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From Ideal Worlds to Ideality

ABSTRACT: *In common treatments of deontic logic, the obligatory is what is true in all deontically ideal possible worlds. In this article, I offer a new semantics for Standard Deontic Logic with Leibnizian intensions rather than possible worlds. Even though the new semantics furnishes models that resemble Venn diagrams, the semantics captures the strong soundness and completeness of Standard Deontic Logic. Since, unlike possible worlds, many Leibnizian intensions are not maximally consistent entities, we can amend the semantics to invalidate the inference rule which ensures that all tautologies are obligatory. I sketch this amended semantics to show how it invalidates the rule in a new way.*

KEYWORD: meaning

Introduction

Possible worlds now enjoy a comfortable monopoly on our modal theorizing. Yet this monopoly was by no means inevitable. We can easily imagine scenarios in which Saul Kripke and others chose different vocations, and, as a result, possible worlds fell into less able hands after idling on the shelf for much longer. But I do not simply mean that possible worlds might have been less influential than they currently are. I mean something more controversial—that something other than possible worlds might have enjoyed a sizable portion of their current influence.

More specifically, I believe that certain resources buried in Leibniz's writings might have provided enough formal flexibility and explanatory power to compete with possible worlds. I will not defend that belief here, since doing so would require, in roughly equal measure, the vast amounts of time and talent devoted to possible worlds. I do hope to mark a tally on its behalf, however. In Warmke (2015), I use these Leibnizian resources in a new semantics for modal logic under a metaphysical interpretation. But I've not yet shown that they can model other kinds of modal discourse. Here, I show that the Leibnizian resources plausibly capture the modal logic of obligation.

The Leibnizian resources consist of primitively intensional entities with a mereological structure. They are primitively intensional in the sense that (i) they are not further defined over possibilia as sets or functions, and (ii) the mereological structure of a property does not reduce to subset or subclass relations among classes of possibilia. In this article, I use them in a new semantics for Standard Deontic Logic whose models resemble Venn diagrams. The semantics says that $\ulcorner \bigcirc \phi \urcorner$ ('it is obligatory that ϕ') is true when the property of being such that ϕ is part of being a deontically ideal world.

I shared parts of this material with students in my spring 2018 seminar on deontic logic, as well as audiences at Bowling Green State University and Northwestern University. Thanks to those in attendance, especially Fabrizio Cariani, Christian Coons, Molly Gardner, Michael Glanzberg, Joe Glover, John Hill, Ben Keoseyan, Brandon Warmke, Michael Weber, and others I've regrettably forgotten. Finally, I'm grateful for constructive feedback from the journal's referees.

In section 1, I briefly cover Standard Deontic Logic and its standard semantics. Then, after I explain my semantic theory's Leibnizian pedigree in section 2, I present the semantics in sections 3 through 6. In Appendix I, I show that its account of logical consequence is extensionally equivalent to the account of logical consequence in the standard semantics. Finally, in Appendix II, I sketch a variation of the semantics to invalidate the inference rule that ensures all tautologies are obligatory.

1 The Standard Picture

Before we cover the new semantics, it will be helpful to revisit Standard Deontic Logic, its language, and the standard semantics. We'll begin with the language.

1.1 Language

The language of Standard Deontic Logic (hereafter, 'SDL') adds the propositional operator $\bigcirc$ to the language of classical propositional logic, whose basic logical symbols include negation ($\neg$) and the material conditional ($\supset$) as well as an infinite stock of basic sentence letters (p_1, p_2, ...). Taking $\neg$, $\supset$, and $\bigcirc$ as the basic symbols, we may define the well-formed formulas of SDL recursively in the usual way. Then, we may introduce the nonbasic symbols of disjunction ($\vee$), conjunction ($\wedge$), and the biconditional ($\equiv$) and define them in terms of our basic symbols in the usual ways. The language of SDL also contains a permission operator P definable in terms of $\neg$ and $\bigcirc$ (i.e., $P\phi =_{def.} \neg \bigcirc \neg\phi$).

To simplify our discussion, I will adopt the obligation reading of $\bigcirc$ ('It is obligatory that…') rather than the ought-to-be reading ('It ought to be that…'). These readings are not equivalent, as Schroeder (2011) and others have shown. Though some prefer the ought-to-be reading for Standard Deontic Logic, nothing consequential hangs on the choice here. Those who prefer the ought-to-be reading may follow Lewis (1973: 100) and stipulate that 'what is obligatory (conditionally or unconditionally) is what ought to be the case, whether or not anyone in particular is obligated to see to it'.

1.2 Logic

I adopt the axiomatization of Standard Deontic Logic in McNamara (2006: 207), with a few cosmetic changes:

TAUT. All tautologous wffs of the language
$\bigcirc$-K. $\bigcirc(\phi \supset \psi) \supset (\bigcirc\phi \supset \bigcirc\psi)$
$\bigcirc$-D. $\bigcirc\phi \supset P\phi$
MP. If $\vdash \phi$ and $\vdash \phi \supset \psi$, then $\vdash \psi$
$\bigcirc$-NEC. If $\vdash \phi$, then $\vdash \bigcirc\phi$

Both TAUT and MP come from classical propositional logic. TAUT grandfathers in as axioms all the tautologies of classical propositional logic. SDL also inherits the main inference rule of classical propositional logic in the form of MP.

The remaining principles govern the $\bigcirc$-operator. $\bigcirc$-K says that if a conditional is obligatory, then if the conditional's antecedent is obligatory, so is its consequent.

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$\bigcirc$-D says that whatever is obligatory is also permissible. Finally, $\bigcirc$-NEC says that if ϕ is a theorem of SDL it is also a theorem that ϕ is obligatory.

1.3 Semantics

According to the standard semantics, $\ulcorner\bigcirc\phi\urcorner$ is true in a possible world w just in case ϕ is true in every world w' such that Rww', where Rww' holds between worlds w and w' when w' is w-acceptable, or acceptable from w's standpoint. Canonical presentations characterize w-acceptability according to the *standard metasemantics*: the w-acceptable worlds are 'deontically perfect' or 'ideal' worlds 'in which all propositions are true which ought to be true in [w]' (Føllesdal and Hilpinen 1970: 17). (Compare Hintikka (1970: 71), Casteñeda (1972: 676).) More recently, Paul McNamara says that a world w' is acceptable to a world w when '[w'] is a world where everything obligatory in [w] holds (i.e., no violations of the obligations holding in [w] occur in [w'])' (2006: 211).

Diving a little deeper, a *model* $\mathcal{M}$ is an ordered triple $\langle$W, R, I$\rangle$, where W is a set of possible worlds and R is the acceptability relation defined over W. In every model, R is *serial*, or such that for all worlds w in W, some w' in W is such that Rww'. In effect, the seriality condition prohibits any model from having a world which accesses no worlds at all. Finally, the interpretation function I assigns truth values to atomic propositions in each world. Then, a *valuation function* $\mathrm{V}_{\mathcal{M}}$ for a model $\mathcal{M} = \langle$W, R, I$\rangle$ uses R and I to assign truth values to every proposition in each world $w \in$ W:

(i) For any atomic sentence ϕ, $\mathrm{V}_{\mathcal{M}}(\phi, w) = 1$ iff $\mathrm{I}(\phi, w) = 1$.
(ii) $\mathrm{V}_{\mathcal{M}}(\neg\phi, w) = 1$ iff $\mathrm{V}_{\mathcal{M}}(\phi, w) = 0$.
(iii) $\mathrm{V}_{\mathcal{M}}(\phi \supset \psi, w) = 1$ iff either $\mathrm{V}_{\mathcal{M}}(\phi, w) = 0$ or $\mathrm{V}_{\mathcal{M}}(\psi, w) = 1$.
(iv) $\mathrm{V}_{\mathcal{M}}(\bigcirc\phi, w) = 1$ iff for all $w' \in \mathrm{W}$ such that $\mathrm{R}w\, w'$, $\mathrm{V}_{\mathcal{M}}(\phi, w') = 1$.

A wff ϕ is *logical consequence* of the wffs in Γ ('$\Gamma \vDash \phi$') when, for any world $w \in$ W in any model $\mathcal{M}$, if every wff ψ in Γ is such that $\mathrm{V}_{\mathcal{M}}(\psi, w) = 1$, then $\mathrm{V}_{\mathcal{M}}(\phi, w) = 1$. ϕ is *valid* when $\mathrm{V}_{\mathcal{M}}(\phi, w) = 1$ for every $w \in$ W in every model $\mathcal{M}$.

SDL is both sound and complete with respect to the standard semantics. In Appendix I, I use these results to show that SDL is sound and complete with respect to my semantics, too. However, these proofs are not trivial. The new semantics does not primarily revolve around possible worlds but around primitively intensional entities with a mereological structure. We cover that structrure next.

