

could perhaps have been made less repetitive. The proof that various characterizations of a complete surface are equivalent (chapter IV, §6) is somewhat vague and in one spot (top of p. 135) definitely misleading. There are too few exercises appended to the chapters of the second part and those that do appear cover only a small part of the material in the text. Several of these contain misprints or incomplete formulations. The discussion of exterior differentiation is extremely short, considering its importance for later topics. In view of the calculated conciseness of most of the presentations it could possibly be argued that too much space is taken up with a discussion of parallel fields of planes and distributions (almost as much as the whole final chapter).

In conclusion, the reviewer feels that this book deserves to be expanded in certain parts and that minor details should be clarified but that it is the best book of its kind available to English readers.

J. R. Vanstone, University of Toronto

A Modern View of Geometry, by L. M. Blumenthal. Freeman, San Francisco, 1961. xii + 191 pages. \$2.25.

Like B. Segre, the author takes the word modern (as applied to geometry) to mean "over a field that is not necessarily commutative." The first six of the eight chapters constitute a carefully prepared account of the rigorous introduction of coordinates in the manner developed by Marshall Hall, Skornyakov, and Bruck. The historical introduction includes Gauss's remark, "I consider the young geometer Bolyai a genius of the first rank," and Hilbert's evaluation of the invention of non-Euclidean geometry as "the most suggestive and notable achievement of the last century." A discussion of infinite sets and truth tables leads naturally to the idea of a system of axioms (or "postulates", as the author prefers to call them). This idea is illustrated by the finite planes  $PG(2, 2)$  and  $EG(2, 3)$ . The author remarks that "the period from 1880 to 1910 saw the publication of 1,385 articles devoted to the foundations of geometry." He cites absolute geometry as "a good example of postulational system that is very rich in consequences and [yet] incomplete" (that is, not categorical).

In Chapter V, he considers the possibility of introducing, into a "rudimentary affine plane," coordinates  $x$  and  $y$  in terms of which a line has a linear equation. He finds a necessary and sufficient condition to be the "first Desargues property" (i. e., Desargues's theorem for triangles that are congruent by translation). For the coordinates to belong to a field (not necessarily commutative), a necessary and sufficient condition is the "third Desargues property" (i. e., Desargues's theorem for homothetic triangles). The author

could have simplified the statement of his theorem on page 96 by remarking that this "third Desargues property" implies the "first" (Coxeter, Introduction to Geometry, Wiley, New York, 1961, p. 193). He introduces an affine form of Pappus' s theorem as a condition for the field to be commutative, and he proves (in the manner of Hessenberg) that this implies the third Desargues property.

Chapter VI contains the analogous treatment of projective planes, such as Veblen-Wedderburn planes and alternative planes. On page 124 he remarks truly that the general theorem of Desargues is valid if and only if its converse is valid, but he gives the wrong reason, namely, that the converse is the dual. The correct reason is that the converse follows from the direct theorem as applied to a different pair of triangles; it is only after this remark has been made that the principle of duality can be seen to remain valid when Desargues' s theorem has been introduced as an extra axiom.

Chapter VII introduces the concept of a metric space. (Here it is, perhaps, unfortunate that points are not denoted by capital letters. For anyone with an old-fashioned schooling, it takes considerable effort to remember that  $pq^2$  means the square of  $PQ$  and not  $p$  times  $q^2$ .) This chapter establishes, for the Euclidean plane, a remarkable set of six axioms, using the bordered determinant of the squared distances of four points. Finally, Chapter VIII shows how these axioms can be modified so as to yield the hyperbolic plane (page 181), the spherical plane (page 185), or the elliptic plane (page 187).

The literary style leaves something to be desired; e. g. , on page 4 we read of "a point  $C$  whose existence is not, nor cannot be, established. . . ," and on page 16, points at infinity are called "infinite points." Particularly irritating is the author' s continual use of the word "pairwise," which has evidently been adapted from the German "paarweise." The only place where this innovation can reasonably be said to effect verbal economy is in the Axiom of Choice at the bottom of page 29. In most other cases it can either be omitted (as in "four pairwise distinct points") or replaced by "in pairs" (as in "joining them pairwise").

The 56 figures have been nicely drawn by Evan Gillespie, and there is an adequate index.

H. S. M. Coxeter, University of Toronto