THE THEORY OF THE MOTION OF HALLEY COMET

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ABSTRACT. The theory of the motion of Halley comet has been built using ground optical positional observations of 1682, 1759, 1835, 1910 and 1982-84. The orbital elements of the comet and two coefficients characterizing non-gravitational perturbations were determined precisely as a result of the statistical treatment of the observations. The estimations of the accuracy of the theory and the data describing its agreement with the observations are presented. The theory has been applied to ballistic-navigational calculations for the Soviet Vega project.

In March 1986 a unique opportunity will bring about many studies of the comet Halley. For carrying out these experiments, it is necessary to ensure a fly-by of the probe near the comet at distance of about 10000 km. At the preparation phase, before the launch of the Vega space-craft, this complicated navigation task requires determination of the comet coordinates at the moment of the encounter. Nowadays, it is necessary to improve the accuracies by two orders of magnitude in comparison to the current ones. In this paper we would like to present the results obtained at the preparation phase, before the launch and at the present time, and also to touch briefly upon the method of parameter improvement.

Comet Halley has a long history of its orbital motion studies. Not dwelling upon that history, we mention the results of two most important investigations of its motion dynamics. Brady and Carpenter (1971) analyzed 5 000 optical observations of the 4 recent Halley apparitions. In 1977 Yeomans published a paper on the investigations of Halley's comet orbital motion. He thoroughly selected observations in the apparitions from 1607 to 1910. The predicted time of the perihelion passage was assered from the results of orbit calculations, using three apparitions of the comet in 1759, 1835, 1910 giving the best r.m.s. fit of observations. In the two above mentioned works, the difference between the perihelion passages in 1986 was about 6^h. This is an unsatisfactory state even for draft calculations. A special program was carried out in the USSR, in constructing numerical theory of the Halley's comet motion. The theory is based on angular optical observations in apparitions from 1682 to 1910.

The studies of Halley's comet motion dynamics put forward a problem of developing a mathematical model of the motion, which could reproduce

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a true motion of the comet to a high degree of accuracy. Difficulties in the creation of such a model are connected with particularities of its orbital motion. The comet's orbit has an eccentricity close to unity, a large period of revolution and close encounters with planets. Besides, non-gravitational forces strongly disturb its motion. A rectangular equatorial coordinate system is used to describe the motion of the comet. Heliocentric coordinates of the comet were obtained by numerically integrating a system of ordinary differential equations of its perturbed motion, the effect of 9 planets being taken into account.

Let us present the equations of the motion of the comet in the Encke form, which describe increments in acceleration. To improve the accuracy of the comet motion, corrections are introduced in the classical Newtonian equations, based on the Schwarzschild solution for a spherically symmetric gravity field of general relativity. Standard coordinate system is used. The non-gravitational acceleration in the right-hand sides of the differential equations is introduced by its radial and transversal terms in the form suggested by Marsden. In order to improve the speed and the accuracy of integration and to save computer memory, we modified the computing procedure for the method of numerical integration of differential equations, suggested by Everhart. The introduction of non-linear extrapolation for the divided differences made it possible to reduce a number of iterations at each integration step, i.e. to reduce a number of references to the right-hand sides of differential equations.

The paper by Kustaanheimo and Lehti (1969) gives the formula and the value of the sidereal period of comet Halley, obtained according to the general relativity with respect to the period of the comet, moving in a Newtonian field. The introduction of relativistic terms into our differential equations leads to similar results. The variation of the sideral period is 0,I days. The allowance for the relativistic term in the differential equations of the comet motion makes it also possible to decrease slightly the r.m.s. discrepancy in right ascension. To develop this theory, astronomical constants, coordinates and velocities of the planets used are those obtained by Oesterwinter and Cohen (1972) over the fifty-year interval of optical observations.

Comet Halley has a long history of observations. In 1682 the angular positions of the comet were obtained by means of a 7-foot sextant, with a sufficiently high (at that time) accuracy. Struve and Bessel made a series of precise measurements during the apparition of 1835. Many angular optical observations from observatories equipped with powerful telescopes were obtained during the apparition of 1910. In our work we used all the unequally accurate observations of the comet, made by different instruments. Optical observations of comet Halley during 1682-1911 are presented as relative observations or observations of right ascension and declination, referred to the mean equator and the epoch of 1950.0. The paper of Rosenberger (1831) deals with reference stars, relative to wich the cometary angular distances have been determined for 37 relative observations of comet Halley, made by Flamsteed in 1682. Rosenberger used 26 observations for which he had obtained a mean error in coincidence, equal to $53''$. It should be noted that Brady and Carpenter (1971) and Yeomans (1977) used observations of 1682 which Halley had reduced to the form α, δ. The use of relative observations makes it possible to avoid

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errors in recomputing relative observations in order to obtain right ascensions and declination and also to preserve natural correlation of observations.

Table I

1759 01 22-1759 03 06			1835 08 21-1836 05 17			1909 10 09-1911 05 24		
N	K	σ	N	K	σ	N	K	σ
143	7	26.4"	812	19	9.4"	2045	54	1.56"
120		28.6"	809		8.8"	2045		1.48"

Rectangular coordinates and velocities of the motion of the comet's center of mass and coefficients A_1, A_2 were chosen as estimated parameters. The least-squares method is used to determine the improved parameters. Derivatives of the right-hand sides of differential equations of the comet motion relative to the central body are calculated as if there were only 9 point perturbations.

