

SKEW POLYNOMIAL RINGS AND
SKEW POWER SERIES RINGS

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The contents of this thesis can broadly be divided into the study of

- (i) the skew polynomial ring $R[x, \alpha]$, and
- (ii) the skew power series ring $R[[x, \alpha]]$.

A skew polynomial ring $R[x, \alpha]$, where R is any ring and α is any endomorphism of R , is defined to be the set of finite sums $r_0 + r_1x + \dots + r_nx^n$, where $r_i \in R$ for each i , with addition carried out as with ordinary polynomial rings and with multiplication determined by $x.r = r^\alpha x$. A skew power series ring $R[[x, \alpha]]$ is defined in an analogous way to that of a skew polynomial ring.

The nilpotents and units in $R[x, \alpha]$ are investigated in terms of the underlying ring R , and where R is commutative and α is an automorphism of finite order, a characterization of the nilpotents and units is discovered.

The Jacobson, prime and upper nil radicals of $R[x, \alpha]$ are studied and results are obtained in terms of the appropriate radical of the underlying ring R . A complete description of these radicals is given when R is commutative and α is an automorphism of finite order.

The Hilbert Basis Theorem is generalized to $R[x, \alpha]$ and ascending chain conditions on principal right (left) ideals of $R[x, \alpha]$ are also considered.

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All the automorphisms of $R[x, \alpha]$ which fix the underlying ring R elementwise are determined, and between different skew polynomial rings whose underlying rings are isomorphic, all isomorphisms which preserve the underlying ring isomorphism are determined.

For the skew power series ring $R[[x, \alpha]]$ topological conditions are discovered for the existence of an endomorphism ϕ_β of $R[[x, \alpha]]$ fixing R elementwise and mapping x to $\beta \in R[[x, \alpha]]$. Under these conditions ϕ_β exists and is unique, and further, ϕ_β is an automorphism precisely when it is a surjective endomorphism.