Aerodynamic Dissipation

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ANALYSES of aerodynamic dissipation in ordinary un-ionized gases are all based upon the Navier-Stokes equations. These equations relate the rate of dissipation to the local gradients in velocity and temperature through the viscosity and heat conduction coefficients. Although it is true that in many flow situations the magnitude of the total dissipation in the gas does not depend on the magnitude of the viscosity coefficient, this coefficient does determine the minimum scale of variations observed in the gas and the form of the Navier-Stokes equations determines the type of phenomena which are observed on a small scale. In order to discuss dissipation in an ionized gas in the presence of a magnetic field, it is therefore necessary to re-examine the derivation of the basic flow equations. This paper attempts to do this for a case of a completely ionized gas and demonstrates that the basic microscopic dissipation mechanism is appreciably different. For example, it is shown that the minimum length in which the properties of the flow field can change noticeably is appreciably less than one mean free path.

For un-ionized gases there are two well-known derivations of the Navier-Stokes equations. The first is a phenomenological approach based upon the experimentally observed fact that the shear stress is directly proportional to the velocity gradients. The second approach is based on an expansion of the Maxwell-Boltzmann equation which describes the history of the individual particle motions in the gas. Since at present there are no experimental data concerning dissipation rates in a completely ionized gas, it is necessary to refer to the Boltzmann equation. The basic equations from which one must start in order to derive hydrodynamic equations are, therefore, two Boltzmann equations, one for the electrons and one for the ions, coupled with the four Maxwell equations which describe the electromagnetic field.

CLASSIFICATION OF REGIONS

In attempting to derive useful hydrodynamic equations, it is worthwhile first to examine the magnitude of various terms in the Boltzmann equation.[1] In this way it is possible to define regions in terms of the gas state where one would expect different terms in the Boltzmann equation to be dominant and therefore different flow phenomena to occur. The Boltzmann equation for the ions is

$$\frac{\partial f}{\partial t}+\mathbf{V}\cdot\nabla f+\frac{e}{M}\left(\mathbf{E}+\frac{\mathbf{V}\times\mathbf{H}}{c}\right)\cdot\nabla_v f=\left(\frac{\partial f}{\partial t}\right)_{\text{coll}}, \quad (1)$$

where f is the number density of ions in six-dimensional phase space, $\mathbf{V}$ is the velocity of an ion, $\mathbf{E}$ is the electric field, e and M are the ionic charge and mass, c is the velocity of light, $\mathbf{H}$ is the magnetic field, t is time, ∇_v is the gradient with respect to the components of the velocity vector and $(\partial f/\partial t)_{\text{coll}}$ is the net influx into six-dimensional phase space due to collisions. The collision term may, for order-of-magnitude purposes, be approximated by[2]

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}}=\frac{f_0-f}{\tau},$$

where f_0 is the Maxwell distribution and τ is the mean free time between collisions. This approximation is based upon Maxwell's conclusion that a nonequilibrium gas adjusts to a Maxwellian distribution in about one mean free time and assumes that the mean free time is of the same order of magnitude for particles of a given type but of all velocities. If l is taken as the characteristic length associated with variations in the flow field, Eq. (1) may be multiplied by $l/\bar{V}'f=t_0/f$ in order to make it nondimensional. This gives

$$t_0\frac{\partial \ln f}{\partial t}+\frac{\mathbf{V}}{\bar{V}'}\cdot l\nabla \ln f+\frac{e\mathbf{E}'l}{M\bar{V}'^2}\bar{V}'\cdot\nabla_v \ln f$$
$$+\frac{l}{r_i}(\mathbf{V}'\times\mathbf{H}_1)\cdot\nabla_v \ln f=\frac{l}{\lambda}\left(\frac{f_0}{f}-1\right), \quad (2)$$

where $\mathbf{H}_1$ is a unit vector in the magnetic field direction; $\mathbf{E}'=\mathbf{E}+(\mathbf{v}\times\mathbf{H})/c$ is the electric field in a coordinate system moving at the gas velocity $\mathbf{v}$; $\mathbf{V}'$ is the thermal velocity of an ion, the bar indicating an average value; r_i is the ion Larmor radius; and λ is the mean free path.

In conventional aerodynamics the characteristic length of the flow field is usually much larger than the mean free path. The coefficient of the collision term is then a very large number as compared to the gradient terms. Therefore $(f_0/f)-1$ must be of order λ/l, so that to zero order in λ/l the distribution function is Maxwellian at all points in the flow field. The Navier-Stokes equations are obtained by substituting a Maxwellian distribution into the terms on the left-hand

[1] A. R. Kantrowitz and H. E. Petschek, "An Introductory Discussion of Magnetohydrodynamics" from *Magnetohydrodynamics*, edited by R. K. M. Landshoff (Stanford University Press, Stanford, California, 1957).

[2] Bhatnagar, Gross, and Krook, Phys. Rev. 94, 511 (1954).

side of the equation and evaluating a first-order correction to the distribution function to be used in the collision term.

