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Dynamics of fossil magnetic fields in massive star interiors

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Abstract. In this talk, I review the different MHD processes, which take place in massive star interiors. First, I describe MHD instabilities, which act on magnetic fields in stellar radiation zones, and the dynamo action in massive stars that give strong indications in favor of a fossil origin of the fields observed at the surface of these stars. Then, I discuss the study of MHD turbulent relaxation processes, which are now examined in stellar interiors, to describe initial conditions for fossil magnetic fields. Finally, I focus on the state of the art of the modeling of the interaction between differential rotation, fossil magnetic field, meridional circulation, and turbulence.

Keywords. magnetohydrodynamics: MHD, plasmas, stars: magnetic fields, stars: rotation, stars: evolution

1. Introduction

Magnetic fields are now detected more and more often at the surface of main-sequence (and Pre Main-Sequence) intermediate mass and active massive stars, which have an external radiative envelope. Indeed, strong fields (300 G to 30 kG) are observed in some fraction of Herbig stars (Alecian *et al.* 2008), A stars (the Ap stars, see Aurière *et al.* 2007), as well as in B stars and in a handful of O stars (see the MiMeS program results discussed by G. Wade in this volume and Grunhut *et al.* 2009). Furthermore, non convective neutron stars display fields strength of $10^8 - 10^{15}$ G. Magnetic fields in stably stratified non convective stellar regions thus deeply modify our vision of massive stars evolution since their formation (Commerçon *et al.* 2010) to their late stages, for example for gravitational supernovae. Thus, these drive stellar internal dynamics for the transport of angular momentum and the resulting rotation history, and chemicals mixing (Maeder & Meynet 2000; Mathis & Zahn 2005).

The large-scale, ordered nature (often approximately dipolar) of such magnetic fields and the scaling of their strengths as a function of their host properties (according to the flux conservation scenario) favour a fossil hypothesis (even if a dynamo is present in the convective core, *c.f.* Brun *et al.* 2005), whose origin has to be investigated. One of the fundamental question is then the understanding of the topology of these large-scale magnetic fields. To have survived since the star's formation or the PMS stage, a field must be stable on a dynamic (Alfvén) timescale. It was suggested by Prendergast (1956) that a stellar magnetic field in stable axisymmetric equilibrium must contain both poloidal (meridional) and toroidal (azimuthal) components, since both are unstable on their own (Tayler 1973; Wright 1973). This was confirmed recently by numerical simulations by Braithwaite & Spruit (2004); Braithwaite & Nordlund (2006); Braithwaite (2008) who showed that initial stochastic helical fields evolve on an Alfvén timescale into stable configurations: axisymmetric and non-axisymmetric mixed poloidal-toroidal fields were found. This phenomenon well known in plasma physics is a MHD turbulent relaxation (*i.e.* a self-organization process involving magnetic reconnections in resistive MHD). In this short paper, we present our present physical understanding of such mechanism in stellar interiors focusing on the axisymmetric case. Then, the field interaction with differential rotation and meridional circulation is discussed.

2. Relaxed non force-free configurations

Here, we focus on the minimum energy non force-free MagnetoHydroStatic (MHS) equilibrium (the balance between gravity, the pressure gradient and the Lorentz force) that a stably stratified radiation zone can reach. Several reasons inclined us to focus on such equilibria instead of force-free ones, which are often studied in plasma laboratory experiments. First, Reisenegger (2009) shows us that no configuration can be force-free everywhere. Although there do exist "force-free" configurations, these induce discontinuities such as current sheets, which are unlikely to appear in nature except in a transient manner. Second, non force-free equilibria have been identified in plasma physics as the result of MHD relaxation (Montgomery & Phillips 1988). Third, as shown by Duez & Mathis (2010), this family of equilibria is a generalization of Taylor states (force-free relaxed equilibria in plasma laboratory experiments; see Taylor 1974) in a stellar context, where the stable stratification of the medium plays a crucial role.

The axisymmetric magnetic field $\boldsymbol{B}(r,\theta)$ is expressed as a function of a poloidal flux $\Psi(r,\theta)$, a toroidal potential $F(r,\theta)$, and the potential vector $\boldsymbol{A}(r,\theta)$ so that it is divergence-free by construction:

$$\boldsymbol{B} = \frac{1}{r\sin\theta}\left(\boldsymbol{\nabla}\Psi \times \hat{\mathbf{e}}_\varphi + F\,\hat{\mathbf{e}}_\varphi\right) = \boldsymbol{\nabla}\times\boldsymbol{A}, \tag{2.1}$$

