

Apart from a few basic results the development differs significantly from that of standard texts dealing with these topics. For a connected Riemannian manifold M the concept of the interior metric $\rho(p, q)$ is introduced almost immediately as follows: for any two points $p, q \in M$ one defines $\rho(p, q)$ as the greatest lower bound of the lengths of all piecewise smooth curves joining p, q ; thus $\rho(p, q)$ transforms M into a metric space. This gives rise to the theory of normal convex neighbourhoods. Results such as the Hopf-Rinow theorem, or the following statements, are typical of these chapters: For any connected Riemannian space M the following properties are equivalent: (1) the space M is complete in the sense that each maximal geodesic on it is defined on the entire real axis [here a geodesic is an autoparallel curve with respect to the usual connection; it is maximal if it is not the restriction of any geodesic defined on a larger interval on the real axis]; (2) the space M is complete with respect to the interior metric ρ (i. e., every fundamental sequence converges to a point of M); (3) every closed subset of M which is bounded in respect of the metric ρ is compact.

The variational properties of geodesics [with fixed end-points (\bar{p}, \bar{q})] are then discussed. This includes a treatment of conjugate points, indices of points and intervals and their evaluation by means of the quadratic forms of Morse and Bott, so that some of the basic tools of Morse theory are automatically included. These results are generalized to the case in which \bar{q} is replaced by some submanifold $N \subset M$, the geodesics concerned being orthogonal to N , which gives rise to a corresponding theory of focal points and their indices. The book concludes with the reduction theorem of Bott.

Because of some unorthodox features of the book one should hesitate before recommending it to a beginner; however, to the specialist interested in differential geometry or in the calculus of variations this work may well prove indispensable.

H. Rund, University of the Witwatersrand, Johannesburg

Multiple integrals in the calculus of variations by C. B. Morrey. Springer-Verlag, New York, 1966. ix + 506 pages.

The theme of this book is, in the author's own words, "the existence and differentiability of the solutions of variational problems involving multiple integrals". Ever since Riemann formulated Dirichlet's principle over a hundred years ago, the connection between the calculus of variations and the analytical theory of differential equations has been an important strand in the fabric of analysis. Several topics of recent emergence in the theory of elliptic differential equations and systems, both linear and non-linear, are assembled and discussed in detail in this volume.

The specific topics are as follows: After introductory chapters on the classical calculus of variations and on the properties of harmonic functions and related inequalities, the author discusses the Dirichlet problem in the context of the classes H_p^m and $H_{p_0}^m$ of L^p functions having distributional derivatives of order m , and introduces quasi-potentials. He then discusses existence theorems for variational quasi-linear elliptic systems using lower semi-continuity results of Serrin. There follows a long chapter on the differentiability of weak solutions, including the De Giorgi - Nash - Moser results and the theory of Ladyshenskaya and Uraltseva. Next the theory of coercive boundary value problems for general elliptic systems is studied, with special reference to differentiability and analyticity for non-linear systems. The author then devotes two chapters to the theory of harmonic integrals, in the real and complex domains respectively. In the real case, the variational techniques of Morrey and Eells are applied to compact manifolds and manifolds with boundary. In the complex case, a study is made of the work of Kohn on the $\bar{\partial}$ -Neumann problem. There follows a chapter on parametric integrals (integrals invariant under diffeomorphisms) with reference to Plateau's problem in the two dimensional case. The volume concludes with a chapter on recent work on higher dimensional plateau problems.

As the foregoing summary suggests, this book is a work of pure mathematics, with primary emphasis upon analytical techniques. A prospective reader should know the elements of functional analysis and elliptic differential equations, and should be prepared for frequent references to other sources which the author must necessarily make on account of limitations of space. The book is carefully written at a high level of technique and sophistication. It will undoubtedly be a most valuable source of information and technique for both students and research workers.

G.F.D. Duff, University of Toronto

Introduction to measure and probability by J.F.C. Kingman and S.J. Taylor. Cambridge, England, 1966. x + 401 pages.

This book falls naturally into two parts, though no formal division is made. The first nine chapters give a reasonably complete and self-contained account of the theory of measure and integration. The remaining six chapters introduce the concepts of probability and apply some of the measure theory in this field.

Both parts of the book are thorough and wide-ranging. The treatment is clear, occasionally over-condensed but always elegant and compact. It is so self-contained that there is even a chapter on set theory and a chapter on point set topology. Surely all readers mature enough to deal with the later parts of the book must inevitably have absorbed the basic concept of set theory a long time before. The same