In estimating the magnitudes of the remaining terms in the ionized gas case let us begin with the electric field term. If plasma oscillations are not set up, the role of the electric field is to insure equal accelerations for the electrons and ions in the gas. The electric field required will be of the order of the acceleration per particle of the entire gas, thus

$$N_i e\mathbf{E}' \approx \rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\boldsymbol{\nabla}\mathbf{v}\right)\approx 0\left(\frac{\rho \bar{V}'^2}{l}\right);$$

$$e\mathbf{E}' = 0\left(\frac{M\bar{V}'^2}{l}\right),$$

where N_i is the ion density and ρ is the mass density. Therefore the coefficient involving the electric field is of order-of-magnitude unity as compared with the first two terms in Eq. (2).

If the Larmor radius is less than the mean free path, the coefficient of the term involving the magnetic field becomes larger than the coefficient of the collision term. If one now considers the case $l/r_i \gg 1$, the magnetic field term exerts the controlling influence on the particle motions and to zero order the distribution function must be such that

$$(\mathbf{V}'\times\mathbf{H}_1)\cdot\boldsymbol{\nabla}_v \ln f = 0. \qquad (3)$$

This requires f to be of the form

$$f = f(|\mathbf{V}'\times\mathbf{H}_1|,\ \mathbf{V}'\cdot\mathbf{H}_1,\ x,\ y,\ z,\ t). \qquad (4)$$

In other words, the particles describe circles about the magnetic field lines but the distribution of velocities are not restricted to be Maxwellian.

In Fig. 1 an attempt is made to indicate the regions in which different terms in the Boltzmann equation will be dominant in terms of the gas state. It is assumed that the gas pressure is equal to the magnetic pressure. For other ratios of these pressures the positions of the bounding lines are somewhat different. At high temperatures and densities (S region) the mean free path is less than the Larmor radius for both the electrons and the ions. In this case the dominant term in both Boltzmann equations is the collision term and the transport properties have a similar form to those in an un-ionized gas. In this region the particle paths are essentially straight between collisions and therefore the electrical conductivity is a scalar.

At somewhat lower densities and higher temperatures (T region) the electron Larmor radius becomes less than the mean free path, resulting in a tensor electrical conductivity. At still lower temperatures and higher densities (M region) the ion Larmor radius also becomes less than the mean free path. In this region, as shown later, the basic dissipation mechanism becomes

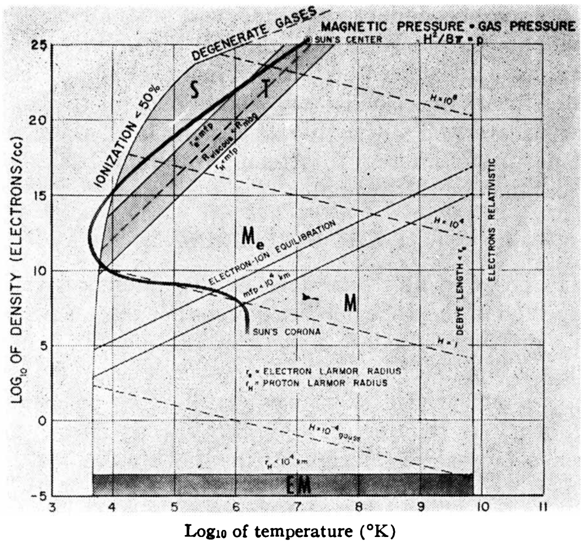

FIG. 1. Magnetohydrodynamics flow regions for fully ionized hydrogen.

appreciably different. If a reasonable length scale for astronomical phenomenon is taken as 10^4 km, there is a density below which the ion Larmor radius becomes larger than this length at extremely low densities. In this region (EM) the electron motion is controlled by the magnetic field, but the ion motion is controlled only by the electric field which insures charge neutrality.

It is to be expected that the change in the dominant term in the Boltzmann equation which occurs at the boundaries of each of the regions in Fig. 1 will produce different basic phenomena in the different regions.

In addition to defining the regions on this map several other lines have been drawn. The conditions under which the mean free path is equal to what has been taken as a typical length is indicated. Above this line one would expect some tendency for particles of one type to assume a Maxwellian distribution. Since it takes many collisions for electrons and ions to adjust to the same temperature, somewhat higher densities are required before one would expect these two temperatures to be essentially equal. On the basis of these two lines one may subdivide the M region into two regions, M_e and M. This is, however, only a subdivision, since the dominant term in both of these regions is the magnetic field term. The Debye length is less than the electron Larmor radius in all of the region covered by the map or as long as the electron thermal velocities are not relativistic. Lines of constant magnetic field strength have been drawn. The line along which the viscous and magnetic Reynolds numbers are equal for a given length has also been drawn. In most of the S and T regions the viscous Reynolds number is larger than the magnetic Reynolds number, whereas most of the conditions under which the magnetic Reynolds number would be larger than the viscous Reynolds number are in the M region where it is not clear that the ordinary

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concepts of conductivity and viscosity apply. For reference purposes the conditions in the interior of the sun have also been indicated. Figure 1 shows that interstellar gas clouds and conditions in the solar corona are well within the M region. It is, therefore, of particular interest to attempt to obtain basic flow equations for this region.