where in spherical coordinates the poloidal component ($\boldsymbol{B}_{\mathrm{P}}$) is in the meridional plane ($\hat{\mathbf{e}}_r, \hat{\mathbf{e}}_\theta$) and the toroidal component ($\boldsymbol{B}_{\mathrm{T}}$) is along the azimuthal direction ($\hat{\mathbf{e}}_\varphi$). Given the field strengths in real stars, the ratio of the Lorentz force to gravity is very low: stellar interiors are thus in a regime where $\beta = P/P_{\mathrm{Mag}} \gg 1$, $P_{\mathrm{Mag}} = B^2/(2\mu_0)$ being the magnetic pressure. Then, we identify the invariants governing the evolution of the reconnection phase, that leads to relaxed states in the non force-free case. The first one is the magnetic helicity $\mathcal{H} = \int_{\mathcal{V}} \boldsymbol{A}\cdot\boldsymbol{B}\,\mathrm{d}\mathcal{V}$, which is an ideal MHD invariant known to be roughly conserved at large scales during relaxation. The second one is the mass encompassed in poloidal magnetic surfaces $M_\Psi = \int_{\mathcal{V}} \Psi\,\rho\,\mathrm{d}\mathcal{V}$ (ρ is the density), conserved because of the stable stratification, which inhibits the radial movements and thus the transport of mass and flux in this direction. Next, we assume a selective decay during relaxation (*c.f.* Biskamp 1997), in which the magnetic energy $E_{\mathrm{mag}} = \int_{\mathcal{V}} \frac{\mathbf{B}^2}{2\mu_0}\mathrm{d}\mathcal{V}$ (μ_0 being the vaccum magnetic permeability), and thus the total energy (internal+gravific+magnetic), decays much faster than $\mathcal{H}$ and M_Ψ, so that they can be considered constant on an energetic decay e-folding time. This is due to the stable stratification and to the different orders of spatial derivatives involved in the variation of E_{mag} and $\mathcal{H}$. The reached equilibrium is thus the one of minimum energy for given magnetic helicity and mass encompassed in magnetic flux tubes. This can be determined applying a variational method where we minimize E with respect to $\mathcal{H}$ and M_Ψ as described by Woltjer (1959) and Duez & Mathis (2010). This leads to the purely dipolar MHS barotropic state (in the *hydrodynamic* meaning of the term, *i.e.* isobar and iso-density surfaces coincide and the field is explicitly coupled with stellar structure through $\boldsymbol{\nabla}\times(\boldsymbol{F}_{\mathcal{L}}/\rho) = \boldsymbol{0}$, where $\boldsymbol{F}_{\mathcal{L}}$ is the

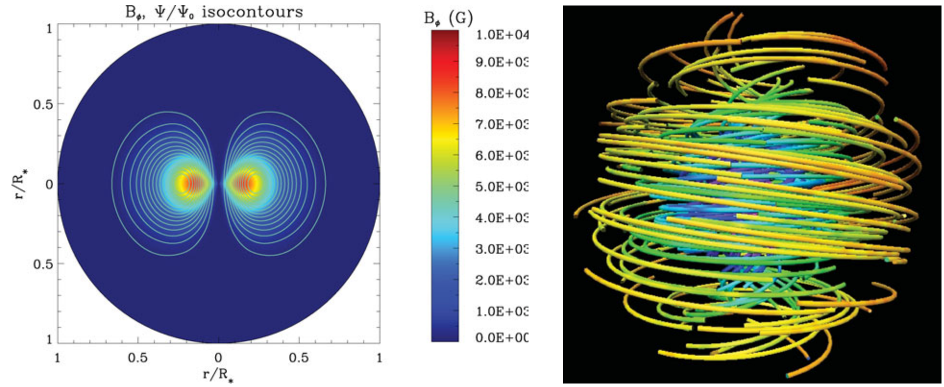

Figure 1. Left: toroidal magnetic field strength in colorscale (arbitrary field's strength) and normalized isocontours of the poloidal flux function (Ψ) in meridional cut for the lowest energy equilibrium configuration ($\lambda_1^1 \simeq 33$); the neutral line is located at $r \simeq 0.23\, R_*$. Right: magnetic field lines representing this mixed field configuration in 3-D looking from the side (the colorscale is a function of the density). Taken from Duez *et al.* (2010a, courtesy The Astrophysical Journal).