2 Leibnizian Intensions

Like Frege and others after him, Leibniz thought meaning had at least two components. (See Lenzen (2004) for discussion and further references.) We can distinguish the two components most clearly in the case of predicates. A predicate's *Leibnizian extension* is the set or class of all possible things that satisfy the predicate. The Leibnizian extension of 'is a dog' is not the set or class of actual dogs but the set or class of all possible dogs. As sets or classes of possibilia, Leibnizian extensions violate the principle of extensionality and therefore qualify as intensional entities by contemporary standards. So Leibnizian extensions differ from extensions now standardly conceived—as sets or classes whose members are actual objects that satisfy a predicate. A predicate's *Leibnizian intension* is an associated divine concept. The Leibnizian intension of 'is a dog', for instance, is the divine concept of being a dog. Importantly, Leibnizian intensions are primitively intensional and do not reduce to Leibnizian extensions or any kind of class or function defined over possibilia.

Leibnizian intensions and extensions provide two ways to treat necessarily true propositions like *all dogs are animals*. On the extensional side, the proposition is true when the class of possible animals contains, i.e., has as a subclass, the class of possible dogs. This treatment should seem familiar to those with backgrounds in contemporary logic. But since Leibniz's intensional treatment may seem foreign to those steeped in contemporary logic, some stage-setting may help familiarize us with it.

Although Leibnizian intensions do not have members or subclasses, many have 'parts'—the concepts that a concept contains. Second, Leibniz seems to endorse the schema that, necessarily, all Fs are Gs if and only if *being G* is part of *being F*. The schema is not a reductive definition for intensional parthood. And, in Warmke (2019), I reject the left-to-right direction. But it does help illustrate the kinds of parthood judgments Leibniz would ordinarily accept. And it remains a useful pedagogical tool since many of its instances are true. For example, all possible squares are possible rectangles (but not vice versa), and all possible dogs are possible animals (but not vice versa). So the schema implies that *being rectangular* is part of *being square* (but not vice versa) and that *being an animal* is part of *being a dog* (but not vice versa). On the intensional side, then, the necessary proposition that all dogs are animals is true when *being an animal* is part of *being a dog*.

Leibniz (1690/1996: 486) noticed that the intensional and extensional treatments run inversely to one another. The extensional treatment says that necessary propositions of the form *all Fs are Gs* are true when the extension of 'G' contains (in the sense of having as a subclass) the extension of 'F'. And the intensional treatment says that the proposition is true when the intension of 'F' contains (in the sense of having as a conceptual part) the intension of 'G'. The diagram below illustrates this inversion in the case of the above dog-animal proposition:

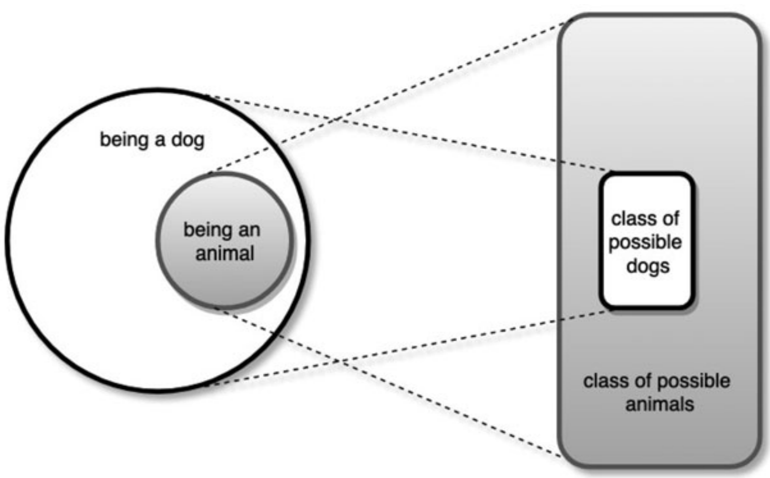

Figure 1. The Leibnizian intensional and extensional inversion.

Leibnizian intensions and extensions behave this way, in part, because an intension's proper parts correspond to the entrance conditions for the intension's own possible extension. So, for example, if *being F*'s proper parts are, at bottom, *being* G_1, *being* G_2, and *being* G_3, then the possible Fs comprise the intersection, not the union, of all the possible G_1s, the G_2s, and the G_3s. On this picture, then, an intension typically has more restrictive entrance conditions for its possible extension than the possible extensions of its proper parts. Generally speaking, the more restrictive the entrance conditions, the fewer things there are that meet them. Since, on Leibniz's picture, *being an animal* is a proper part of *being a dog*, the entrance conditions of the former's extension are less restrictive than the entrance conditions of the latter's extension. As a result, the class of possible animals subsumes and outstrips the class of possible dogs.

These considerations inspire a new semantics for modal logic, in general, and a new semantics for Standard Deontic Logic, in particular. A contemporary Leibnizian could de-theologize the intensional entities involved by replacing divine concepts with properties. Then she could say, first, that the Leibnizian extension of 'is a world' is the class of possible worlds and, second, that its Leibnizian intension is, intuitively, the property whose parts are the properties that any possible world would exemplify if it were actual. Importantly, this is not a definition of the property, but an intuitive characterization of it. We'll call it the *property of being a world in general* (hereafter, $\mathcal{W}$). Of course, properties do not have parts like a table might have spatiotemporal parts. Following Leibniz's view in Parkinson (1966: 135) that an intension's parts are its conjuncts, we may also conceive of $\mathcal{W}$ as a conjunctive propositional property (the property of being such that this, *and* such that this, and so on). So, in what follows, one may substitute 'is a conjunct of' for 'is part of' and 'has as a conjunct' for 'includes (as a part)'.

In the background metaphysics, the propositional property *being such that* ϕ (hereafter, $[\phi]$) is part of $\mathcal{W}$ just in case any possible world w would be such that ϕ if w were actual. Again, this is not a definition of $\mathcal{W}$ but a simple biconditional bridge from the more to the less familiar. Against the intensional backdrop I develop in Warmke (2015; 2019), 'possible worlds' are themselves intensional entities and the commonalities among them owe to their having $\mathcal{W}$ as a part rather than the other way around. So instead of using possible worlds to define the modal notions, we can use $\mathcal{W}$, the property anything must exemplify to be a world in the first place. According to the modal semantics, then, $\ulcorner\Box\phi\urcorner$ is true just in case $[\phi]$ is part of $\mathcal{W}$ (Warmke 2015). We will return to this idea shortly.

Our contemporary Leibnizian could then go on to say that the Leibnizian extension of 'is a deontically ideal world' is the class of deontically ideal possible worlds and that its Leibnizian intension is the property whose parts are the properties that any deontically ideal possible world would exemplify, if it were actual. We'll call this Leibnizian intension the *property of being deontically ideal* (hereafter, $\mathcal{O}$). Now, the standard metasemantics for the possible worlds approach says that a world w' is deontically ideal relative to w when everything that is obligatory in w is true in w'. So the standard metasemantics uses a list of obligations in the metalanguage to define the notion of a deontically ideal world. Fred Feldman (1986: 182) and James Forrester (1996: 132–134) have argued that this move is viciously circular. But the move is not circular. And the Leibnizian can do something similar. With such a list, we can define the property of being a deontically ideal world by ensuring that $\mathcal{O}$'s parts correspond to obligations on the list. Then, instead of saying that $\ulcorner \bigcirc \phi \urcorner$ is true in w when $\ulcorner \phi \urcorner$ is true in all deontically ideal worlds relative to w, we can say that $\ulcorner \bigcirc \phi \urcorner$ is true when $[\phi]$ is part of $\mathcal{O}$. As those familiar with SDL should expect, we will not need to relativize $\mathcal{O}$, the property of being deontically ideal, to different lists of obligations in the metalanguage in order to validate all and only the theorems of SDL.

Ordinarily, we restrict the quantifiers to narrow the possible worlds under consideration from all possible worlds to the subclass of deontically ideal possible worlds. With Leibnizian intensions, we do the inverse. We add properties to the property of being a world in general to form the property of being a deontically ideal world. As a result, the relationship between ideal worlds and all possible worlds, on the one hand, and the relationship between *being a world in general* and *being a deontically ideal world*, on the other, form the familiar Leibnizian pattern:

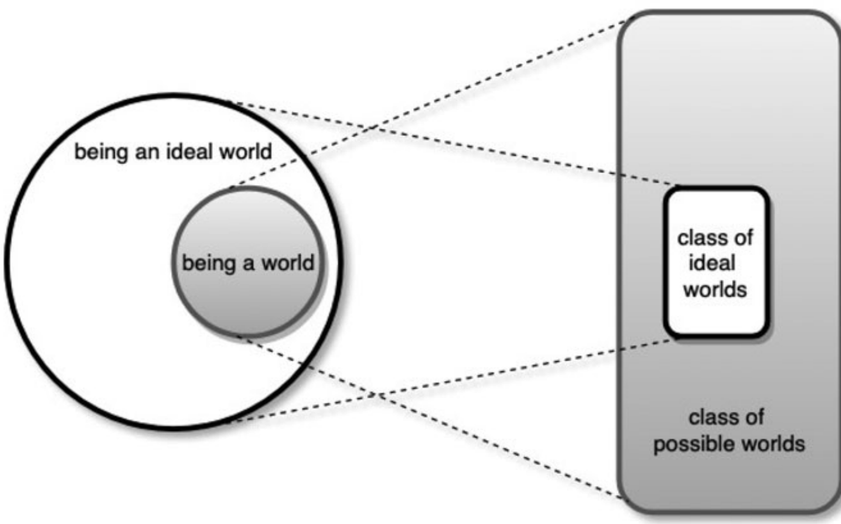

Figure 2. The property addition and quantifier restriction inversion.