PULSE STEEPENING

In order to illustrate some of the differences to be expected in the M region, let us consider the steepening of a pressure pulse into a shock wave. In an un-ionized gas the nonlinearity of the flow equations produces a steepening which continues until the steepening tendency is counteracted by the viscous effect. The final steady-state thickness of the shock wave is of the order one mean free path.

For the **M** region gas, let us consider a particular one-dimensional, time-dependent problem. We assume that a broad pulse has been produced in the fluid by, for example, the motion of a piston in the x direction. The magnetic field is taken in the z direction. Since quantities vary only with x, this choice of magnetic field automatically satisfies the equation for the divergence of the magnetic field. The electric field in the z direction can be chosen as a boundary condition and set equal to zero. An electric field exists in the x direction in order to maintain equal acceleration of the electrons and the ions. An electric field is also induced in the y direction due to changes of the magnetic field with time.

The procedure adopted in order to determine hydrodynamic flow equations is similar to that which is used in the Chapman-Enskog method of deriving flow equations from the Boltzmann equation. We first take moments of the Boltzmann equation corresponding to conservation of mass, momentum, and energy. These moment equations involve particular moments of the distribution function. The latter moments are evaluated by going back to the Maxwellian-Boltzmann equation. In doing this we assume that the typical scale length of the pulse is much larger than both the Debye length and the electron Larmor radius. We keep terms containing the Larmor radius of the ions. These terms are dropped for the calculation of the steepening of the pulse; however, they are of use in the next section where an attempt is made to derive the final structure of a shock wave.

Integrating the Maxwell-Boltzmann equation over the velocity coordinates at a fixed position in time results in a continuity equation for each species

$$\frac{\partial N_e}{\partial t}+\frac{\partial N_e U_e}{\partial x}=0,$$

$$\frac{\partial N_i}{\partial t}+\frac{\partial N_i U_i}{\partial x}=0,$$

where N_e and N_i are the electron and ion densities, and U_e and U_i are the electron and ion mean flow velocities in the x direction. Since the Debye length is taken as very small, the gas to a good approximation has essentially equal densities of electrons and ions at all points. In other words, the Poisson equation may be replaced by

$$N_e = N_i.$$

Substitution of this condition in the above continuity equations show that $\partial[N_e(u_e-u_i)]/\partial x=0$, or that the x component of the current $eN_e(u_i-u_e)/c$ must be independent of position. Therefore if we exclude a uniform current in the x direction by the choice of suitable boundary conditions at plus and minus infinity, the two velocities must be equal. The continuity equations may then be combined to give

$$(\partial\rho/\partial t)+(\partial\rho u/\partial x)=0, \tag{5}$$

where the subscript on the velocity has been dropped.

The assumption that the electron Larmor radius is much less than the characteristic scale of the pulse implies that to a good approximation the magnetic field term in the electron Boltzmann equation must be equal to zero or that the electron distribution function satisfies Eq. (4). Since the electrons are closely coupled to the magnetic field and since there are no gradients in the y direction, their mean velocity in the x direction is at all points equal to the so-called velocity of the magnetic field, cE_y/H. The equation for the curl of the electric field may then be written as

$$c\frac{\partial E_y}{\partial x}+\frac{\partial H}{\partial t}=\frac{\partial uH}{\partial x}+\frac{\partial H}{\partial t}=0.$$

Combining this with the continuity Eq. (5) implies

$$\frac{H}{\rho}=\text{const.} \tag{6}$$

Equation (6) is a result of the fact that the gas is in the M region and that the pulse is large compared to the electron Larmor radius. The infinite conductivity assumption has not been made directly.

Multiplying each of the Boltzmann equations by the momentum in the x direction of each particle, integrating over all velocities, adding the two equations, and making use of the equation for the curl of the magnetic field

$$\frac{\partial H}{\partial x}=-4\pi j=-4\pi\frac{Ne}{c}(v_i-v_e),$$

where j is the current density and v_i and v_e are the mean velocities in the y direction for the ions and electrons,