Lorentz force) for Ψ:

$$\Psi\,(r,\theta) \;=\; -\mu_0\beta_0\lambda_1^1\frac{r}{R}\left\{j_1\left(\lambda_1^1\frac{r}{R}\right)\int_r^R\left[y_1\left(\lambda_1^1\frac{\xi}{R}\right)\overline{\rho}\xi^3\right]\mathrm{d}\xi\right.$$
$$\left.+y_1\left(\lambda_1^1\frac{r}{R}\right)\int_0^r\left[j_1\left(\lambda_1^1\frac{\xi}{R}\right)\overline{\rho}\xi^3\right]\mathrm{d}\xi\right\}\sin^2\theta, \tag{2.2}$$

where $\overline{\rho}$ is the density in the non-magnetic case, R the upper boundary confining the magnetic field, and β_0 is related to the surface field intensity. λ_1^1 is the first eigenvalue allowing to verify the boundary conditions at R where we cancel both radial and latitudinal components of the field to avoid any current sheets. The functions j_l and y_l are respectively the spherical Bessel functions of the first and the second kind. The toroidal magnetic field is then given by $F(\Psi) = \lambda_1^1\Psi_1/R$. Furthermore, this state is ruled by the following helicity-energy relation $\mathcal{H} = \frac{2\mu_0 R}{\lambda_1^1}\left(E_{\rm mag} - \frac{1}{2}\beta_0 M_\Psi\right)$, which generalizes the one known in plasma physics for Taylor states (which are recovered if we do not take into account M_Ψ) to the stellar non force-free case. In the case of a stably stratified $n = 3$ polytrope (a good approximation to an upper main-sequence star radiative envelope) where we set $R = 0.85\, R_*$ (R_* is the radius of the star), we have $\lambda_1^1 \simeq 32.95$ (represented in Fig. 1). This is a generalization of Prendergast's equilibrium taking into account compressibility.

Let us now compare this analytical configuration to those obtained using numerical simulations (see Braithwaite & Spruit 2004; Braithwaite & Nordlund 2006; Braithwaite 2008). Braithwaite and collaborators performed numerical magnetohydrodynamical simulations of the relaxation of an initially random magnetic field in a stably stratified star. Then, this initial magnetic field is found to relax on the Alfvén time scale into a stable MHS equilibrium mixed configuration consisting of twisted flux tube(s). Two families are then identified: in the first, the equilibria configurations are roughly axisymmetric with one flux tube forming a circle around the equator, such as the present analytical configuration; in the second family, the relaxed fields are non-axisymmetric consisting of one or more flux tubes forming a complex structure. Whether an axisymmetric or non-axisymmetric equilibrium forms depends on the initial condition chosen for the

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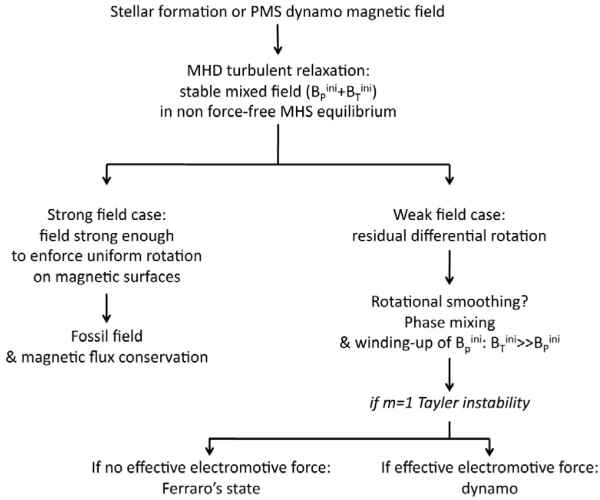

Figure 2. Interaction of fossil magnetic field and differential rotation.

radial profile of the initial stochastic field strength $||\mathbf{B}|| \propto \overline{\rho}^p$: a centrally concentrated one evolves into an axisymmetric equilibrium as in our configuration while a more spread-out field with a stronger connection to the atmosphere relaxes into a non-axisymmetric one; using an ideal-gas star modeled initially with a polytrope of index $n = 3$, the threshold is $p \approx 1/2$. Moreover, as shown in Fig. 7 in Braithwaite (2008), the selective decay of the magnetic helicity ($\mathcal{H}$) and of the magnetic energy ($E_{\rm mag}$) assumed here occurs and the transport of flux and mass in the radial direction is inhibited because of the stable stratification and the mass encompassed in poloidal magnetic surfaces is conserved (*i.e.* M_Ψ). The obtained configuration is of course non force-free.

Finally, note that this analytical configuration for which $E_{\rm mag;P}/E_{\rm mag} \approx 5.23 \times 10^{-2}$ (where $E_{\rm mag;P} = \int_{\mathcal{V}} \boldsymbol{B}_{\rm P}^2/(2\mu_0)\,{\rm d}\mathcal{V}$) verifies the stability criterion derived by Braithwaite (2009) for axisymmetric configurations: $\mathcal{A}\,E_{\rm mag}/E_{\rm grav} < E_{\rm mag;P}/E_{\rm mag} \leqslant 0.8$, where $E_{\rm grav}$ is the gravitational energy in the star, and $\mathcal{A}$ a dimensionless factor whose value is ~ 10 in a main-sequence star and $\sim 10^3$ in a neutron star, while we expect $E_{\rm mag}/E_{\rm grav} < 10^{-6}$ in a realistic star (see for example Duez *et al.* 2010b). This analytical solution is thus similar to the axisymmetric non force-free relaxed solution family obtained by Braithwaite & Spruit (2004) and Braithwaite & Nordlund (2006). Its stability has now been demonstrated by Duez *et al.* (2010a, see also Duez *et al.* in this volume).