This pattern suggests that the sort of flexibility obtained by possible worlds approaches with the tool of quantifier restriction might also be obtained by a Leibnizian approach with the tool of property addition. I cannot assess the suggestion in a single article. But the semantics I provide here does provide some evidence for it.

In Warmke (2015), $\ulcorner \Box \phi \urcorner$ is true when $[\phi]$ is part of $\mathcal{W}$, the property of being a world in general. The model structure in the semantics consists of the ordered triple $\langle \mathcal{A}, \mathcal{W}, \mathcal{P} \rangle$. Where 'Alpha' names the actual world, $\mathcal{A}$ is the property of being Alpha. $\mathcal{W}$ is the property of being a world in general, and $\mathcal{P}$ is a relation of

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property parthood. In the semantics, $\mathcal{A}$'s parts correspond to true (nonmodal) propositions and $\mathcal{W}$'s parts correspond to necessarily true propositions. The pure formalism this semantics permits can be reinterpreted and applied to SDL.

Two small changes help rig the semantics for SDL. First, we reinterpret the property that plays the $\mathcal{W}$-role so that its parts correspond to what is obligatory rather than what is necessary. Second, we configure $\mathcal{P}$ to invalidate formulas which are not theorems of SDL. For example, we will invalidate the deontic analogue of the (T) axiom schema (i.e., $\bigcirc\phi \supset \phi$) to avoid saying that everything which is obligatory is the case. In the resulting model structure $\langle\mathcal{A}, \mathcal{O}, \mathcal{P}\rangle$, $\mathcal{A}$ remains the property of being Alpha. Our new friend, $\mathcal{O}$, is the property of being a deontically ideal world. I'll call it 'the property of being an ideal world' for short. On the intended interpretation, $\mathcal{O}$ has swallowed up $\mathcal{W}$ by taking over its job and its parts.

We may partially represent the model structure in a Venn-like diagram. A model structure that plausibly represents our actual state of affairs would capture the fulfillment of some obligations and the failure of many more. The Venn-like diagram below depicts such a situation:

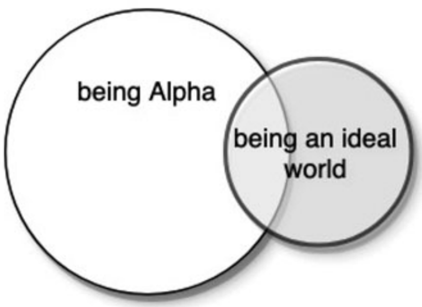

Figure 3. A partially represented model structure.

The diagram has areas for the obligatory and true, the obligatory and false, and the non-obligatory. A propositional property $[\phi]$ is:

(a) in the overlapping space when $\ulcorner\phi\urcorner$ and $\ulcorner\bigcirc\phi\urcorner$ are both true,
(b) in the non-overlapping grey space when $\ulcorner\bigcirc\phi\urcorner$ is true but $\ulcorner\phi\urcorner$ is not, and
(c) in the non-overlapping white space when $\ulcorner\phi\urcorner$ is true but $\ulcorner\bigcirc\phi\urcorner$ is not.

Any model with at least one propositional property in the non-overlapping grey space falsifies the deontic analogue of the (T) axiom schema (i.e., $\bigcirc\phi \supset \phi$) because at least one ϕ is such that $\ulcorner\phi\urcorner$ is false even though $\ulcorner\bigcirc\phi\urcorner$ is true.

In summary, the new semantics for SDL has three main components. $\mathcal{A}$, the property of being Alpha, accounts for nonmodal truths. $\mathcal{O}$, the property of being deontically ideal, accounts for what is obligatory. And $\mathcal{P}$, the parthood relation,

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helps validate all and only the theorems of SDL. Let us examine each of these components more closely.

3 On Being Alpha

$\mathcal{A}$ is a propositional property whose conjuncts or parts capture the actual world's character. It is true that Fred is tall iff *being such that Fred is tall* is part of $\mathcal{A}$. The proposition that Fred is tall is *nonmodal* since it embeds no modal operators. Since $\mathcal{A}$'s parts capture how the world actually is, we may use them to capture a nonmodal proposition's truth:

(A) $\ulcorner\phi\urcorner$ is true when $[\phi]$ is part of $\mathcal{A}$. (For a similar treatment, see Zalta [1993: 410–421].)

We will have to assume as a matter of course that $\mathcal{A}$ has various features for it to play its role in a semantics for SDL. Properties appropriate for the $\mathcal{A}$-role I will call *$\mathcal{A}$-suitable*, and we have two routes to them. We may start with maximal consistent sets and define $\mathcal{A}$-suitable properties as those whose parts correspond to the members of such a set. Or we may say that a property is $\mathcal{A}$-suitable when it satisfies principles analogous to those which define maximally consistent sets. Though nothing important hangs on this choice, I will go with the first and more convenient route.

A proposition ϕ and the propositional property $[\phi]$ each *correspond* to the other. And a set S and property Q *derivatively correspond* when each of S's members corresponds to one of Q's parts and vice versa. I will soon define $\mathcal{A}$-suitable properties as corresponding to maximal consistent sets. And this requires more precise notions of maximality and consistency. A set Γ of propositions is *maximal*^{pc} when it is such that for every nonmodal ϕ, either $\phi \in \Gamma$ or $\neg\phi \in \Gamma$. And a set Γ of propositions is *consistent*^{pc} when no finite subset $\{\phi_1, \ldots, \phi_n\}$ of Γ is such that that $\neg(\phi_1 \wedge \ldots \wedge \phi_n)$ is provable in the propositional calculus. A set is *maximal-consistent*^{pc} when it is both maximal^{pc} and consistent^{pc}.

A property Q is $\mathcal{A}$-suitable when some maximal-consistent^{pc} set S is such that $\phi \in$ S iff $[\phi]$ is part of Q. Given any maximal-consistent^{pc} set S, and given the correspondence between propositions and their propositional properties, there is a set of propositional properties S′ such that $\phi \in$ S iff $[\phi] \in$ S′. The property mereology in Warmke (2015: 320–323) and (2019: 9-11) includes an unrestricted composition principle according to which, for any specifiable set of properties, there is a sum, a property, composed of those properties. Given such a principle, there is a sum, a property, composed of those properties in S′. Thus, for each maximal-consistent^{pc} set S, there is an $\mathcal{A}$-suitable property Q such that $\phi \in$ S iff $[\phi]$ is part of Q. There are no other $\mathcal{A}$-suitable properties. We'll call this *Principle 1*.

$\mathcal{A}$-suitable properties unsurprisingly satisfy principles analogous to those which define maximally consistent sets. Every $\mathcal{A}$-suitable property is *maximal*$_{pc}$, or such that for any nonmodal ϕ, either $[\phi]$ or $[\neg\phi]$ is part of it. And every such property is *consistent*$_{pc}$, or such that there is no finite set of its parts $\{[\phi_1], \ldots, [\phi_n]\}$ such that $\neg(\phi_1 \wedge \ldots \wedge \phi_n)$ is provable in the propositional calculus. So every $\mathcal{A}$-suitable

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property is *maximal-consistent*$_{pc}$, i.e., both maximal$_{pc}$ and consistent$_{pc}$. Since the theorems of the propositional calculus hold in every maximal-consistent^{pc} set, the propositional properties corresponding to those theorems are parts of every $\mathcal{A}$-suitable property. Therefore, given (A), in any model based on any model structure $\langle \mathcal{A}, \mathcal{O}, \mathcal{P} \rangle$, every theorem of the propositional calculus is true. So Principle 1 helps validate TAUT. However, it does not validate TAUT by itself. The validity of some tautologies in SDL, say $\bigcirc p \supset \bigcirc p$, depends on $\mathcal{O}$'s features, too.

Principle 1 also ensures that $\mathcal{A}$-suitable properties satisfy an intuitive conception of the material conditional. Since $\phi \supset \psi$ is a member of a maximal-consistent^{pc} set S iff either $\neg\phi$ or ψ is a member of S, $[\phi \supset \psi]$ is part of an $\mathcal{A}$-suitable property Q iff either $[\neg\phi]$ or $[\psi]$ is part of Q. And so the parts of any $\mathcal{A}$-suitable property behave in accordance with a property analogue of modus ponens. For suppose that $[\phi]$ and $[\phi \supset \psi]$ are both part of an $\mathcal{A}$-suitable property Q. Since $[\phi \supset \psi]$ is part of Q, either $[\neg\phi]$ or $[\psi]$ is part of Q. Because $[\phi]$ is part of Q and Q is consistent$_{pc}$, $[\neg\phi]$ is not part of Q . Therefore, $[\psi]$ is part of Q. So for any nonmodal ϕ and ψ, if $[\phi]$ and $[\phi \supset \psi]$ are both part of Q, so is $[\psi]$.