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one obtains

$$\rho\frac{Du}{Dt}+\frac{\partial[p_{ixx}+p_{exx}+(H^2/8\pi)]}{\partial x}=0, \qquad (7)$$

where

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}; \quad p_{ixx}=m_i\int U'^2 f_i dv';$$

p_{exx} is a similar integral over the electron distribution function; and U' is the x component of the velocity relative to the mean flow velocity. The equation for conservation of momentum in the y direction can be obtained by multiplying by the y component of velocity. The equation expressing conservation of energy may be obtained similarly by multiplying the two Boltzmann equations by the total kinetic energy per particle:

$$\rho\frac{D}{Dt}\left(\frac{p_{ixx}+p_{iyy}+p_{izz}+p_{exx}+p_{eyy}+p_{ezz}+H^2/4\pi}{2\rho}\right)$$
$$+\left(p_{ixx}+p_{exx}+\frac{H^2}{8\pi}\right)\frac{\partial u}{\partial x}+p_{exy}\frac{\partial v_e}{\partial x}$$
$$+p_{ixy}\frac{\partial v_i}{\partial x}+\frac{\partial q}{\partial x}=0, \qquad (8)$$

where the pressures are defined as above, and

$$q=\tfrac{1}{2}m_i\int U'(U'^2+V'^2+W'^2)f_i d\mathrm{V}'$$
$$+\tfrac{1}{2}m_e\int U'(U'^2+V'^2+W^2)f_e d\mathrm{V}'.$$

Formally, the collision terms appear to have dropped out of these equations since mass, momentum, and energy are all conserved on collision. In ordinary gas dynamics the effect of collisions comes in the expressions which define the moments of the distribution function such as q. However, if the magnetic field terms are dominant in the Boltzmann equation ($r_i \ll \lambda$) and if we assume for the time being that the scale of the pulse is still very large compared to the ion Larmor radius, one may to a good approximation write the ion and the electron distribution functions in the form given by Eq. (4).[3] In this case the symmetry of the distribution functions reduces Eqs. (7) and (8) to the form,

$$\rho\frac{Du}{Dt}+\frac{\partial[p+(H^2/8\pi)]}{\partial x}=0 \qquad (9)$$

and

$$\rho\frac{D}{Dt}\left(C_v\frac{p}{\rho}+\frac{H^2}{8\pi\rho}\right)+\left(p+\frac{H^2}{8\pi}\right)\frac{\partial u}{\partial x}=0, \qquad (10)$$

where

$$p=p_{exx}+p_{ixx}=p_{eyy}+p_{iyy}$$

and

$$C_v=1+\tfrac{1}{2}\frac{p_{ezz}+p_{izz}}{p_{exx}+p_{ixx}}.$$

Equations (5), (6), (9), and (10) form a set of hydrodynamic equations which are very similar in form to the ordinary equations for inviscid flow. The set is not quite complete as yet, since the magnitude of C_v has not been specified. This quantity can be determined easily in two limiting cases. If the scale of the pulse is large compared to the mean free path, collisions will insure that the distribution function is isotropic in three directions, so that $C_v=\frac{3}{2}$. If the scale of the pulse is smaller than the mean free path there will not be enough collisions to affect the particle motion in the z direction; and since the electric and magnetic fields do not accelerate particles in this direction, the kinetic energy per particle due to motion in the z direction will be a constant; that is, $D/Dt(C_v-1)p/\rho=0$. In this case one may effectively take $C_v=1$ in Eq. (10). The only effect of the collision term in these equations is therefore to change the ratio of the internal energy of the gas to the pressure, or the effective specific heat of the gas, by a small factor.

Assuming that the pulse started with a scale large compared to the mean free path, the equations are initially identical with the inviscid flow equations and the pulse will tend to steepen towards a shock wave. When the pulse width becomes comparable with the mean free path, the flow equations change very slightly because of the change in specific heat, but the essential basis for the steepening process is still present. This steepening then continues until the assumption that the pulse width is much larger than the ion Larmor radius breaks down. We conclude that a shock wave will steepen until its thickness is comparable with an ion Larmor radius or possibly even less.

There is one minor exception to the above conclusion. The speed of sound in the gas with the assumed geometry of the magnetic field is

$$a=\left(\frac{C_v+1}{C_v}\frac{p}{\rho}+\frac{H^2}{4\pi\rho}\right)^{\frac{1}{2}}. \qquad (11)$$

This speed increases slightly as C_v decreases. It is therefore possible to have a very weak wave whose velocity would be supersonic when collisions adjust the three degrees of freedom of particle motion, but whose velocity is subsonic if the pulse width becomes so small that collisions are unimportant. Such a wave would therefore steepen if its width is longer than a mean free path, but the steepening would not continue beyond the point where its local velocity becomes sonic. With the exception of this very small range of shock velocities, the shock thickness is limited by the ion Larmor radius and possibly even by a smaller dimension.

[3] This type of approach was suggested for the case where there are no collisions by Chew, Goldberger, and Low, Proc. Roy. Soc. (London) **A236**, 112 (1956); K. M. Watson, Phys. Rev. **102**, 12 (1956); and K. M. Watson and K. A. Brueckner, Phys. Rev. **102**, 19 (1956).

