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COMET HALLEY AND NONGRAVITATIONAL FORCES

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The motion of Comet Halley is investigated over the 1607-1911 interval. The required nongravitational force model was found to be most consistent with a rocket-type thrust from the vaporization of water-ice in the comet's nucleus. The nongravitational effects are time-independent over the investigated interval.

For the majority of short-period comets, their motions are effected by substantial nongravitational perturbations. By assuming that the nongravitational accelerations are due to the rocket effect of outgassing volatiles from an icy-conglomerate nucleus (Whipple 1950), these nongravitational accelerations have been successfully modeled by Marsden *et al.* (1973). In the orbit computation procedure, the cometary equations of motion are written

$$\frac{d^2\vec{r}}{dt^2} = -\mu \frac{\vec{r}}{r^3} + \frac{\partial R}{\partial \vec{r}} + A_1 \; g(r) \; \hat{r} + A_2 \; g(r) \; \hat{T}$$

where

$$g(r) = \alpha\left(\frac{r}{ro}\right)^{-m}\left[1 + \left(\frac{r}{ro}\right)^{n}\right]^{-k} .$$

The mathematical form of the nongravitational acceleration term g(r) represents an empirical fit to a theoretical plot of water snow vaporization flux versus heliocentric distance. The scale distance r_o is the heliocentric distance when reradiation of solar energy begins to dominate the use of this energy for vaporizing the comet's nuclear ices. For water ice r_o = 2.808 a.u. and the normalizing constant α = 0.111262. The acceleration is given in astronomical units per (40 ephemeris days)^2, μ is the product of the gravitational constant and the solar mass, while R is the planetary disturbing function. The exponents m, n, k equal 2.15, 5.093 and 4.6142 respectively. The nongravitational acceleration is represented by a radial, A_1 g(r) and a transverse A_2 g(r) term in the equations of motion. The radial unit vector ($\hat{r}$) is defined outward along the radius vector while the transverse unit vector ($\hat{T}$) is directed normal to $\hat{r}$, in the orbit plane and in the direction of the comet's motion. An acceleration component normal to the orbit plane was found to have a negligible effect upon the orbital motion of Comet Halley - a result that is consistent with the re-

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sults of other short period comets. Using 885 observations over the 1607-1911 interval, the parameters A_1 and A_2 as well as the six initial conditions can be solved for in the least squares differential correction procedure. Integrated variational orbits were used to form the necessary partial derivatives and the employed numerical integrator was a ninth-order predictor - corrector scheme running at a constant 1/2 day step size. At each step, the required planetary coordinates, from all nine planets, have been read from magnetic tape. All computations were performed in double precision (18 significant figures) on UNIVAC 1108 computers.

While the nongravitational acceleration term g(r) was originally established for water ice, Marsden *et al.* (1973) have shown that if the Bond albedo in the visible range is assumed equal to the infrared albedo of the cometary ices, then the scale distance r_o is inversely proportional to the square of the vaporization heat of the volatile substance. The vaporization heat (L) in calories per mole is related to r_o by

$$r_o \simeq \frac{4.0 \times 10^8}{L^2} \text{ a.u.}$$

In an effort to determine the optimum value of r_o, several differential corrections were successfully computed using the observations from the 1835 and 1910 returns. For this test, only the six initial conditions were solved for in the differential corrections. With r_o fixed at a particular value and A_1 set equal to zero, A_2 was empirically adjusted until an orbit was found that would fit the 1909-11 and 1835-36 observations and also successfully predict the 1759 perihelion passage time to within 0.1 day. For each value of r_o, A_2 was adjusted until the resulting orbit could predict the 1759 perihelion passage time to within 0.1 day. The resulting mean residual of the 1835-1911 observations was taken as a measure of success for the input value of r_o. Table I demonstrates that r_o = 2.808 results in the lowest mean residual. Hence it appears that the outgassing responsible for Comet Halley's nongravitational effects is most consistent with the vaporization of *water* ice.

Table II represents three successful solutions for the six initial conditions and the two nongravitational parameters A_1 and A_2. In all three cases r_o was set equal to 2.808. The probable errors associated with A_1 and A_2 indicate

TABLE I

DIFFERENTIAL CORRECTIONS (6 PARAMETER SOLUTIONS)

USING 1835-1911 OBSERVATIONS

Input Quantities			Mean Residual
r_o	A_1	$A_2 \times 10^8$	
1.6	0	0.0130	12''122
2.808	0	0.0159	6''756
3.5	0	0.0157	6''765
4.0	0	0.0155	6''805
5.0	0	0.0151	6''984
6.0	0	0.0150	6''995

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TABLE II

DIFFERENTIAL CORRECTIONS (8 PARAMETER SOLUTIONS)

Observation Interval	No. of Obsn.	Mean Residual	$A_1 \times 10^8$	$A_2 \times 10^8$
1911-1759	863	11″0	0.2799±0.0346	0.0159±0.0000
1836-1682	437	19″1	1.1746±0.0796	0.0150±0.0000
1759-1607	161	48″6	0.2767±0.0710	0.0150±0.0000

that A_2 is determined far better than A_1. The ratio A_2/A_1 indicates a small lag angle (< 4^o) between the subsolar meridian and the direction of maximum mass ejection. The lack of a secular trend in the nongravitational acceleration is evident from the constancy of A_2 over the 1607-1911 interval. A more complete discussion of these results is in preparation.

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DISCUSSION

DELSEMME: I had pointed out (1972) that the dependence on distance of the non-gravitational force of Comet P/Schwassmann-Wachmann 2 corresponds closely to the law for sublimation of water. Following my suggestion, Marsden et al. *(1973) had shown that this was also true for all those short-period comets whose non-gravitational effects are sufficiently well known. I am glad that Yeomans has now been able to demonstrate that this result can be extended to Comet Halley.*

I want first to give another argument confirming this deduction. The brightness law of any comet is traditionally represented by $B_r = B_o r^{-n}$ *where* B_r *is the reduced brightness at heliocentric distance r* (reduced, *means re-computed as if the comet were always at* $\Delta = 1$ *AU from the earth).* B_o *is the absolute brightness* ($\Delta = r = 1$ *AU) and an average n can be computed over the observed range of the comet. For Comet Halley (1910 II) Vsekhsvyatskii (1958) gives n = 5.4, and the range extends from r = 0.6 AU to r = 4 AU. In the vaporization theory, n varies with the heliocentric distance but an average* <n> *can be computed for this range. Here are the results:*

Sublimation controlled by:	H_2O	CO_2	*CO*	CH_4	*Halley*
<n> (from 0.6 to 4.0 AU)	*5.5*	*2.2*	*2.0*	*2.0*	*5.4*

Now, we have therefore a cross confirmation that the sublimation of Comet Halley is indeed also controlled by water. A most general conclusion seems to emerge: statistically, all short-period comets loose all gases more volatile than water very fast, - at least from the outer layers of the nucleus that are heated enough to participate to the sublimations; therefore the most pristine objects, that have not lost the most volatile fraction at least of their outer

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layers, must be found in new or long period comets, like Comet Bennett, which rather seems to vaporize CO_2 (Delsemme and Combi 1977). This is at variance with the proposals that reverse the sense of evolution of comets (Vsekhsvyatskii, Lyttleton, Alfven). It makes sense that the arrow of time does not appear from studies in celestial mechanics,but it does clearly appear here, where entropy is at work.

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