The $\mathcal{A}$-suitable properties in models that represent the actual world bear important connections to the worlds in the standard semantics for SDL. Given the usual treatment of possible worlds in the standard semantics for SDL as maximal and consistent sets, each maximal-consistent^{pc} set is a subset of a possible world in some model. And each possible world in any model has a maximal-consistent^{pc} set as a subset. So Principle 1 secures a tight connection between the $\mathcal{A}$-suitable properties defined in terms of maximal-consistent^{pc} sets and the possible worlds in the standard semantics.

(1-a) For any model of the new semantics based on any model structure $\langle \mathcal{A}, \mathcal{O}, \mathcal{P} \rangle$, there is a maximal-consistent^{pc} subset S of a world w in some possible worlds model such that $\phi \in$ S iff $[\phi]$ is part of $\mathcal{A}$.

(1-b) For the maximal-consistent^{pc} subset S of any world w in any possible worlds model, there is a model in the new semantics based on some model structure $\langle \mathcal{A}, \mathcal{O}, \mathcal{P} \rangle$ such that $\phi \in$ S iff $[\phi]$ is part of $\mathcal{A}$.

$\mathcal{A}$-suitable properties certainly resemble possible worlds. And if we call them that, as many possible worlds appear across the collection of *all* models in the new semantics as appear in any *single* model of possible worlds semantics. But the point of this essay is not that we can forego worlds entirely. Rather, the point is that as long as they have a certain mereological structure, each individual model for SDL needs at most one, the one assigned to characterize the actual world. We do not need them to account for modal truths.

4 On Being an Ideal World

$\mathcal{O}$ is a propositional property whose conjuncts or parts determine the truth values of $\bigcirc$-statements:

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(O) $\ulcorner \bigcirc\phi \urcorner$ is true when $[\phi]$ is part of $\mathcal{O}$.

(O) and the permission operator P's abbreviation of $\neg \bigcirc \neg\phi$ together supply truth conditions for P-statements:

(P) $\ulcorner \mathsf{P}\phi \urcorner$ is true when $[\neg\phi]$ is not part of $\mathcal{O}$.

I will incorporate these truth conditions into the valuation function in the next section. It is important to note that I will attribute to $\mathcal{O}$ the features necessary to help capture SDL. I myself do not think $\mathcal{O}$ should have all these features, and this is all to the good. The very features I would like to deny of $\mathcal{O}$ correspond to theorems of SDL which would not be theorems in my preferred deontic logic on my preferred reading of the modal operators. Just as the standard semantics for SDL gave rise to further variations, the new semantics here can give rise to further variations. However, unlike the standard semantics, the feature in my semantics that provides the truth conditions for $\bigcirc$-statements does not involve maximally consistent entities. So the semantics gives way to a flexible array of possible variations with surprising results. In appendix II, I sketch a variation which invalidates $\bigcirc$-NEC.

To get the full buffet of $\mathcal{O}$-suitable properties, I once again go the easy route of sets. A set S of propositions is *closed under modus ponens* when it is such that if $\phi \in$ S and $(\phi \supset \psi) \in$ S, then $\psi \in$ S. And a set S of propositions is *closed under necessitation* when it is such that $\bigcirc\phi \in$ S if $\phi \in$ S. The set Λ is closed under monus ponens and necessitation and contains

(PC) All the tautologies of the propositional calculus,

as well as every instance of the schemas

(K) $\bigcirc(\phi \supset \psi) \supset (\bigcirc\phi \supset \bigcirc\psi)$, and
(D) $\bigcirc\phi \supset \mathsf{P}\phi$.

So, by definition, Λ contains all and only the theorems of SDL. An *extension of* Λ is the union of Λ and any (possibly non-empty) set of propositions, again closed under modus ponens, and for which none of its finite subsets $\{\phi_1, \ldots, \phi_n\}$ is such that that $\neg(\phi_1 \wedge \ldots \wedge \phi_n)$ is provable in SDL. We'll call this last feature *consistency*^{sdl}.

A property Q is *$\mathcal{O}$-suitable* when an extension of Λ, S, is such that $\phi \in$ S iff $[\phi]$ is part of Q. Given any extension of Λ, and given the correspondence between propositions and their propositional properties, there is a set of propositional properties S′ such that $\phi \in \Lambda$ iff $[\phi] \in$ S′. Given the background property mereology in Warmke (2015: 320–323) and (2019: 9-11), which includes the aforementioned unrestricted composition principle, there is a sum, a property, composed of those properties in S′. Thus, for each extension S of Λ there is an $\mathcal{O}$-suitable property such that $\phi \in$ S iff $[\phi]$ is part of Q. There are no other $\mathcal{O}$-suitable properties.

Given our definition of $\mathcal{O}$-suitability, every $\mathcal{O}$-suitable property includes a propositional property corresponding to each logical truth of the propositional calculus. And since every extension of Λ is closed under modus ponens, the parts of each $\mathcal{O}$-suitable property collectively obey an analogue of modus ponens. For suppose that $[\phi]$ and $[\phi \supset \psi]$ are both part of an $\mathcal{O}$-suitable property Q. Then ϕ and $\phi \supset \psi$ are members of Q's corresponding extension of Λ, S. Since S is closed under modus ponens, ψ is also a member of S. Therefore, since Q corresponds to S, $[\psi]$ is part of Q. So if $[\phi]$ and $[\phi \supset \psi]$ are parts of some $\mathcal{O}$-suitable property Q, $[\psi]$ is part of Q, too.

Already, we see why the (K) axiom is valid. Suppose for reductio that $\ulcorner \bigcirc(\phi \supset \psi)\urcorner$ is true but $\ulcorner(\bigcirc\phi \supset \bigcirc\psi)\urcorner$ is false. Then, by (O), $[\phi \supset \psi]$ and $[\phi]$ are parts of $\mathcal{O}$ but $[\psi]$ is not. Since the modus ponens analogue guarantees that $[\psi]$ is part of $\mathcal{O}$, we have reached the desired contradiction and shown that the (K) axiom is true in every model.

Since every extension of Λ is consistent^{sdl}, every $\mathcal{O}$-suitable property Q is *consistent*$_{sdl}$, or such that no finite set of its parts $\{[\phi_1], \ldots , [\phi_n]\}$ is such that that $\neg(\phi_1 \wedge \ldots \wedge \phi_n)$ is provable in SDL. This consistency guarantees that for any ϕ, at most one of $[\phi]$ or $[\neg\phi]$ is part of $\mathcal{O}$. As I explain in the next section, this feature of $\mathcal{P}$ validates $\bigcirc$-D, the characteristic axiom of SDL.

Given our account of $\mathcal{O}$-suitability, $\mathcal{O}$-suitable properties also include the instances of

$(\mathcal{O}$-K$)[\bigcirc(\phi \supset \psi) \supset (\bigcirc\phi \supset \bigcirc\psi]$, and
$(\mathcal{O}$-D$)[\bigcirc\phi \supset \mathsf{P}\phi]$.

Notice, however, that $\mathcal{O}$'s inclusion of ($\mathcal{O}$-K) and ($\mathcal{O}$-D) are not the features of $\mathcal{O}$ that validate the axioms $\bigcirc$-K and $\bigcirc$-D, respectively. Rather, the modus ponens analogue for $\mathcal{O}$ validates $\bigcirc$-K, and $\mathcal{O}$'s consistency validates $\bigcirc$-D. Why, then, should $\mathcal{O}$ also include ($\mathcal{O}$-K) and ($\mathcal{O}$-D)?

$\mathcal{O}$ includes the propositional properties corresponding to logical truths, and, in the current context, the logical truths encompass the theorems of SDL. So our definition of $\mathcal{O}$-suitability ensures that $\mathcal{O}$-suitable properties contain propositional properties corresponding to the theorems of SDL. Since each $\mathcal{O}$-suitable property has a part corresponding to each member of Λ, and since Λ itself is closed under necessitation, we will find a similar feature among the parts of $\mathcal{O}$-suitable properties. Every $\mathcal{O}$-suitable property contains (i) propositional properties corresponding to the logical truths of the propositional calculus, (ii) propositional properties corresponding to instances of ($\mathcal{O}$-K) and ($\mathcal{O}$-D), and (iii) the properties which follow from these via the modus ponens and necessitation analogues. So if $[\phi]$ is any of these properties, then, $[\bigcirc\phi]$ is also part of each $\mathcal{O}$-suitable property. We'll call this the *necessitation feature*, since it helps ensure that the inference rule $\bigcirc$-NEC preserves validity. The necessitation feature ensures that, for each theorem ϕ of SDL, $[\phi]$ is part of $\mathcal{O}$ in every model, which ensures that $\ulcorner\bigcirc\phi\urcorner$ is true in every model, given (O).