This conclusion is in contradiction with calculations of the shock structure which had been made by Marshall[4] and Sen,[5] who both conclude that the mean free path is the important dimension. Both of these calculations made use of a viscosity coefficient quoted in Chapman and Cowling.[6] Their coefficient differs only by a numerical factor between the cases where the ratio of Larmor radius to mean free path is extremely small and where it is extremely large. The source of the error in the Chapman and Cowling result has not been located, since the detailed calculations are not presented. It is, however, clear from the above arguments that the viscosity is effectively reduced in the presence of a strong magnetic field. Physically one may explain this reduction in viscosity by the fact that in the presence of a magnetic field the mean velocity of an ion is adjusted continuously between collisions by the electric and magnetic fields, whereas in the absence of the magnetic field the particle velocity remains constant between collisions. The calculations of Marshall and Sen are therefore only valid in the S and T regions where the magnetic terms do not dominate the Boltzmann equation.

MAGNETIC STORMS

One example of an astrophysical phenomenon which seems to indicate the existence of a shock wave which is much thinner than a mean free path is the sudden commencement of magnetic storms on the earth. It was suggested by Gold[7] that this sudden commencement was due to a shock wave arising from a disturbance on the sun. The objection which has been raised to this suggestion was that a temperature corresponding to the velocity at which these waves travel, $\sim 2\times 10^8$ cm/sec, and assuming an interplanetary gas density of about 10^3 particles per cubic centimeter, the mean free path is much greater than one astronomical unit. Therefore, a shock wave one mean free path thick could not be formed between the earth and the sun. However, if one now assumes that the shock thickness is comparable with the ion Larmor radius, then for an interplanetary magnetic field of 10^{-5} gauss, the time required for a shock wave at this velocity to pass a particular point will be only of the order of 10 sec. Since this time is less than the observed two-minute time associated with the commencement of magnetic storms, it seems very likely that these storms may indeed indicate the arrival of a shock wave from the sun. The fact that the observed signal has a slower rise time than the incident shock wave is probably caused by delays in the transmission of the signal through the ionosphere.

[4] W. Marshall, Proc. Roy. Soc. (London) **A233**, 367 (1955).
[5] H. K. Sen, Phys. Rev. **102**, 5 (1956).
[6] S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases* (Cambridge University Press, New York, 1953), p. 337.
[7] T. Gold, "Discussion on shock waves and rarefied gases," from *Gas Dynamics of Cosmic Clouds*, edited by H. C. van de Hulst and J. M. Burgers (North Holland Publishing Company, Amsterdam, The Netherlands, 1955).

STEADY-STATE SHOCK STRUCTURE

An attempt to compute the final steady-state shock structure utilizing a method similar to the Chapman-Enskog expansion has been attempted. This method consists basically of computing corrections to the zero-order distribution function given by Eq. (4) from the Boltzmann equation and using the corrected distribution function to evaluate the moments required in Eqs. (9) and (10). This procedure assumes that the distribution function differs only slightly from the zero-order distribution function. One therefore, expects it to be valid only for the case of fairly weak shock waves where one might expect, by analogy with the structure of a shock wave in an un-ionized gas, that the thickness of a shock wave would be at least several Larmor radii.

Assuming that the ion Larmor radius is very much shorter than the mean free path, there will be virtually no collisions in the shock front and, therefore, the collision terms may be neglected. On this basis the first-order correction to the distribution function is given by

$$\frac{e}{m_i c}(\mathbf{V}'\times\mathbf{H})\nabla_v f_1 = -\mathbf{V}'\nabla f_0 - \frac{e\mathbf{E}'}{m_i}\cdot\nabla_v f_0,$$

where the subscripts 0 and 1 represent the order in the expansion. The corresponding correction to the electron distribution function is much smaller because the electron Larmor radius is so much smaller, and has therefore been neglected. Substituting the distribution function to first order into the momentum and energy equations (7) and (8) reduces them again to the form given in Eqs. (9) and (10). It is therefore necessary to continue the expansion to second order, before one obtains a pressure tensor which contains terms analogous to the viscous stresses and before one obtains a heat flux vector. Making use of the second-order terms and making the approximation of weak shock waves, one obtains a second-order differential equation for the variation of the density in a steady-state pulse. The analogous equation for an un-ionized gas is a first-order differential equation which permits a smooth variation of gas density from the conditions in the supersonic stream to those in the subsonic stream. The second-order equation obtained in this case does not permit such a solution. The solution obtained from this equation describes a pulse in which the gas density increases, goes beyond the density required to satisfy the Rankine-Hugionot equations in the subsonic stream and increases to a maximum, and finally decreases again returning to the initial density in the supersonic stream. The width of this pulse is $\pi/4[r_i/(M-1)^{\frac{1}{2}}]$, where r_i is the Larmor radius based on the flow velocity and M is the ratio of the flow velocity to the sound speed in Eq. (11). This pulse is *not* a shock wave, since it returns to the initial condition. If the collision term had not been dropped completely, the final state behind the pulse would have been slightly different from the initial condition and a series of pulses would follow. These pulses would eventually damp in a distance of the order of a mean free path and leave the gas in the appropriate condition for the subsonic stream. However, if the mean free path is very much longer than the ion Larmor radius, there would be very many pulses in this series and it is questionable that such a long train of pulses would be stable.

