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BOOK REVIEWS

CHATTERS, A. W. and HAJARNAVIS, C. R., *Rings with chain conditions* (Pitman, 1980), £8·00.

In the last decade quite a number of books dealing with non-commutative ring theory have been published. However, much of the material in this book has not previously been published in book form and therefore it is a welcome addition to the literature. It is concerned with certain aspects of the theory of Noetherian rings (i.e. rings which satisfy the ascending chain condition on right ideals and left ideals) though as the title suggests more general rings are also considered.

What aspects of ring theory are considered? A recurring theme is the existence of classical quotient rings Q for rings R and the interplay between R and Q. As well as presenting the early theorems of Goldie and Small an account is given of the recent work of Stafford on Noetherian rings R with $R=Q$. Among other topics discussed are Noetherian rings of Krull dimension 1 (although Krull dimension is not defined in general), Jacobson's conjecture whether the intersection of the finite powers of the Jacobson radical of a Noetherian ring is zero, serial rings, fully bounded rings, hereditary rings and more generally rings with every principal right or left ideal projective, rings with finite global dimension, the Artin Rees property and simple rings. Always the rings satisfy some chain condition on one side or on both the right and left. Many examples (often of 2×2 matrices) are given to illustrate the results and show they can't be generalised and there are a number of open questions raised by the discussion which take the form of remarks at the end of each chapter.

As one might expect the proofs use the techniques developed by Goldie and others in studying quotient rings and in particular uniform and essential modules are prominent. But a feature of the book is the repeated use of reduced rank and the Artinian radical. If R is a semiprime right Noetherian ring, with classical right quotient ring Q (which is semiprime Artinian) and M is a finitely generated right R-module, then the reduced rank is the composition length of the right Q-module $M\otimes_R Q$ and this definition can be extended to the case when R is not semiprime. Reduced rank is used in various ways perhaps the most notable being in the proof of the following generalisation of the Principal Ideal Theorem of commutative algebra: if P is a prime ideal of a right Noetherian ring R minimal over an invertible ideal then rank $P\leqq 1$. The Artinian radical of a Noetherian ring R is the sum of all Artinian right ideals of R and so is an ideal and is Artinian as a right R-module. The other striking feature of the proofs is the remarkable facility with element manipulations displayed throughout. Indeed the book is not only a tribute to the "Leeds school" of ring theory but also an expression of the "Leeds style" combining clarity, elegance and ingenuity.

This book is called a research note and as such is aimed in the first instance at experts in the field. What about other readers? If the book has a weakness it is that the style is a little terse in parts. For example, the short slick proofs of Goldie's theorem don't give as much insight as the much longer original one. Also in at least two points it is assumed that the reader knows that a non-zero ideal in a prime ring with polynomial identity contains a non-zero central element. But these are minor quibbles and potential readers should not be unduly alarmed! Anyone interested in non-commutative Noetherian rings will find this very readable and attractive book worth consulting.

P. F. SMITH

STROMBERG, KARL R., *An Introduction to Classical Real Analysis* (Wadsworth International Group, 1981), 576 pp., cloth, U.S. $29.95.

This book includes not only the material usually presented in a first rigorous course in Analysis but also much that will interest the mature mathematician.