A set is maximal^{sdl} when it contains, for every ϕ in the language of SDL, either ϕ or $\neg\phi$. Some extensions of Λ are maximal^{sdl}. Therefore, some $\mathcal{O}$-suitable properties are

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maximal$_{sdl}$, or such that for every wff ϕ of SDL, either $[\phi]$ or $[\neg\phi]$ is part of $\mathcal{O}$. But $\mathcal{O}$-suitability does not require maximality. Given (O), models in which $\mathcal{O}$ is maximal^{sdl} guarantee that, for every ϕ, either $\ulcorner\bigcirc\phi\urcorner$ or $\ulcorner\bigcirc\neg\phi\urcorner$ is true. In these models, nothing is permissible unless it is obligatory. Since this principle is invalid in SDL, I do not restrict the parthood relation to exclude non-maximal$_{sdl}$ properties from serving as $\mathcal{O}$ in a model.

5 On Parthood

The model structure's third component is $\mathcal{P}$, the parthood relation. We can restrict $\mathcal{P}$ in a number of ways. To express those restrictions in the metalanguage, I will use '<' for the *is part of* relation. Some possible restrictions are more complicated than others. For example, consider the propositional property of being such that $[\phi]$ is part of $\mathcal{O}$. In the metalanguage, I will use '$[[\phi] < \mathcal{O}]$' to refer to the propositional property of being such that $[\phi]$ is part of $\mathcal{O}$. This propositional property has another propositional property, $[\phi]$, embedded within it. But this embedding is not parthood, in my sense. Finally, let $\nless$ abbreviate 'is not part of', and let '$[[\phi] \nless \mathcal{O}]$' refer to the propositional property of being such that $[\phi]$ is not part of $\mathcal{O}$. Some possible restrictions on $\mathcal{P}$ include:

$\mathcal{O}$-consistency. If $[\phi] < \mathcal{O}$, then $[\neg\phi] \nless \mathcal{O}$.
Connectedness. If $[\phi] < \mathcal{O}$, then $[\phi] < \mathcal{A}$.
Fortification. If $[\phi] < \mathcal{A}$, then $[[\neg\phi] \nless \mathcal{O}] < \mathcal{O}$.
Ininclusivity. If $[\phi] < \mathcal{O}$, then $[[\phi] < \mathcal{O}] < \mathcal{O}$.
Inexclusivity. If $[\phi] \nless \mathcal{O}$, then $[[\phi] \nless \mathcal{O}] < \mathcal{O}$.

To make the semantics adequate for SDL, we flip $\mathcal{P}$'s 'on' switch for $\mathcal{O}$-consistency and leave undisturbed the switches for Connectedness, Fortification, Ininclusivity, and Inexclusivity. When $\mathcal{P}$ is $\mathcal{O}$-consistent but otherwise unrestricted, we validate all and only the theorems of SDL.

We have already seen how the semantics validates $\bigcirc$-K. But let us see how it validates $\bigcirc$-D, on the one hand, and how it invalidates a clear non-theorem of SDL, on the other. First, consider $\bigcirc$-D: $\bigcirc\phi \supset \mathsf{P}\phi$. Suppose that $\ulcorner\bigcirc\phi\urcorner$ is true. Then, $[\phi]$ is part of $\mathcal{O}$, via (O). $\mathcal{O}$-consistency then ensures that $[\neg\phi]$ is not part of $\mathcal{O}$. Consequently, $\ulcorner\mathsf{P}\phi\urcorner$ is true, via (P). So $\bigcirc$-D is true in all $\mathcal{O}$-consistent models.

Second, consider the (T) axiom schema from alethic modal logic, i.e., $\Box\phi \supset \phi$. It says that whatever is necessary is the case. While it is an intuitive principle in alethic modal logic, its deontic analogue, $\bigcirc$-T ($\bigcirc\phi \supset \phi$), is not. Since $\bigcirc$-T is not a theorem of SDL, the new semantics should capture its falsity in at least one model. $\bigcirc$-T is false when some propositional property $[\phi]$ is part of $\mathcal{O}$ but not part of $\mathcal{A}$. Since we've not restricted $\mathcal{P}$ to satisfy Connectedness, there are many models in which some parts of $\mathcal{O}$ are not parts of $\mathcal{A}$. Why think this?

For an atomic proposition ϕ and its negation $\neg\phi$, there are pairs of extensions of Λ (section 4) for which ϕ but not $\neg\phi$ is a member of one and $\neg\phi$ but not ϕ is a member of the other. From this, it follows from the definition of $\mathcal{O}$-suitable properties that the parts of at least one $\mathcal{O}$-suitable property include $[\phi]$ but not $[\neg\phi]$ and that the parts of at least one other $\mathcal{O}$-suitable property include $[\neg\phi]$ but not $[\phi]$. Since ϕ is atomic, we may use Lindenbaum's Lemma and the previously mentioned pairs of extensions of Λ to infer that there are pairs of maximal-consistent^{pc} sets for which ϕ but not $\neg\phi$ is a member of one and $\neg\phi$ but not ϕ is a member of the other. From this, it follows from the definition of $\mathcal{A}$-suitable properties that the parts of at least one $\mathcal{A}$-suitable property include $[\phi]$ but not $[\neg\phi]$ and that the parts of at least one other $\mathcal{A}$-suitable property include $[\neg\phi]$ but not $[\phi]$.

Our models for SDL are $\mathcal{O}$-consistent and impose no further constraint on how $\mathcal{A}$ and $\mathcal{O}$ relate to one another in a model. Therefore, $\mathcal{A}$-suitable and $\mathcal{O}$-suitable properties may freely combine with one another in models for SDL. That is, each $\mathcal{O}$-suitable property pairs with each $\mathcal{A}$-suitable property in some $\mathcal{O}$-consistent model. I will call this the *free combination feature*. Given this feature, each of the above $\mathcal{O}$-suitable properties whose parts include $[\phi]$ but not $[\neg\phi]$ cohabit in a model with one of the above $\mathcal{A}$-suitable properties whose parts include $[\neg\phi]$ but not $[\phi]$. (We can also infer that the above $\mathcal{O}$-suitable properties whose parts include $[\neg\phi]$ but not $[\phi]$ each cohabit in a model with one of the above $\mathcal{A}$-suitable properties whose parts include $[\phi]$ but not $[\neg\phi]$.) As a result, there are models in which some parts of $\mathcal{O}$ are not parts of $\mathcal{A}$.

Now, there are also models in which every part of $\mathcal{O}$ is also part of $\mathcal{A}$. This, too, follows from the definitions of $\mathcal{O}$-suitable and $\mathcal{A}$-suitable properties and the free combination feature. These are models in which $\bigcirc$-T is true. But these models cannot make $\bigcirc$-T valid because the previously mentioned models (in which some parts of $\mathcal{O}$ are not parts of $\mathcal{A}$) suffice to invalidate $\bigcirc$-T. Like the standard semantics, then, the semantics here has models in which $\bigcirc$-T is true and others in which it is false and serve to invalidate $\bigcirc$-T.

The remaining restrictions correspond to deontic analogues of axioms from other well-known modal systems. Fortification would secure the deontic analogue of the (B) axiom of modal logic. And Ininclusivity and Inexclusivity would secure deontic analogues of the (S4) and (S5) axioms of modal logic, respectively. These are not theorems of SDL. So the semantics here does not restrict $\mathcal{P}$ in these ways.

6 The Semantics, Formally

In the new semantics, a *model structure* is the triple $\langle\mathcal{A}, \mathcal{O}, \mathcal{P}\rangle$, where $\mathcal{A}$ is the property of being Alpha, $\mathcal{O}$ is the property of being an ideal world, and $\mathcal{P}$ specifies restrictions on the parts of $\mathcal{A}$ and $\mathcal{O}$. We restrict ourselves to models in which $\mathcal{P}$ satisfies $\mathcal{O}$-consistency.

A *model* $\mathcal{M}$ is an ordered quadruple $\langle\mathcal{A}, \mathcal{O}, \mathcal{P}, \mathrm{V}\rangle$, which adds a valuation function V to the model structure $\langle\mathcal{A}, \mathcal{O}, \mathcal{P}\rangle$ on which it is based. The valuation function V for a model $\mathcal{M}$ ('$\mathrm{V}_{\mathcal{M}}$') meets the following conditions:

(i*) For any atomic proposition ϕ, $\mathrm{V}_{\mathcal{M}}(\phi) = 1$ iff $[\phi] < \mathcal{A}$.
(ii*) $\mathrm{V}_{\mathcal{M}}(\neg\phi) = 1$ iff $\mathrm{V}_{\mathcal{M}}(\phi) = 0$.
(iii*) $\mathrm{V}_{\mathcal{M}}(\phi \supset \psi) = 1$ iff either $\mathrm{V}_{\mathcal{M}}(\phi) = 0$ or $\mathrm{V}_{\mathcal{M}}(\psi) = 1$.
(iv*) $\mathrm{V}_{\mathcal{M}}(\bigcirc\phi) = 1$ iff $[\phi] < \mathcal{O}$.