Longmire and Rosenbluth and Colgate[8] suggested that a shock wave of this type should not have a steady-state structure, but would oscillate in time even in a coordinate system moving with the shock wave. Colgate has assumed without justification that the important length associated with this oscillation is the electron Larmor radius. However, the above calculation indicates that some effects begin to occur with a scale comparable to the ion Larmor radius. One might therefore be more justified in assuming that the ion Larmor radius is the important length.

At present it is not clear what is the final structure of such a shock wave. This leaves open the question of whether the dissipation associated with a shock wave produces a high ion temperature or a high electron temperature immediately behind the shock wave. It is interesting to speculate on the possibility that such a shock wave is in fact unsteady in time and may therefore lead to the emission of radio waves. Also, it is conceivable that the form of the dissipation mechanism is such that a few particles are accelerated to very high energies and thus a shock wave might be a source of cosmic rays. This acceleration would seem plausible if it is true that the shock structure is time dependent and if the frequency associated with oscillation is the cyclotron frequency of the ions.

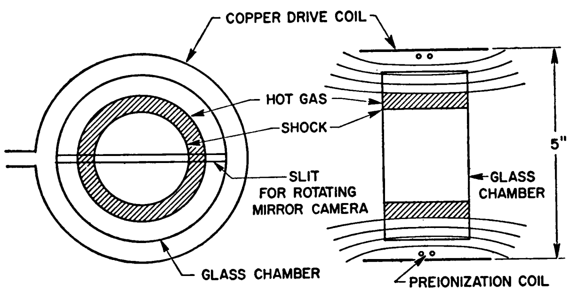
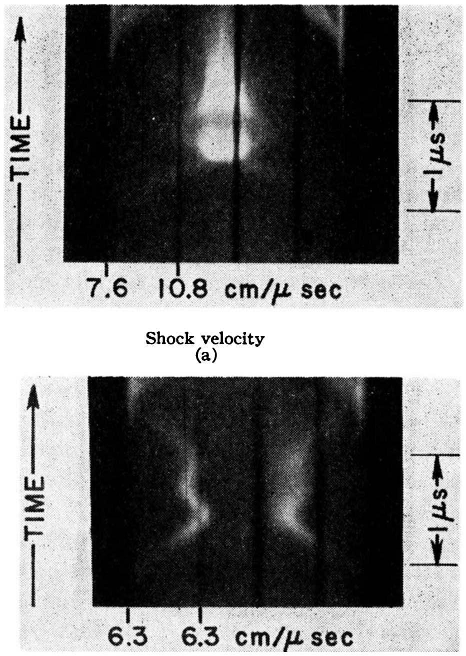


FIG. 2. Schematic diagram of gas accelerator to produce cylindrically converging shock waves.



Shock velocity
(b)

FIG. 3. Mirror camera pictures of cylindrically converging shock waves in hydrogen. Horizontal axis indicates distance along the diameter of the chamber (see slit indicated in Fig. 2) and vertical axis indicates time. The initial pressure was (a) 0.2 mm Hg and (b) 0.3 mm Hg. In (a) the gas was preionized so that there was no magnetic field in the center. In (b) it was not preionized.

EXPERIMENTAL*

Before concluding, I will briefly mention experiments which are being performed at the AVCO Research Laboratory with an aim of studying gas dynamics in the M region. In order to produce a laboratory sample of gas in the M region with a magnetic pressure of the order of the gas pressure one requires a sample of gas at about 10^6 °K and a density of the order of 10^{16} particles per cubic centimeter. At the present time, temperatures of the order of 3×10^5 °K have been achieved.

A schematic diagram of the experimental setup is shown in Fig. 2. A condenser bank is discharged suddenly into the drive coil. This produces an axial magnetic field inside the coil. The gas inside the chamber which has been preionized by a low-energy discharge excludes the magnetic field from the center of the chamber by a current on the surface. The magnetic field then acts as a piston pushing on the outer radius of the gas and produces a cylindrically converging shock wave. Shock velocities as high as 12×10^6 cm/sec in deuterium have been obtained. Using the Rankine-Hugoniot conditions across the shock wave, this velocity corresponds to a temperature of 3×10^5 °K. The experimentally observed shock velocities are in agreement with a theoretical prediction based upon setting the magnetic pressure equal to the gas pressure behind the shock wave.

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[8] S. A. Colgate, University of California Radiation Laboratory Report, UCRL 4829 (1957).

* This experimental program is being carried out primarily by G. Sargent Janes.

