

# Study accretion and ejection using a new GPU-accelerated GRMHD code

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**Abstract.** We study disks and jets in various accretion states (SANE and MAD) using novel, GPU-accelerated general-relativistic magneto-hydrodynamic (GR-MHD) code which we developed, based on HARM. This code, written in CUDA-c and uses OpenMP to parallelize multi-GPU setups, allows high resolution simulations of accretion disks and the formation and structure of jets without the need of multi-node supercomputer infrastructure. A  $256^3$  simulation is well within the reach of an Nvidia DGX-V100 server, with the computation being a factor about 100 times faster if only the CPU was used.

We use this code to examine several disk structures, wind and jet properties in the MAD and SANE states. In the MAD state, we find that the magnetic flux threading the horizon mostly depends on the spin of the BH. This implies that the jet structure and power are strong functions of the spin, with non-spinning BHs have the widest jets.

**Keywords.** Accretion, Black hole physics, Computational methods, GPU computing, Magneto-hydrodynamical simulations

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## 1. Introduction

Accretion disks around compact objects are central to many astronomical objects of interest, including active galactic nuclei (AGNs), X-ray-emitting binaries (XRBs), and even gamma-ray bursts (GRBs). Theoretically, several types of disks have been identified, whose structures mainly depend on (i) the mass accretion rate and the resulting optical depth and on (ii) the configuration of the magnetic field inside the disk (for reviews, see, e.g., [Narayan & McClintock 2008](#); [Abramowicz & Fragile 2013](#)).

Two main types of steady state disk configurations are commonly identified in the literature. They differ by the role of magnetic field in enabling and regulating the accretion. The first is magnetically arrested disks (MAD; see [Narayan et al. 2003](#)). In the MAD state, magnetic field lines are accumulated near the horizon, and the resulting magnetic pressure regulates the accretion rate by delaying the accreting material. As a consequence of the strong magnetic field close to the black hole, in the inner parts of the accretion disk, the infalling gas accretes in filaments ([Foucart et al. 2017](#); [Wong et al. 2021](#)). This configuration is often accompanied by ejection of matter in the form of strong, relativistic jets. The second type of disk configuration is termed “standard and normal evolution (SANE; [Narayan et al. 2012](#)). In this configuration, the magnetic flux threading the horizon is relatively weak, enabling smooth accretion. Magnetic fields therefore contribute mainly to the transport of angular momentum to larger radii. While relativistic jets are observed, they are typically weaker than those seen in MAD state.

In the past two decades, the rapid advance in parallel computation enables to study these complex systems with more and more advanced global numerical simulations. Such simulations are used for time-dependent studies of these complex accreting (and ejecting)

dynamical systems in full general relativity (e.g., using the Kerr metric), shedding light on the magnetohydrodynamical processes in the close vicinity of a black hole (see, e.g., Mizuno 2022 for a recent review). Over the years, several codes solving the general relativistic magnetohydrodynamic (hereinafter GRMHD) equations have been developed (see, e.g., De Villiers et al. 2003; Gammie et al. 2003; Del Zanna et al. 2007; Stone et al. 2008; Porth et al. 2017).

Clearly, such simulations, despite their great success, are very time-consuming and computationally expensive, necessitating access to high-performance computing (HPC) facilities. This naturally provides a physical limitation on the ability to perform such calculations. In the past decade, HPC has seen the development of accelerators (e.g., Giles & Reguly 2014) with the generalization of graphics processing units (GPUs) on computing nodes. Although development of applications for GPUs is a very time-consuming task, it is also very rewarding: time-explicit grid-based MHD simulations can effectively run on GPU accelerators, with a large time gain (typically more than an order of magnitude) compared to their CPU counterparts. Today, we are aware of three general-relativistic codes aimed at studying accretion disks with GPU capabilities: grim (Chandra et al. 2017), H-AMR (Liska et al. 2018; Chatterjee et al. 2019), which is based on the hierarchical use of MPI, OpenMP, and CUDA and K-ATHENA (Grete et al. 2021). These codes are not publicly available.