These configurations can thus be relevant to model initial equilibrium conditions for evolutionary calculations involving large-scale fossil fields in stellar radiation zones (see for example Mathis & Zahn 2005). We here restrict ourselves to the non-rotating case, but results also apply to radiative regions in a state where rotation is uniform (Woltjer 1959); the case of MHD relaxation with differential rotation has now to be studied.

3. Interaction with differential rotation and meridional circulation

Once the initial non force-free magnetic configuration (axi or non-axisymmetric) has been established by the initial MHD turbulent relaxation processes, this interacts with

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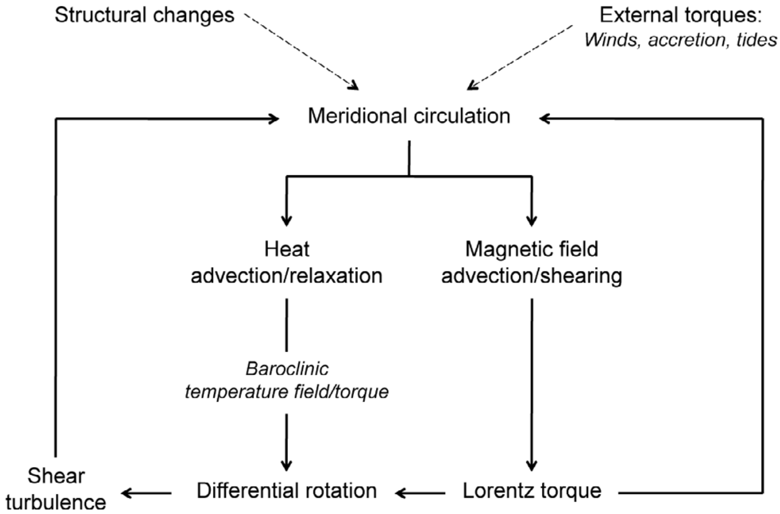

Figure 3. The transport loop in a differentially rotating magnetized stellar radiation zone

differential rotation. Then, two cases are possible as described by Spruit (1999). In the first case, if the field is strong, the rotation becomes uniform on magnetic surfaces due to Alfvén waves phase mixing, which damps the differential rotation; in the axisymmetric case this leads to the Ferraro's state where $\Omega = f(\Psi)$ and to a uniform rotation in the non-axisymmetric case (the oblique rotators case for example). Then, the field can just be modified by structural adjustments. In the second case, if the field is weak, it could first become axisymmetric if it is non-axisymmetric because of rotational smoothing and then, because of phase mixing, this leads to the Ferraro's state. This picture could be modified by magnetic instabilities, if during the first step of the phase mixing, the residual differential rotation on each magnetic surface is able to generate a strong toroidal component of the field that becomes unstable and if this instability becomes able to trigger a dynamo action through an α-effect; this question remains open (Spruit 2002; Braithwaite 2006; Zahn *et al.* 2007). The critical value of the field that gives the limit between the weak and the strong field regimes has been given in Moss (1992); Aurière *et al.* (2007). A summary is given in Fig. 2.

Let us now take into account the meridional circulation. To understand its interaction with the other dynamical processes (the differential rotation and the shear induced turbulence) in presence of a fossil magnetic field, we shall adopt the picture of rotational transport as described by Busse (1981); Zahn (1992); Rieutord (2006) and Decressin *et al.* (2009) and generalise it to the magnetic case. As described in those works, meridional circulation in radiation zones are driven by applied torques (internal like the Lorentz torque or external like those induced by stellar winds), structural adjustments during stellar evolution, and turbulent transport. In the case where all these sources vanish, the meridional circulation dies after an Eddington-Sweet time and the star settles in a baroclinic state described by the thermal wind equation. If we apply this picture to the case of radiation zones with a fossil magnetic field, we thus understand that the meridional circulation (if we consider a star without structural adjustments and external torques) will be mainly driven by the residual magnetic torque until the phase mixing leads the star to a torque-free state. Then, the meridional circulation advection of angular momentum balances the residual Lorentz torque (see Mestel *et al.* 1988).

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Discussion

M. Gagné: We know that $5-8\,M_\odot$ pre-main sequence stars go through a fully convective phase before they establish a radiative outer envelope on the main sequence. I wonder how long these stars might be in such convective state and how that might change the initial fossil field.

S. Mathis: This is one point that must be strongly investigated in a near future. Moreover, we also know that pre-neutron stars go through a convective phase and the phenomenon is thus the same for such objects. Note that the variational method presented here does not depend on the initial condition that are chosen.

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