Since, in Appendix I, I will refer to the standard models and the accounts of logical consequence and validity that they generate, I will subscript the newer accounts of these notions with an 'n' to avoid confusion. For $\mathcal{O}$-consistent models, a wff ϕ is a *logical consequence*$_n$ of the wffs in Γ ('$\Gamma \vDash_n \phi$') when $V_{\mathcal{M}_n}(\phi) = 1$ if every wff ψ in Γ is such that $V_{\mathcal{M}_n}(\psi) = 1$, for every model $\mathcal{M}_n$. A wff ϕ is *valid*$_n$ when $V_{\mathcal{M}_n}(\phi) = 1$ for every model $\mathcal{M}_n$.

In Appendix I, I show that my semantic theory's account of logical consequence is extensionally equivalent to the account of logical consequence in the traditional semantics.

7 Conclusion

Some may wonder whether a new semantics for a benchmark system like SDL is worth anything more than a notch on a small scorecard for Leibnizian intensions. But I cannot imagine getting a bigger bang for my buck. The traditional semantics served as a springboard for further semantic theories. Without it, we would not now have the vast array of semantic theories that arose from amending or expanding it or from amending or expanding the amendments and expansions. Given this history, we might reasonably regard an alternative as a substantial down payment for future innovation. In Appendix II, I provide some evidence for this optimistic outlook by amending the semantic theory to invalidate $\bigcirc$-NEC in a new way. Overall, I hope to have made it slightly less reasonable to dismiss Leibnizian intensions simply because they have not yet been as successful as possible worlds. This disparity rests almost entirely on the vast resources thrown at one rather than the other. And this latter disparity is, in my view, an accident of history.

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Appendix I

To avoid ambiguity, I will subscript the labels for various notions of the standard semantics in section 1.3. I'll use the subscript 't' for models in the traditional semantics and for the related notions of consequence and validity.

Where '$\Gamma \vdash \phi$' says that ϕ is provable in SDL from the wffs in Γ and '$\Gamma \vDash_t \phi$' says that ϕ is a consequence$_t$ of the wffs in Γ, the principles below are already well-established:

Soundness-T. If $\Gamma \vdash \phi$, then $\Gamma \vDash_t \phi$.
Completeness-T. If $\Gamma \vDash_t \phi$, then $\Gamma \vdash \phi$.

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Where '$\Gamma \vDash_n \phi$' says that ϕ is a consequence$_n$ (in the new sense) of the wffs in Γ, we will prove:

> SOUNDNESS-N. If $\Gamma \vdash \phi$, then $\Gamma \vDash_n \phi$.
> COMPLETENESS-N. If $\Gamma \vDash_n \phi$, then $\Gamma \vdash \phi$.

We can prove both if we first prove the following:

> EQUIVALENCE-1. If $\Gamma \vDash_t \phi$, then $\Gamma \vDash_n \phi$.
> EQUIVALENCE-2. If $\Gamma \vDash_n \phi$, then $\Gamma \vDash_t \phi$.

For EQUIVALENCE-1 and SOUNDNESS-T imply SOUNDNESS-N, and EQUIVALENCE-2 and COMPLETENESS-T imply COMPLETENESS-N. Proving EQUIVALENCE-1 and -2 only requires proving the two principles below:

> MODELS-TN. For any world w in any model $\mathcal{M}_t$, some newer model $\mathcal{M}_n$ is such that $V_{\mathcal{M}_t}(\phi, w) = 1$ iff $V_{\mathcal{M}_n}(\phi) = 1$.
> MODELS-NT. For each newer model $\mathcal{M}_n$, some world w in some model $\mathcal{M}_t$ is such that $V_{\mathcal{M}_t}(\phi, w) = 1$ iff $V_{\mathcal{M}_n}(\phi) = 1$.

But to prove MODELS-TN and MODELS-NT, I will first prove two claims about $\mathcal{O}$-suitable properties:

(2-a) For each model $\mathcal{M}_n$, there is a world w in some model $\mathcal{M}_t$ such that for any ϕ, $\phi \in w'$ for every w' such that Rww' iff $[\phi]$ is part of $\mathcal{O}$.

(2-b) For any world w in any model $\mathcal{M}_t$, there is a model $\mathcal{M}_n$ such that for any ϕ, $\phi \in w'$ for every w' such that Rww' iff $[\phi]$ is part of $\mathcal{O}$.

With respect to the traditional semantics, let a world w's *ideal set* S_w contain ϕ iff $\phi \in w'$ for every w' such that Rww'. And with respect to the new semantics, a property is $\mathcal{O}$-suitable iff it serves as $\mathcal{O}$ in some model. Therefore, proving 2-a and 2-b only requires proving that (i) for any $\mathcal{O}$-suitable property Q, there is a world w in some traditional model such that $\phi \in S_w$ iff $[\phi]$ is part of Q, and (ii) for any world w in any traditional model, there is an $\mathcal{O}$-suitable property Q such that $\phi \in S_w$ iff $[\phi]$ is part of Q.

To prove (i), and by extension (2-a), assume for reductio that some $\mathcal{O}$-suitable property Q is such that there is no world w in any traditional model for which $\phi \in S_w$ iff $[\phi]$ is part of $\mathcal{O}$. By definition (section 4), Q is such that there is some extension S of Λ for which $\phi \in S$ iff $[\phi]$ is part of Q. Now let A be the set of SDL wffs, and let B be the relative complement of S with respect to A (i.e., the set of wffs in A which are not in S). Then let set C be the set of all wffs ϕ such that, first, $\phi \in B$ and, second, the union of S and ϕ is consistent^{sdl}. Set C, then, is the set of SDL wffs which are not in S and are individually consistent with S. We will then order C's members into a list, D:

$D = \phi_1, \phi_2, \ldots$

With D, we construct a sequence of sets consisting of the union of S and each wff in D:

$D' = S \cup \phi_1, S \cup \phi_2, \ldots$

Each set in this sequence is a consistent set of SDL wffs. So, by Lindenbaum's Lemma, each set in the sequence is a subset of some maximal consistent set of SDL wffs.

For each $x_n \in D'$, a non-empty set d_n of maximal-consistent^{sdl} sets is such that (i) for all $w \in d_n$, $x_n \subseteq w$, and (ii) every $w \in d_n$ is such that $\bigcirc\phi \in w$ iff $\phi \in w$. There is a set D'' of these sets d_n of maximal-consistent^{sdl} sets. By the Axiom of Choice, there is a set W containing a member from each member d_n of D''. And, furthermore, $\cap W = S$.

Intuitively, we've just defined a set a worlds such that each world is acceptable to itself alone and whose intersection is the extension S of Λ with which we began. Now we can simply build a traditional model involving these worlds and one more, a world w' for which all and only the worlds in W are w'-acceptable, and we will have ensured that the ideal set of w' and the $\mathcal{O}$-suitable property Q are such that $\phi \in S_{w'}$ iff $[\phi]$ ¡ Q.

Let w' be a maximal consistent set of SDL wffs such that $\bigcirc\phi \in w\prime$ iff ϕ is a member of every member of W. Then let $W' = W \cup \{w\prime\}$. And let R be a set of ordered pairs defined over W' such that (a) $\langle w', w' \rangle \notin R$, (b) for every $w_n \in W'$ such that $w_n \neq w\prime$, $\langle w', w_n \rangle \in R$, and (c) for every $w_n \in W'$ such that $w_n \neq w'$, $\langle w_n, w_n \rangle \in R$. (a) and (b) ensure that every world is acceptable to w' except w' itself, and (c) ensures that every world besides w' is acceptable to itself. Together, this means that R is serial but not reflexive. And we ensure that for every ϕ, $\bigcirc\phi \in w\prime$ iff $\phi \in S$, our extension of Λ. Hence, the extension S of Λ is identical to the ideal set $S_{w'}$. Therefore, there is a world w in some traditional model such that $\phi \in S_w$ iff $[\phi]$ is part of Q. Contradiction. So we have proved 2-a.

To prove (ii), and by extension (2-b), assume for reductio that for a world w in a traditional model, there is no $\mathcal{O}$-suitable property Q such that $\phi \in S_w$ iff $[\phi]$ is part of Q. S_w is an intersection of w-acceptable worlds. Every world in the traditional model contains Λ as subset. So every w-acceptable world contains Λ as subset. Furthermore, if every w-acceptable world contains a formula ϕ not in Λ, then since each such world is consistent and closed under modus ponens, each such world will contain any formulas derivable from the wffs in Λ together with ϕ. As a result, the intersection of w-acceptable worlds is the union of Λ and some possible non-empty set of wffs closed under modus ponens. Hence, S_w is an extension of Λ. Therefore, by the definition of $\mathcal{O}$-suitability, some $\mathcal{O}$-suitable property Q is such that $[\phi]$ is part of Q iff ϕ is in S_w. Contradiction. So we've established (ii) and 2-b.

With (1-a), (1-b), (2-a), and (2-b) in hand, we can prove Models-TN and Models-NT. And with these latter two, we can prove Equivalence-1 and -2.

