, we might say that his constructions were designed to be admired rather than used as a serious astronomical tool. And so the mathematical astronomer was soon to part company with the natural philosopher.

The parting happened all the more rapidly because in the next generations Greek astronomers had access to the arithmetical astronomy of the Babylonians. The finest period of Babylonian astronomy is in the last three centuries before Christ -- later, that is, than Eudoxus. But their records of eclipses went back to the eighth century. Among the motivations for their interests in astronomy were the lunar calendar they used, and the widespread commitment to astrology which soon spread to the Greeks and led them too to demand accurate planetary tables. The Babylonians were blessed with the sexagesimal notation for numbers, the base of sixty which the Greeks soon took over into astronomy and handed down to us. This enabled the Babylonians to handle complex arithmetical operations with ease, and their astronomy involved using their arithmetical skills to predict new moons, eclipses and so forth. Where the Greeks sought to reproduce the motion of a planet at all times by their geometrical models, the Babylonians used their arithmetic to calculate special events; and, as far as the evidence goes, where the Greeks sought to understand, the Babylonians served practical needs. But what the Babylonians did, they did with ruthless brilliance. For their lunar calendar they needed to control the speed of the Sun along the ecliptic. This speed varies throughout the year, increasing for a time and then decreasing again. The Babylonians approximated to this by supposing the Sun to move for half the year with one constant speed, and then for half the year with another constant speed, with an instantaneous jump between the two; or by supposing the Sun to increase speed uniformly for six months and then decrease for six months, in what we would think of as a zig-zag function; and they then used their arithmetical skills to make the resultant calculations. Clearly they did not suppose the real Sun moved in either of these ways. They were simply arriving at a desired practical result with sufficient accuracy.

Accuracy was just what Eudoxus's nests of spheres did not provide. His approach prevented the Sun or the Moon (or the other planets) from varying their distance from Earth, which they seem to do. The planetary loops we actually observe do not have the regularity of the loops generated by Eudoxus's spheres. And a mathematical investigation shows that while parameters can be chosen that work reasonably well for Jupiter and Saturn, the situation is hopeless for Mars, Venus and Mercury. And so it is not surprising that Greek astronomy moved away from concentric spheres, while still honouring the rule that the motions used must be uniform and circular. It seems that around 200 BC, Apollonius, one of the greatest mathematicians of Antiquity, proposed the use of eccentric circles -- circles on which the planet moves with uniform angular velocity, but where the centre of the circle is no longer the Earth itself (which is therefore 'eccentric'). This obviously would allow the planet to vary in distance from the Earth. Secondly, Apollonius proposed to allow the planet to move uniformly on a little circle (or 'epicycle'), whose centre moved uniformly on a large circle centred on the Earth. Here, of course, Greek astronomy hit upon a most promising line of development. For Venus, say, orbits the Sun in an ellipse that is not far from being a true circle, and the Sun appears to orbit the Earth in an ellipse that is not far from being a true circle; so that the orbit of Venus viewed from Earth is indeed epicyclic to the first approximation.

In the short term, however, it was the Sun and Moon that were to be treated with the new mathematical tools. Hipparchus of Rhodes, who lived in the second century before Christ, was the first Greek to be both a careful observer and a competent mathematical astronomer. He compiled a catalogue of over eight hundred stars, and Pliny tells us this was because Hipparchus believed a new star had appeared but he had no existing catalogue by which to prove its novelty. He is said to have discovered the precession of the equinoxes. In order to facilitate the determination of the difference in geographical longitudes between two places, he prepared a table of eclipses for the next six hundred years, so that observers in each place would be ready to record the local time at which the eclipse occurred and so determine the time difference between the two places. And he used the tools of Apollonius in order to construct the first reasonably satisfactory geometrical models for the motions of the Sun and Moon.