## 2. cuHARM: an overview

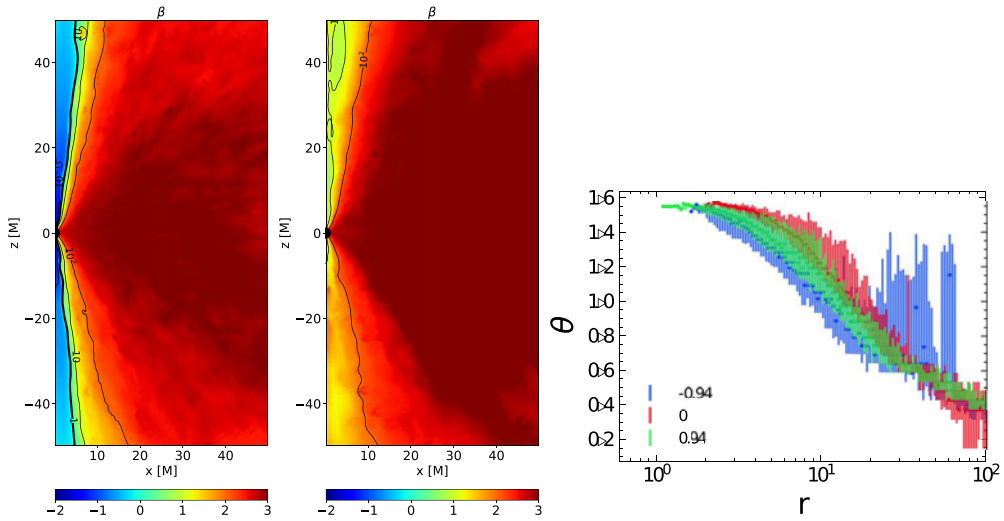
In the past few years we developed a new, GPU-based numerical solver for GRMHD equations in Kerr metric. This code is written in cuda-C, and is based, in many ways, on the publicly available code HARM (Gammie et al. 2003; Noble et al. 2006). We therefore call it cuHARM (cuda-HARM; Bégué et al. 2023). In its current form, cuHARM is designed to run on a single multi-GPU workstation by splitting the volume into equal shares between each GPU. This already allows us to run 3D simulations with resolution of  $\sim 192^3$ , taking about 72 hr to reach  $t = 10^4$  M on an Nvidia DGX- 8xV100 server.

cuHARM solves the ideal, non-resistive GRMHD equations. This system of equations describes the motion of ideal magnetized plasma in arbitrary space-time. The equations that are being solved are the mass, energy, and momentum conservation as well as the homogeneous Maxwell's equations. Here we assume stationary space-time (Kerr metric). These equations characterize the time and space evolution of the gas properties: density,  $\rho$ , internal energy density  $u$ , four-velocity,  $u^\mu$ , gas pressure  $p_g$ , entropy,  $S$ , and magnetic field.

These equations are solved using the finite volume methodology, similar to the original HARM algorithm. We assume that the value of each quantity is uniform within the entire volume of each cell, and calculate the flux of quantities (such as matter, momentum and energy) at the cell's boundaries. The values of the quantities are reconstructed at the boundaries using piecewise linear interpolation method (PLM). In order to satisfy the solenoid condition ( $\vec{\nabla} \cdot \vec{B} = 0$ ), we implement the constrained transport algorithm using the “flux-CT” method of Tóth (2000) and Gammie et al. (2003).

The code is highly optimized, reaching computational efficiency of  $\sim 20\%$ . With modern GPU workstation we use (Nvidia DGX-V100 server, with 8 V100 GPUs, each with dual Nvlink-2 setup), it updates  $\approx 1.5 \times 10^8$  cells per second per card. This results in a typical run time of 2-10 days for resolution  $256 \times 256 \times 128$ .

**Coordinate system.** In the results presented below, All the runs were performed using modified, horizon-penetrating Kerr-Schild coordinate system. The grid was refined towards the equator, and cylindrical close to the polar axes (Tchekhovskoy et al. 2011). This configuration allows a for a faster computation by reducing the Courant condition on the time step. Full description of the code appears in Bégué et al. (2023).



**Figure 1.** Left. Results of disk/jet in SANE state, showing that when the magnetic flux threading the horizon (left) is high, a jet is formed, while when it drops below a critical value (middle) the polar region is filled with matter, and the jet disappear (from Bégué et al. 2023). Right. The structure of the jet ( $r - \theta$ ) in the MAD state shows widest jet (red) for non-spinning BH. The error bars represent the fluctuations. Figure from Zhang et. al. (in prep.).