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Figure 3 shows mirror camera pictures of shock waves produced in this manner. In Fig. 3(a) the shock wave starts from the outside of the chamber and continues to the center where it is reflected. In this case there was no magnetic field in the center of the chamber before the shock wave was initiated. In Fig. 3(b) the shock wave does not proceed to the center of the chamber but the gas appears to be reflected before it reaches the center. In this case no preionization was used so that some of the magnetic field leaked through the gas to the center of the chamber before breakdown actually occurred. The gas is then presumably reflected by the compression of the magnetic field in the center. In this picture the gas has been slowed down gradually by the magnetic field and that because of the small extent of the gas sample no reflected shock has been formed. This is to be contrasted with Fig. 3(a) where the gas is decelerated rapidly at the center and a reflected shock wave is formed. These pictures are an example of one way in which dissipation in a gas can be reduced by the presence of a magnetic field.

DISCUSSION

H. K. SEN, *GRD, AFCRC, Hanscom Field, Bedford, Massachusetts:* In my paper [Phys. Rev. **102**, 5 (1956)] on magnetohydrodynamic shock structure, I found that the pressure tensor as derived by Chapman and Cowling (*Mathematical Theory of Non-uniform Gases*) reduces to the usual magnetohydrodynamic extension of the Navier-Stokes equation for two asymptotic cases: $\omega/\nu \ll 1$ and $\omega/\nu \gg 1$, where ω is the gyrofrequency (in deference to Laporte, I would not call it the Larmor frequency) and ν is the collisional frequency. The first case is the hydrodynamic analysis with the magnetic field as a perturbation. The second case, curiously enough, turns out to be similar to the first, with a pseudo-viscosity$\simeq\frac{1}{4}$ times the ordinary viscosity. The Chapman-Cowling treatment, probably, is no longer valid in this case.

A comprehensive analysis should, however, be based on the nondimensional ratio ω/ν as a parameter, so that it could yield the two asymptotic limits mentioned above and at the same time be valid for the physically interesting transition region where $\omega/\nu \simeq 1$. This remark is not trivial, inasmuch as uncritical neglect of parameters has not infrequently led to singularities with no physical basis whatsoever. The implication is that there is no *a priori* reason to expect that the results obtained from a treatment with complete neglect of collisions would closely approximate or even be similar to those that obtain for weak collisions ($\omega/\nu \ll 1$).

H. E. PETSCHEK, *Avco Research Laboratory, Everett, Massachusetts:* The results that are quoted in Chapman and Cowling do give the result that the viscosity is essentially the same in the two limits of small and large ω/ν. However, I believe that this is incorrect. It is difficult to follow exactly where the error is in Chapman and Cowling, since they do not give a detailed discussion for this particular case but only quote the results. Physically, I think it is quite clear that for the case where the cyclotron frequency is much larger than the collision frequency, the viscosity will be appreciably reduced and shock waves will steepen.

H. W. LIEPMANN, *Daniel Guggenheim Aeronautical Laboratory, California Institute of Technology, Pasadena, California:* I did not see any dissipation in your model, so I do not see how you can get a thickness of a shock wave at all.

H. E. PETSCHEK: This is exactly the problem—that for the steady-state situation there appears to be no dissipation. If one looks at higher orders in this expansion, they also indicate no dissipation. Now the type of dissipation one *may* get is if the flow becomes unstable, and makes a general mess, which will be a dissipation.

H. W. LIEPMANN: I do not even see that. How do you get dissipation from instability? You must somehow have a mechanism like viscosity or some sort of randomization. You have to increase the entropy, and the rate of increase of entropy determines the shock thickness.

H. E. PETSCHEK: The randomness introduced by the instability is already an increase in entropy.

H. W. LIEPMANN: I think we are getting into information theory.

L. SPITZER, JR., *Princeton University Observatory, Princeton, New Jersey:* Several mechanisms can be invoked. In the first place, we may refer to the quantity: square of the velocity perpendicular to the magnetic field divided by the magnetic field. This quantity is an adiabatic invariant, which is constant for slow changes. In a shock this quantity changes, and I believe this change may lead to a change of entropy of the system. In the second place, a change of entropy may occur through fine-scale mixing. Let us suppose that behind a shock there are oscillations which involve wiggles in the velocity distribution function. With increasing distance behind the shock, these wiggles or irregularities would have shorter and shorter wavelengths, and must

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ultimately be damped out even by very weak collisions. I believe these two mechanisms may be the ones that must be invoked for the dissipation in a shock.

E. C. BULLARD, *Department of Geodesy and Geophysics, Cambridge University, Cambridge, England:* Does dissipation mean getting the energy into heat, and, therefore, that you must have collisions sooner or later? Or can the field be irregular enough to randomize the motions without collisions?

L. SPITZER, JR.: Finally, of course, collisions are required to yield a situation where the entropy can be computed by classical means.