Apart from one short work, all the writings of Hipparchus have been lost -- except, that is, for those which have been assimilated into and developed in the work of Ptolemy. Ptolemy, writing in the middle of the second century after Christ, compiled in his Megale Syntaxis an elaborate treatment of all the planets, a treatment that Copernicus himself admits could be amended to generate acceptably accurate planetary tables. The Almagest, as we know it, rendered all previous works in the field obsolete, causing them to vanish, leaving the Almagest appearing all the more astonishing an achievement for the absence of visible predecessors. Of what had happened in astronomy in the interim we know little. In his lunar theory Ptolemy bases himself on Hipparchus, but he finds that the earlier theory cannot account for evection which Ptolemy discovered from observations. Ptolemy's own theory of the Moon is too elaborate to discuss here, but if taken at face value it would involve the Moon varying its distance from the Earth by a factor of two, with of course a similar variation in its apparent radius. Obviously this does not happen in reality, and so we see here the over-riding concern of the practising astrologer (Ptolemy is the author of an elaborate textbook of astrology) for the position of the Moon against the background of the stars.

It was no doubt because of his pressing practical need for a geometrical machinery that would quickly and accurately compute planetary

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positions, that Ptolemy allowed himself to depart from the philosophers' rule that only uniform circular motions can occur in the heavens. If we take advantage of hindsight we can understand why the Ptolemaic device termed the 'equant' was so economical. Consider Kepler's laws that tell us that the Earth moves in an elliptical orbit around the Sun, with the Sun at one focus of the ellipse, and that the radius vector from the Sun to the Earth traces out equal areas in equal times. As far as the observational appearances go we can interchange the Sun and the Earth, and think of the Sun moving round the Earth in an ellipse with the Earth at a focus, the radius vector tracing out equal areas in equal times as before. Now consider the view from the empty focus of the ellipse. When the Sun in its orbit is near the empty focus, its linear velocity is least, but for an observer at the empty focus this is compensated for by the nearness of the Sun. Conversely, when the Sun is nearest to the Earth in its orbit, its linear velocity is greatest, but for the observer at the empty focus this is compensated for by the greater distance of the Sun. To sum up, the passage of the Sun across the sky will appear to an observer at the empty focus to be very close to uniform angular velocity.

Now the fact that the true orbit of the Sun around the Earth is an ellipse rather than a circle is of secondary importance; it is the variations in speed that are more obvious. And so we conclude that we can approximate well to the observed motion of the Sun across the sky if we consider the Sun to be moving on a circle whose centre is someway from the Earth, and to be moving with such a varying speed that when viewed from a point equidistant at the other side of the centre of the circle, the Sun appears to be moving with uniform angular velocity. Such a point is an equant point, and we note that (i) this requires the Sun to move with variable velocity, contrary to the prescriptions of the philosophers, and (ii) thanks to Kepler, we can see why this device was useful. By using it as one of his permitted mathematical tools, Ptolemy achieved his practical aim of facilitating the calculation of planetary tables; but he created a gulf between the philosophers, or cosmologists, and the mathematical astronomers. The philosophers were less than happy with circles and circular motions that were not centred on the Earth; with non-uniform motions introduced by Ptolemy, it was clear beyond doubt that the real cosmos was not to be found in the work of the mathematical astronomers. The unease at this must have been felt by every student in the late medieval universities, and it is a source of pride to Copernicus that he has succeeded in developing accurate and economical geometrical models without the device of the equant.

Ptolemy's achievement in creating for each planet a geometrical model that enabled its position to be computed with accuracy for centuries to come was extraordinary. But in the *Almagest* there is no unified system. There are common techniques, of course, and a period of one year keeps cropping up, to arouse the curiosity of later medieval astronomers, but there is no integrated system and certainly no physical model of the heavens. This Ptolemy provided in a late work, the *Planetary Hypotheses*, that was unknown to Copernicus and his contemporaries. Here we are indeed given a physical model of the universe, in which the epicycles are like ball bearings running in grooves. Ptolemy adopts a particular order for the planets (Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, fixed stars), and he assumes that at every height from the Earth it is possible to have one and only one planet. That is to say, the maximum height of the Moon above the Earth equals the minimum height of Mercury, the maximum height of Mercury equals the minimum height of Venus, and so on.