### 3. First results

We performed several runs, both in the MAD and the SANE states. In all runs, we started with initial torus in hydrostatic equilibrium, as described by Fishbone & Moncrief (1976). In this solution, the density scale is not specified, and we normalize it at its maximum value to  $\rho = 1$  (McKinney & Gammie 2004). The steady state configuration of the accretion flow (SANE or MAD) is determined by the initial magnetic field configuration, which is set by specifying the magnetic vector potential,  $A_\phi$  (this automatically ensures  $\vec{\nabla} \cdot \vec{B} = 0$ ). Furthermore, the initial intensity of the magnetic field is determined by specifying the maximum value of the gas to magnetic field pressure ratio,  $\beta_0 \equiv p_{g,\max}/p_{b,\max} \gg 1$ . The initially sub-dominant magnetic field is amplified by the development of magneto-rotational instability (MRI) which enables transport of angular momentum through the disk, and accretion of matter into the black hole.

For runs in the SANE configuration, we find that the accretion rate nearly only depends on the initial mass of the disk. An empirical fit gives  $\log \dot{M} = 0.84 \log M - 4.1$ , demonstrating that the initial magnetic field and the adiabatic index do not substantially influence the mass accretion rate for the SANE/ RIAF disks considered. The initial magnetization parameter has no significant effect on the accretion rate, with the general trend that the accretion rate weakly increases with  $\beta_0$ . We find that the larger the initial  $\beta_0$ , the faster the spread of the disk. This is to be expected as the Maxwell viscous coefficient scales as  $\propto b^r b^\phi$ , namely with the magnetic field components. Therefore, larger  $\beta_0$  leads to a slower initial disk spreading causing a slight increase in the accretion rate. As for the jet production, we find that once the magnetic flux below the horizon drops below a critical value (MAD parameter  $\langle \Phi_B(t) \rangle < 1.5$ ), the magnetic flux threading the black hole is too weak to support the launch of a jet, which disappears. When this occurs, the polar region is filled with low to average density plasma (see Figure 1, left and middle).

Disks in the MAD state are characterized by strong, relativistic jets. For such disks, we explored the dependence of the jet properties on the spin of the central black hole (BH). We found that prograde disks produce wider jets than retrograde disks, and that

the widest jets occur for non-spinning BH ( $a = 0$ ; see Figure 1, right). We found that the magnetic pressure dominates the gas pressure in the inner disk regions,  $r \lesssim 3r_g$ . As the jet boundaries depend on the disk pressure distribution, the magnetic field in the innermost regions is the key to understanding the jet geometry. While the  $B_\phi$  component is the dominant component for most disk configuration, it is found to be negligible compared to  $B_r$  at small radii for  $a = 0$ , which could be associated with the widest jets.

#### 4. Conclusions

We introduce a new, GPU-based global GR-MHD simulation which we call cuHARM. The use of GPUs enable it to speed the calculation by a factor  $\gtrsim 20$  relative to similar, CPU-based codes, thereby enabling us both to increase the resolution, as well as explore the evolution of the disk/jet system in a larger parameter space region than previously. Currently, we presented results obtained for disks in the SANE and MAD states. Full description appears in Bégué et al. (2023), as well as in Zhang et. al. (in prep.). As this code is now fully developed, more results are expected in the nearby future.

