where the product runs over all prime divisors of d and an empty product is taken to be 1.

L. Moser

P 15. (Conjecture) Every point on the perimeter of an ellipse is a vertex of an inscribed triangle of maximum area. There are other closed convex curves with this property, e.g. parallelograms. Is it true that the ellipse is the only closed strictly convex (no proper segments) plane curve with this property [cf. R.P. Bambah, Proc. Nat. Inst. Sci. India, Part A 23 (1957), 540-543]?

H. Helfenstein

SOLUTIONS

P 2. Put $S_k(n) = \sum_{a \leq n} a^k$, $T_k(n) = \sum a^k$, the second sum extending over all $a \leq n$ such that $(a,n) \neq 1$ and a does not divide n. Let $\mu(n)$ denote Moebius' function. Prove that

(i) $$T_k(n) = -\sum_{d|n,\, d>1} (1 + \mu(d) S_k(n/d)),$$

(ii) n divides $S_1(n)$ (that is, n is multiply perfect) if and only if

$$T_1(n) \equiv 1 \pmod n \text{ if n is odd, } (1+n/2) \pmod n \text{ if n is even.}$$

J.C. Hayes and P. Scherk

Solution by the proposers. Let $\varphi_k(n) = \sum_{a \leq n,\,(a,n)=1} a^k$. Then $S_k(n) = \sum_{d|n} \sum_{a \leq n,\, (a,n)=d} a^k = \sum_{d|n} d^k \varphi_k(n/d)$ and by Moebius inversion $\varphi_k(n) = \sum_{d|n} \mu(d) d^k S_k(n/d)$. Now put $\sigma_k(n) = \sum_{d|n} d^k$ Then

(1) $$T_k(n) = S_k(n) - \varphi_k(n) - \sigma_k(n) + 1$$

$$= S_k(n) - \sum_{d|n} \mu(d) d^k S_k(n/d) - \sum_{d|n} d^k + 1.$$

This yields the first assertion $T_k(n) = -\sum_{d|n,\, d>1} d^k(1 + \mu(d)\, S_k(n/d))$.

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