To calibrate this system Ptolemy needs the distance of the Moon from the Earth. Several earlier writers had tackled this problem, and from Aristarchus in the third century BC we possess a mathematical treatise showing how it might be solved. Aristarchus's method involved measuring the angle Moon-Earth-Sun when the Moon is at quadrature, that is, when the angle Sun-Moon-Earth is known to be right angle. The observation gives him the shape of the triangle and so the relative distances from Earth of the Sun and the Moon; and further information about the apparent diameters of the Sun and Moon and about the width of the Earth's shadow as compared to the diameter of the Moon gives him the distances of the Sun and Moon as multiples of the Earth's diameter. This was a brilliant demonstration of the ability of mathematics to master cosmic dimensions, but hardly the technique of a practising astronomer. In the _Almagest_ Ptolemy uses parallax observations to derive the distance of the Moon, and from this he calibrates the planetary system set out in the _Planetary Hypotheses_. By an extraordinary coincidence, the distance of the Sun then turns out to be close to that he derives by a totally independent method in the _Almagest_, which does nothing to diminish Ptolemy's confidence in the validity of his system. The fixed stars prove to lie at some twenty thousand Earth radii, or well over one hundred million kilometres, so that the Ptolemaic universe is not small.

For fourteen hundred years, Ptolemy's _Almagest_ was without rival, until the invention of printing in the fifteenth century gave a boost to the study of technical astronomy. Copernicus, working the early sixteenth century, saw in the _Almagest_ an extraordinary mathematical achievement that was capable of revision to an acceptable standard of accuracy, but was nevertheless defective in important ways. In particular, it treated each planet independently and on an ad hoc basis, giving the same period of one year to Sun, Venus and Mercury, and leaving even the very order of the planets open to argument. He realised that if the Earth is considered a typical planet, and the other planets are observed from this moving platform, then the true periods of the planets and their distances from the Sun can be established and these two sets of numbers shown to fall into a single harmonious pattern. The division of the planets into those always near the Sun and those not so is no longer surprising and puzzling but actually to be expected. And the curious retrograde motions of the planets, and when and for how long they occur, are all explained, once one accepts that we observe them from a moving platform. All this he shows in Book I of _De revolutionibus_, and no doubt Plato would have been astonished but delighted with this response to his challenge, with the true harmony of the universe revealed.

Then, in the remaining books of De revolutionibus, Copernicus shows how the heliocentric approach can be made the basis of a technical astronomy more accurate than Ptolemy's, with numerous epicycles and eccentres but without the illegal equant.

De revolutionibus is clearly a mathematical treatment of the apparent motions of the planets in the tradition of the Almagest, though free of some of the defects that had made the Almagest itself unacceptable to the natural philosophers. Unlike its predecessor, it offers a heliocentric cosmology, and even some hints on the physics of the Earth's motion. It is, one might say, as far as Greek astronomy could go. Within two generations came the obsessive search of Tycho Brahe for observational accuracy; the telescope of Galileo, and his theory of uniform motion as a state similar to the state of rest; and the dynamical astronomy of Kepler with its consequent emphasis on the actual orbit of a planet rather than on possible components from which the orbit might be constructed. Between Copernicus and Kepler is a greater intellectual gap than between Ptolemy and Copernicus. Greek astronomy, culminating in Ptolemy and Copernicus, had shown the power of mathematical models in explaining and predicting the motions of the planets, and had indicated in De revolutionibus the need for a new physics in which the motion of the Earth was no longer absurd. Greek astronomy by itself was not a sufficient condition for the development of modern science; but in historical terms it was a necessary one.