#### References

- Abramowicz, M. A. & Fragile, P. C. 2013, Foundations of Black Hole Accretion Disk Theory. *Living Reviews in Relativity*, 16, 1.
- Bégué, D., Pe'er, A., Zhang, G. Q., Zhang, B. B., & Pevzner, B. 2023, cuHARM: A New GPU-accelerated GRMHD Code and Its Application to ADAF Disks. *ApJS*, 264(2), 32.
- Chandra, M., Foucart, F., & Gammie, C. F. 2017, grim: A Flexible, Conservative Scheme for Relativistic Fluid Theories. *ApJ*, 837(1), 92.
- Chatterjee, K., Liska, M., Tchekhovskoy, A., & Markoff, S. B. 2019, Accelerating AGN jets to parsec scales using general relativistic MHD simulations. *MNRAS*, 490(2), 2200–2218.
- De Villiers, J.-P., Hawley, J. F., & Krolik, J. H. 2003, Magnetically Driven Accretion Flows in the Kerr Metric. I. Models and Overall Structure. *ApJ*, 599(2), 1238–1253.
- Del Zanna, L., Zanotti, O., Bucciantini, N., & Londrillo, P. 2007, ECHO: a Eulerian conservative high-order scheme for general relativistic magnetohydrodynamics and magnetodynamics. *A&A*, 473(1), 11–30.
- Fishbone, L. G. & Moncrief, V. 1976, Relativistic fluid disks in orbit around Kerr black holes. *ApJ*, 207, 962–976.
- Foucart, F., Chandra, M., Gammie, C. F., Quataert, E., & Tchekhovskoy, A. 2017, How important is non-ideal physics in simulations of sub-Eddington accretion on to spinning black holes? *MNRAS*, 470(2), 2240–2252.
- Gammie, C. F., McKinney, J. C., & Tóth, G. 2003, HARM: A Numerical Scheme for General Relativistic Magnetohydrodynamics. *ApJ*, 589, 444–457.
- Giles, M. B. & Reguly, I. 2014, Trends in high-performance computing for engineering calculations. *Philosophical Transactions of the Royal Society of London Series A*, 372(2022), 20130319–20130319.
- Grete, P., Glines, F. W., & O’Shea, B. W. 2021, K-Athena: a performance portable structured grid finite volume magnetohydrodynamics code. *IEEE Transactions on Parallel and Distributed Systems*, 32(1), 85–907.
- Liska, M., Hesp, C., Tchekhovskoy, A., Ingram, A., van der Klis, M., & Markoff, S. 2018, Formation of precessing jets by tilted black hole discs in 3D general relativistic MHD simulations. *MNRAS*, 474(1), L81–L85.
- McKinney, J. C. & Gammie, C. F. 2004, A Measurement of the Electromagnetic Luminosity of a Kerr Black Hole. *ApJ*, 611, 977–995.
- Mizuno, Y. 2022, GRMHD Simulations and Modeling for Jet Formation and Acceleration Region in AGNs. *Universe*, 8(2), 85.
- Narayan, R., Igumenshchev, I. V., & Abramowicz, M. A. 2003, Magnetically Arrested Disk: an Energetically Efficient Accretion Flow. *PASJ*, 55, L69–L72.

- Narayan, R. & McClintock, J. E. 2008, Advection-dominated accretion and the black hole event horizon. *NewAR*, 51(10-12), 733–751.
- Narayan, R., Śądowski, A., Penna, R. F., & Kulkarni, A. K. 2012, GRMHD simulations of magnetized advection-dominated accretion on a non-spinning black hole: role of outflows. *MNRAS*, 426, 3241–3259.
- Noble, S. C., Gammie, C. F., McKinney, J. C., & Del Zanna, L. 2006, Primitive Variable Solvers for Conservative General Relativistic Magnetohydrodynamics. *ApJ*, 641, 626–637.
- Porth, O., Olivares, H., Mizuno, Y., Younsi, Z., Rezzolla, L., Moscibrodzka, M., Falcke, H., & Kramer, M. 2017, The black hole accretion code. *Computational Astrophysics and Cosmology*, 4(1), 1.
- Stone, J. M., Gardiner, T. A., Teuben, P., Hawley, J. F., & Simon, J. B. 2008, Athena: A New Code for Astrophysical MHD. *ApJS*, 178(1), 137–177.
- Tchekhovskoy, A., Narayan, R., & McKinney, J. C. 2011, Efficient generation of jets from magnetically arrested accretion on a rapidly spinning black hole. *MNRAS*, 418, L79–L83.
- Tóth, G. 2000, The  $\nabla \cdot \mathbf{B}=0$  Constraint in Shock-Capturing Magnetohydrodynamics Codes. *Journal of Computational Physics*, 161(2), 605–652.
- Wong, G. N., Du, Y., Prather, B. S., & Gammie, C. F. 2021, The Jet-disk Boundary Layer in Black Hole Accretion. *ApJ*, 914(1), 55.