H. E. PETSCHEK: It is not clear that the change in magnetic moment when there is a sharp change in the field is an entropy change. For the case where $\gamma=2$, this is the usual isentropic relation. And, as Spitzer has pointed out, this is not necessarily valid if the gradient becomes steep compared to a Larmor radius. One can show, for example, that if one has a sudden change in magnetic field which will produce a change in this quantity for a particle going across, and if this change is reversed, at the distance which is precisely the distance the guiding center has moved in one Larmor orbit, the gas particles will all come out in the condition in which they started. Therefore, this is not an irreversible process.

A. SCHLÜTER, *Max Planck Institut Für Physik, Böttingerstrasse 4, Göttingen, Germany:* The fact that the magnetic moment of the spiraling motion of the particles is not constant does not in itself determine the shock width, because one really needs a mechanism which produces entropy, and this mechanism does not produce entropy even if the magnetic moment changes. So the only process which generates entropy is collisions, and if the rate of collisions is small, then the deviations from thermodynamic equilibrium must be so large that the few collisions can do the job.

L. SPITZER, JR.: That is certainly entirely true. However, we have been wondering whether one should perhaps define a more generalized entropy, to discuss conditions with fine scale mixing, which on the macroscopic scale produces without collisions, essentially the same effect that collisions would produce.

A. SCHLÜTER: Do I understand you correctly in saying that the rate of change of the magnetic moment depends upon the relative phase of the particle in its spiraling motion relative to the phase of the shock wave traveling through the gas and that, therefore, you get something which corresponds to the effect of collisions? If so, I see the point [cf. F. Hertweck and A. Schlüter, Z. Naturforsch. (to be published)].

R. LANDSHOFF, *Missile Systems Division, Lockheed Aircraft Corporation, Sunnyvale, California:* The comment I want to make is quite similar to what Spitzer has said. Actually, the charged particles certainly interact with each other all the time. But artificially, in order to treat the interaction so as to fit the mathematical formation of the Boltzmann equation, we divide it into two parts. A smooth part we treat as field, invoking the Maxwell equations; the rest whose cross sections go down at high velocities as $(e^2/mv^2)^2$ we call collisions. But nevertheless, interactions are there, and that we divide them up in this fashion does not mean that they cannot provide a transfer of energy. I also wanted to ask a question. The equation describing the sharpening of the pulse given by the method of characteristics,

$$\frac{d}{dt}\left[\frac{\partial \ln\rho}{\partial x}\right]=\left(\frac{\partial \ln\rho}{\partial x}\right)^2 a\frac{n+1}{n}$$

(a=small disturbance velocity; n=number of degrees of freedom), seems to indicate that no matter what, there will be an increase in the density or the pressure. This looks as if we do not have to worry about dissipation at all.

H. E. PETSCHEK: This equation comes from the equations where the length has been taken large compared to the Larmor radius and one does not have any of the terms corresponding to viscous stresses. Eventually the gradient becomes steep enough so that these terms have an effect. Also, as to the first comment you made, one has separated the collision of gas particles with other particles into two regions, the more or less uniform field and the collision field due to a particle. The question, I think, is how complicated does the nonparticle field with which a particle interacts have to become before one gets an entropy increase?

S. I. PAI, *Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland:* I am not familiar with Petschek's analysis, but I am quite familiar with studying shock waves from ordinary Navier-Stokes equations. For instance, if we neglect the viscosity and just consider the Euler equations, we may analyze the ordinary steepening of the compression waves into a shock, and will get exactly the same equation as was given by Landshoff, but instead of ordinary sound speed, you get the expression with effective sound speed. So as far as this equation is concerned, you really don't consider dissipation. You just calculate how the waves steepen. If we really go into details of the shock structure we have to put in the viscosity, etc., which produce dissipation.

H. E. PETSCHEK: The equation for the rate of steepening is a standard result. The point is that the nonviscous equation is valid up through the region where the mean thickness becomes comparable with the mean free path, and the steepening continues to this limit. This approach does not say anything about the steady-state structure.

H. W. LIEPMANN: A consideration of the basic thermodynamics of the model should clarify the problem of the dissipation. For example, you can make one of your shock waves in a type of a Gay-Lussac experiment by breaking a diaphragm between two gaseous regions at different pressure in a container. In this way you can make a strong shock in a magnetic field, and in the beginning and the end you can apply thermodynamic reasoning. Then you either have dissipation or you do not. The entropy goes up or it does not. So I see how you can get a steep front, but I cannot possibly see how you can get the whole shock disturbance without getting some form of collision. I think Schlüter agrees with me.

A. R. KANTROWITZ, *Avco Research Laboratories, Everett, Massachusetts:* I would like to emphasize another form of dissipation that can appear in this problem: concentration of energy in particles on the tail of the distribution function would be a very nice way to do it. We have looked hard for this effect and we haven't been able to find it theoretically. Experimentally, as Petschek pointed out, it has been observed. For example, it is observed in the astronomical situation in the relationship of cosmic rays to solar disturbances. It is also observed in the laboratory. And this, it seems to me, is the most likely place to look for a powerful dissipation mechanism.

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