The Proof. To prove Models-TN, assume for reductio that some world w in some model $\mathcal{M}_t$ is such that there is no newer model $\mathcal{M}_n$ for which $V_{\mathcal{M}_t}(A, w) = 1$ iff

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$V_{\mathcal{M}_n}(A) = 1$. To reach a contradiction, we prove by induction that $\mathcal{M}_t$ and some newer model $\mathcal{M}_n$ are such that for any wff A, $V_{\mathcal{M}_t}(A, w) = 1$ iff $V_{\mathcal{M}_n}(A) = 1$. When a world w in model $\mathcal{M}_t$ and $\mathcal{A}$ in a newer model $\mathcal{M}_n$ are such that for every atomic wff ϕ, $\phi \in w$ iff $[\phi]$ ¡ $\mathcal{A}$, w in $\mathcal{M}_t$ and $\mathcal{M}_n$ are *atomic equivalent*. The proof will rely on atomic equivalence and begins with the case of atomic wffs.

Suppose A is an atomic wff ϕ. Then, 1-b guarantees that there is some atomic equivalent model $\mathcal{M}_n$ such that $\phi \in w$ in $\mathcal{M}_t$ iff $[\phi]$ is part of $\mathcal{A}$ in $\mathcal{M}_n$. By (i) and (i*), the valuation clauses for atomic wffs from the traditional (Sec. 2) and new (Sec. 7) theories, respectively, $V_{\mathcal{M}_t}(\phi, w) = 1$ iff $V_{\mathcal{M}_n}(\phi) = 1$.

For the inductive hypothesis, assume that for every formula B less complex than formula A below, the world w in model $\mathcal{M}_t$ is such that for some atomic equivalent model $\mathcal{M}_n$, $V_{\mathcal{M}_t}(A, w) = 1$ iff $V_{\mathcal{M}_n}(A) = 1$.

Suppose that A is $\neg\phi$. By (ii) and (ii*), $V_{\mathcal{M}_t}(\neg\phi, w) = 1$ iff $V_{\mathcal{M}_t}(\phi, w) = 0$, and $V_{\mathcal{M}_n}(\neg\phi) = 1$ iff $V_{\mathcal{M}_n}(\phi) = 0$, respectively. By the inductive hypothesis, $V_{\mathcal{M}_t}(\phi, w) = 0$ iff $V_{\mathcal{M}_n}(\phi) = 0$. Therefore, $V_{\mathcal{M}_t}(\neg\phi, w) = 1$ iff $V_{\mathcal{M}_n}(\neg\phi) = 1$.

Suppose that A is $\phi \supset \psi$. By (iii) and (iii*), $V_{\mathcal{M}_t}(\phi \supset \psi, w) = 1$ iff either $V_{\mathcal{M}_t}(\phi, w) = 0$ or $V_{\mathcal{M}_t}(\psi, w) = 1$, and $V_{\mathcal{M}_n}(\phi \supset \psi) = 1$ iff either $V_{\mathcal{M}_n}(\phi) = 0$ or $V_{\mathcal{M}_n}(\psi) = 1$, respectively. By the inductive hypothesis, $V_{\mathcal{M}_t}(\phi, w) = 0$ or $V_{\mathcal{M}_t}(\psi, w) = 1$ iff $V_{\mathcal{M}_n}(\phi) = 0$ or $V_{\mathcal{M}_n}(\psi) = 1$. Therefore, $V_{\mathcal{M}_t}(\phi \supset \psi, w) = 1$ iff $V_{\mathcal{M}_n}(\phi \supset \psi) = 1$.

Suppose that A is $\bigcirc\phi$. (iv) says that $V_{\mathcal{M}_t}(\bigcirc\phi, w) = 1$ iff for all $w' \in W$ such that Rww', $V_{\mathcal{M}_t}(\phi, w') = 1$. And for all $w' \in W$ such that $Rw\,w'$, $V_{\mathcal{M}_t}(\phi, w') = 1$ iff for every w' such that Rww', $\phi \in w'$. 2-b says that a world w is such that for every w' for which Rww', $\phi \in w'$ iff $[\phi] < \mathcal{O}$ in some newer model $\mathcal{M}_n$. By the free combination feature (section 5), any $\mathcal{A}$-suitable property satisfying the clauses above cohabits in some new model $\mathcal{M}_n$ with the $\mathcal{O}$-suitable property here. By (iv*), then, some atomic equivalent model $\mathcal{M}_n$ is such that $V_{\mathcal{M}_t}(\bigcirc\phi, w) = 1$ iff $V_{\mathcal{M}_n}(\bigcirc\phi) = 1$.

Therefore, model $\mathcal{M}_t$ and some newer model $\mathcal{M}_n$ are such that for any wff A, $V_{\mathcal{M}_t}(A, w) = 1$ iff $V_{\mathcal{M}_n}(A) = 1$. Contradiction. This completes the reductio and establishes Models-TN.

To prove Models-NT, assume for reductio that some newer model $\mathcal{M}_n$ is such that there is no world w in any model $\mathcal{M}_t$ for which $V_{\mathcal{M}_t}(\phi, w) = 1$ iff $V_{\mathcal{M}_n}(\phi) = 1$. To reach a contradiction, we will prove by induction that $\mathcal{M}_n$ and some w in some model $\mathcal{M}_t$ are such that for any wff A, $V_{\mathcal{M}_t}(A, w) = 1$ iff $V_{\mathcal{M}_n}(A) = 1$. We begin again with the base case of atomic wffs.

Suppose A is an atomic wff ϕ. Then, 1-a guarantees that there is some atomic equivalent model $\mathcal{M}_t$ such that $\phi \in w$ in $\mathcal{M}_t$ iff $[\phi]$ is part of $\mathcal{A}$ in $\mathcal{M}_n$. By (i) and (i*), $V_{\mathcal{M}_t}(\phi, w) = 1$ iff $V_{\mathcal{M}_n}(\phi) = 1$.

For the inductive hypothesis, assume that for every formula B less complex than formula A below, model $\mathcal{M}_n$ is such that there is some world w in an atomic equivalent model $\mathcal{M}_t$ for which $V_{\mathcal{M}_t}(A, w) = 1$ iff $V_{\mathcal{M}_n}(A) = 1$.

Suppose that A is $\neg\phi$. By (ii) and (ii*), $V_{\mathcal{M}_t}(\neg\phi, w) = 1$ iff $V_{\mathcal{M}_t}(\phi, w) = 0$, and $V_{\mathcal{M}_n}(\neg\phi) = 1$ iff $V_{\mathcal{M}_n}(\phi) = 0$, respectively. By the inductive hypothesis, $V_{\mathcal{M}_t}(\phi, w) = 0$ iff $V_{\mathcal{M}_n}(\phi) = 0$. Therefore, $V_{\mathcal{M}_t}(\neg\phi, w) = 1$ iff $V_{\mathcal{M}_n}(\neg\phi) = 1$.

Suppose that A is $\phi \supset \psi$. By (iii) and (iii*), $V_{\mathcal{M}_t}(\phi \supset \psi, w) = 1$ iff either $V_{\mathcal{M}_t}(\phi, w) = 0$ or $V_{\mathcal{M}_t}(\psi, w) = 1$, and $V_{\mathcal{M}_n}(\phi \supset \psi) = 1$ iff either $V_{\mathcal{M}_n}(\phi) = 0$ or $V_{\mathcal{M}_n}(\psi) = 1$,

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respectively. By the inductive hypothesis, $V_{\mathcal{M}_t}(\phi, w) = 0$ or $V_{\mathcal{M}_t}(\psi, w) = 1$ iff $V_{\mathcal{M}_n}(\phi) = 0$ or $V_{\mathcal{M}_n}(\psi) = 1$. Therefore, $V_{\mathcal{M}_t}(\phi \supset \psi, w) = 1$ iff $V_{\mathcal{M}_n}(\phi \supset \psi) = 1$.

Suppose that A is $\bigcirc\phi$. By (iv*), $V_{\mathcal{M}_n}(\bigcirc\phi) = 1$ iff $[\phi] < \mathcal{O}$. 2-a says that $[\phi]$ is part of $\mathcal{O}$ in $\mathcal{M}_n$ iff there is a world w in some model $\mathcal{M}_t$ such that $\phi \in w'$ for every w' such that Rww'. Now, the atomic equivalence of w in a model $\mathcal{M}_t$ with $\mathcal{A}$ in $\mathcal{M}_n$ does not determine anything about w's ideal set S_w (because R is only required to be serial). Therefore, by (iv), some world w in some atomic equivalent model $\mathcal{M}_t$ is such that $V_{\mathcal{M}_t}(\bigcirc\phi, w) = 1$ iff $V_{\mathcal{M}_n}(\bigcirc\phi) = 1$.

So the model $\mathcal{M}_n$ and some model $\mathcal{M}_t$ are such that for any wff A, $V_{\mathcal{M}_t}(A, w) = 1$ iff $V_{\mathcal{M}_n}(A) = 1$. Contradiction. This completes the reductio and establishes Models-NT. Since we've established both Models-TN and Models-NT, we've proved both Equivalence-1 and Equivalence-2 and therefore also Soundness-N and Completeness-N.