Roemer (1961) disputed the presence of non-gravitational perturbations. According to her opinion, the discrepancies in orbital elements of the comet in different apparitions are explained by difficulties in the observations and studies of the motion. Among them we have the systematic shift of the optical center of the cometary nucleus from the gravitation center, the asymmetry of the comet's image near the Sun, the accumulation of errors of integration at the pericenter and in the regions of close approach to the planets, the errors in the approximation of the Earth-Moon mass center motion and the inaccurate knowledge of planetary masses. In our opinion the shift of the optical center though being very important is known very poorly.

The accuracy of the vector of parameters to be determined, depends on how the weight characteristics of some observations were assigned. The condition of the comet's visibility during its apparition in 1910 allows to conclude, that the rather high r.m.s. discrepancies are caused by the difficulty of identifying the comet's center of mass. A month after the pericenter passage, comet Halley was observed as a light spot with a diameter of 30 arseconds without central condensation. The shift of the optical center from the center of gravitation can lead to major errors in improving kinematic parameters. As the reflection properties are not studied well enough even for the atmospheres of planets, we have adopted the simplest law of light reflection-the law of mirror reflection. The coma size is assumed to be inversely proportional to the comet distance from the Sun. The curve giving the change of the absolute value of the right ascension deviation is represented in Fig.1. This deviation is due to the optical center shift with respect to the comet mass center. It is visible that the adopted model of the optical center shift is in sufficient agreement with the observational data (Fig. 2).

While developing the theory, the weights of the observations were assigned on the basis of two independent errors in the observations. One of them is the fluctuation error caused by random errors. The second error is caused by the shift of the optical center. Table II presents os-

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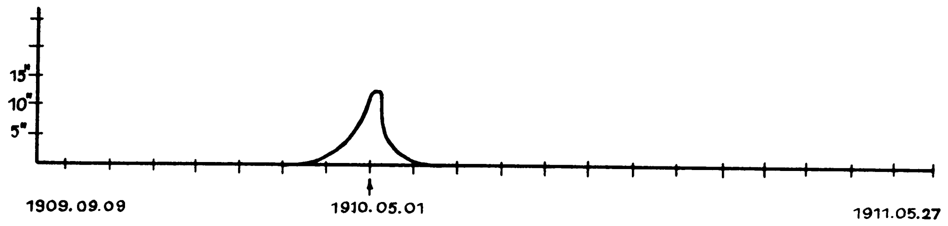

Figure 1 : Absolute value of the elevation in right ascension of comet Halley.

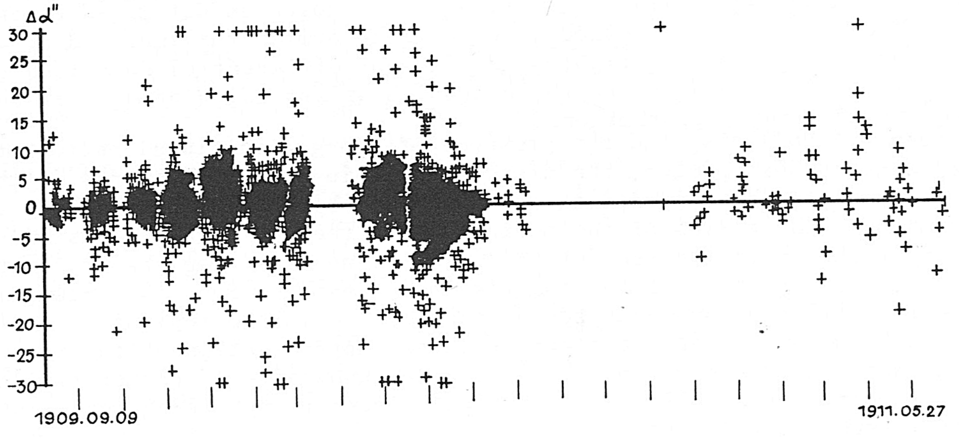

Figure 2 : Residuals Δα in 1909-1911.

TABLE II

τ	q	e	ω	Ω	i	D_{osc}
19860209.4696	0.5871023	0.9672708	111.8474	58.1456	162.2394	19860219
19100420.1783	0.5872044	0.9672982	111.7168	57.8455	162.2159	19100509
18351116.4393	0.5865597	0.9673883	110.6850	56.8014	162.2560	18351118
17590313.0125	0.5844623	0.9676814	110.6901	56.5284	162.3697	17590321
16820915.2830	0.5826139	0.9679250	109.2044	54.8497	162.2620	16820831
16071027.7630	0.5836320	0.9674914	107.5303	53.0511	162.8983	16071024

TABLE III

Observation type	Total number of observations	Number of processed observations	σ
α	3000	2484	7".8
δ	2974	2492	7".3
S	37	26	34".1

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culating elements obtained on the basis of the developed theory. The number of observations to be processed and the r.m.s, deviations are given in Table III. The comet's motion parameters obtained here make it possible to calculate discrepancies between measured and estimated values. The prediction for 1982 gives a good agreement between theoretical and observational data. The deviations in the mesured angles do not exceed 2", i.e., they are within the limits of the measurement errors. Similar discrepancies were obtained from observations of Kitt Peak Observatory and European Southern Observatory. There are 82 comet Halley observations from 1982 to the beginning of 1984. The orbit obtained with the data between 1682 and 1984 slightly differs from that whose parameters are given in Table III.

The last control operations with the VEGA stations will take place a month prior to encounters. The position of the comet will be determined from all measurements including the last January observations. An analysis was made in order to estimate the accuracy of coordinates determination at the encounter moment and it permitted us to conclude the expected accuracy (1000 km) will meet the objectives of the experiment with the VEGA stations.

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