Appendix II

Some have have expressed concerns about $\bigcirc$-NEC because it ensures that all tautologies are obligatory (von Wright 1963: 154; al-Hibri 1978: 15; Jones and Pörn 1985; McNamara 2006: 227). Channeling G. H. Von Wright (1981: 8), Charles Pigden (1989: 139) writes: 'The kindest thing to be said about this is that it is an "absurdity" which logicians have been induced to "swallow" for the sake of "formal elegance and simplicity"'. That 1 is or isn't an even number is true and necessarily so. And some have reinterpreted the $\bigcirc$-operator plausibly so that $\ulcorner\bigcirc\phi\urcorner$ is true when ϕ is an arbitrary tautology (Anderson 1956 and Wedewood 2006; 2007). But the concerns above presuppose plausible senses of 'ought' and 'obligation' on which not all tautologies ought to be or are obligatory. On any of these senses, a single non-obligatory tautology complicates matters greatly for possible worlds approaches. Since each tautology is true in all possible worlds, each tautology is true in any subset of them, including the subset of ideal possible worlds. But being true in all ideal possible worlds suffices for being obligatory in the standard semantics.

The semantic theories in which $\bigcirc$-NEC fails use possible worlds, and they avoid $\bigcirc$-NEC either by rendering all obligations as conditional obligations or by adding nonnormal worlds in which nothing is obligatory and everything is permissible (al-Hibri 1978; Chellas 1980; Jones and Pörn 1985; Goble 1990). But it seems worthwhile to have a semantic theory that can avoid $\bigcirc$-NEC without making all obligations conditional or by appealing to worlds which, for all we know, are actually impossible.

Neighborhood semantics is perhaps the most impressive alternative framework that invalidates $\bigcirc$-NEC. But why should worlds in which everything is permissible and nothing is obligatory enter into the truth conditions about what is actually obligatory or what actually ought to be the case? Questions like this motivate work under the 'truthmaker semantics' label. One could argue that the my theories should fall under the same label. (See Fine (2014; 2017).) Furthermore, neighborhood semantics has strange consequences for those who would like to invalidate $\bigcirc$-NEC. As Eric Pacuit (2017: 54) shows, the following holds for the class of neighborhood frames:

> **$\bigcirc$-RE.** if $\phi \equiv \psi$ is valid, then $\bigcirc\phi \equiv \bigcirc\psi$ is valid.

Since $\phi \equiv (\phi \wedge \top)$ is also valid, it follows that if $\ulcorner\bigcirc\phi\urcorner$ is true in a world, so is $\ulcorner\bigcirc(\phi \wedge \top)\urcorner$, even if $\ulcorner\bigcirc\top\urcorner$ is not. (See Goble (1990: 198, n. 8) for a similar feature.) That is, in any world in which something is obligatory, any tautology that is not obligatory will still be a conjunct of an obligatory conjunction. So neighborhood semantics invalidates $\bigcirc$-NEC but does not quite respect the motivating intuition for denying $\bigcirc$-NEC in the first place, especially if one thinks that obligations or oughts should distribute over conjunction. (Pigden (1989: 140) has similar critiques of the systems in Schotch and Jennings (1981) and Chellas (1980: 272–276).)

Now, if we want our semantics to invalidate $\bigcirc$-NEC and thereby allow for some non-obligatory tautologies, we will also need to invalidate either $\bigcirc$-RE or the following:

> **&-Distribution.** $\bigcirc(\phi \wedge \psi) \supset (\bigcirc\phi \wedge \bigcirc\psi)$

Neighborhood semantics invalidates &-Distribution rather than $\bigcirc$-RE. But I believe this is less intuitive than doing the converse on some readings of the $\bigcirc$-operator. On these readings, if a conjunction is obligatory, so are its conjuncts. Although $\bigcirc$-RE is widely regarded as 'the most fundamental and least controversial rule of inference in deontic logic' (McNamara 2006: 205), I believe that $\bigcirc$-RE is deeply unintuitive. I'm also not alone: Hansson (2006: 322) says that it 'gives rise to most of the major deontic paradoxes'. (Compare Schroeder (2011: 19–21).) For it turns out that, as long as &-Distribution holds, and as long as $\bigcirc$-RE is valid, in any model in which anything is obligatory, every tautology is obligatory. So if we find &-Distribution intuitive, we must invalidate both $\bigcirc$-NEC and $\bigcirc$-RE. As far as I'm aware, no deontic semantics plausibly does this because the theories on offer overwhelmingly rely on maximally consistent entities like possible worlds to treat $\bigcirc$-statements.

Given the background metaphysics in Warmke (2015; 2019)according to which says that properties obey the general sum principle, we can define a Leibnizian intension with nearly whatever parts we like. So we can invalidate $\bigcirc$-NEC and $\bigcirc$-RE in a new way. As we saw in section 4, $\mathcal{O}$ (the property of being an ideal world) intuitively includes, in every SDL model, the propositional properties corresponding to all the tautologies. But, now, instead of using $\mathcal{O}$ to account for $\bigcirc$-statements, we might opt for the smaller $\mathcal{O}^-$. In many models, $\mathcal{O}^-$ is a property which lacks many but not all of the tautologous parts which belong to $\mathcal{O}$ in every SDL model.

Let $\mathcal{O}^-$ satisfy ($\mathcal{O}$-K) and ($\mathcal{O}$-D) (from section 4), as well as:

(a) $[\phi \wedge \psi]$ is part of $\mathcal{O}^-$ iff $[\phi]$ and $[\psi]$ are parts of $\mathcal{O}^-$
(b) if $[\phi]$ is part of $\mathcal{O}^-$, $[\phi \vee \psi]$ is part of $\mathcal{O}^-$
(c) $[\phi]$ is part of $\mathcal{O}^-$ iff $[\neg\neg\phi]$ is part of $\mathcal{O}^-$

And let $\mathcal{O}^-$ also satisfy cross-connective equivalencies, such as:

(a) $[\phi \wedge \psi]$ is part of $\mathcal{O}^-$ iff $[\neg(\neg\phi \vee \neg\psi)]$ is part of $\mathcal{O}^-$
(b) $[\phi \supset \psi]$ is part of $\mathcal{O}^-$ iff $[\neg\phi \vee \psi]$ is part of $\mathcal{O}^-$

Given that $\mathcal{O}^-$ now replaces $\mathcal{O}$, it is trivial to show that the resulting semantics invalidates $\bigcirc$-NEC and $\bigcirc$-RE but validates $\bigcirc$-K via ($\mathcal{O}$-K), $\bigcirc$-D via ($\mathcal{O}$-D), and principles like &-Distribution via (a) and the $\vee$-Weakening principle, $\bigcirc\phi \supset \bigcirc(\phi \vee \psi)$, via (b). Although $\vee$-Weakening inspires Ross's paradox, I follow Wedgewood (2007: 115) and believe that pragmatic considerations dissolve it.

The semantics has three more features worth highlighting. First, $\mathcal{O}^-$ arguably involves a more complicated set-up than $\mathcal{O}$. Absent the constraints that would validate both $\bigcirc$-NEC and $\bigcirc$-RE, we need extra principles to ensure that we can draw inferences about what is obligatory from what is obligatory without guaranteeing that all tautologies are obligatory, too. We seem to have sacrificed some elegance, as a result.

Second, given the above constraints, $\mathcal{O}^-$ need not have any parts at all. In models in which $\mathcal{O}^-$ has no parts, nothing is obligatory and, in my view, '$\mathcal{O}^-$' refers to no property at all in the metalanguage. But we should expect models without obligations in a semantics that invalidates both $\bigcirc$-NEC and $\bigcirc$-RE. In fact, some suggest that the main purpose for including $\bigcirc$-NEC is to guarantee that something or other is obligatory in every model. For example, Brian Chellas (1974: 23) even claims that 'the presence of [$\bigcirc$-NEC] is equivalent to the thesis that obligations exist at every possible world'. However, I would have thought that if there were obligations necessarily in force, they would resemble 'it is obligatory that no one murders' more than 'it is obligatory that 1 is even or 1 is not even'. So if we want to preclude obligation-less models without recourse to tautologies, we may introduce δ_1, ..., δ_n as nonlogical, propositional constants, restrict $\mathcal{O}^-$ to have $[\delta_1]$, ..., $[\delta_n]$ as parts in every model, and, where $1 \leq m \leq n$, add each $\bigcirc\delta_m$ to the logic as an axiom.

Third, although the semantics does not require tautologies to be obligatory, it still allows for obligatory tautologies. For example, if $\ulcorner\bigcirc\phi\urcorner$ is true, because $[\phi]$ is part of $\mathcal{O}^-$, then, by (b), $[\phi \vee \neg\phi]$ is also part of $\mathcal{O}^-$, which makes $\ulcorner\bigcirc\phi \vee \neg\phi\urcorner$ true. This result seems plausible to me.

As far as I know, no other semantics for deontic modal logic invalidates $\bigcirc$-NEC and $\bigcirc$-RE and validates both &-Distribution and $\vee$-Weakening. Hence, the modal semantics described in section 2 not only provides the raw materials for the new deontic semantics in sections 3 through 6. It also provides opportunities to approach problems in deontic logic from a new angle.