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CORRECTION TO: ANALYTIC TAF ALGEBRAS

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We would like to inform the reader of an error in [1].

First, Remark 1.10 should read as follows:

REMARK 1.10. Suppose $\mathcal{T} = \mathcal{A}(\mathcal{P})$ is a strongly maximal triangular subalgebra of $\mathfrak{A}$ with diagonal $\mathfrak{D}$. For each n, let $\mathfrak{B}_n = C^*(\mathfrak{A}_n, \mathfrak{D})$. Then there exists a clopen subset $\mathcal{R}_n$ of $\mathcal{R}$ such that $\mathcal{A}(\mathcal{R}_n) = \mathfrak{B}_n$. Note that $\mathcal{R} = \bigcup_n \mathcal{R}_n$. Let $\mathcal{P}_n = \mathcal{P} \cap \mathcal{R}_n$ and $\mathcal{P}_n^+ = \mathcal{P}_n \setminus X$. Since $\mathcal{T}$ is strongly maximal, we have $\mathcal{R}_n = \mathcal{P}_n^+ \cup X \cup (\mathcal{P}_n^+)^{-1}$. Now suppose that for each n, we can define a cocycle c_n on $\mathcal{R}_n$ such that $c_n(x,y) > 0$ if $(x,y) \in \mathcal{P}_n^+$. Suppose also that for each $(x,y) \in \mathcal{R}$, there is some m such that $(x,y) \in \mathcal{R}_m$ and $c_n(x,y) = c_{n+1}(x,y)$ for all $n \geq m$. Then $d(x,y) = \lim_{n\to\infty} c_n(x,y)$ exists (as a finite number) for every $(x,y) \in \mathcal{R}$, d satisfies the cocycle condition $d(x,z) = d(x,y) + d(y,z)$ of Definition 1.7, and $\mathcal{P} = d^{-1}[0,\infty)$ since $\mathcal{P} = \bigcup \mathcal{P}_n$. Conversely, if $\mathcal{T} = \mathcal{T}_d$ for some cocycle d, then $c_n = d|_{\mathcal{R}_n}$ is a cocycle on $\mathcal{R}_n$ such that $c_n(x,y) > 0$ for $(x,y) \in \mathcal{P}_n^+$, and $d(x,y) = c_n(x,y)$ for all $n \geq$ some m since $\mathcal{R} = \bigcup \mathcal{R}_n$.

Note that if the c_n's are given and d is defined by $d(x,y) = \lim_{n\to\infty} c_n(x,y)$, then d satisfies the cocycle condition, but in general d need not be continuous. This leads to a revised Theorem 2.2.

THEOREM 2.2. *Let $\mathcal{T} = \mathcal{A}(\mathcal{P})$ be a strongly maximal triangular subalgebra of $\mathfrak{A}$. If $\mathcal{T}$ is $\mathbb{Z}$-analytic, then $\lim_{n\to\infty} d_n(x,y) = \hat{d}(x,y)$ exists (as a finite number) for each $(x,y) \in \mathcal{R}$, $\hat{d}$ satisfies the cocycle condition on $\mathcal{R}$, and $\mathcal{P} = \hat{d}^{-1}[0,\infty)$. Conversely, if $\hat{d}$ is finite and continuous (i.e., locally constant), then $\hat{d}$ is a cocycle on $\mathcal{R}$ and $\mathcal{T}$ is $\mathbb{Z}$-analytic with $\mathcal{T} = \mathcal{T}_{\hat{d}}$.*

PROOF. Suppose $\mathcal{T} = \mathcal{T}_d$ for an integer-valued cocycle d. If $(x,y) \in \mathcal{P}_m$, then $\{d_n(x,y) : m \leq n < \infty\}$ is an increasing sequence bounded above by $d(x,y)$, so $\hat{d}(x,y)$ exists, and it satisfies the cocycle condition on $\mathcal{R}$ by Remark 1.10. Also, if $(x,y) \in \mathcal{R}$, then

$$\begin{aligned} d(x,y) \geq 0 &\Leftrightarrow (x,y) \in \hat{v} \quad \text{for some matrix unit } v \in \mathcal{T} \cap \mathfrak{A}_n \\ &\Leftrightarrow d_n(x,y) \geq 0 \quad \text{for some } n \\ &\Leftrightarrow \hat{d}(x,y) \geq 0. \end{aligned}$$

Therefore, $d^{-1}[0,\infty) = \hat{d}^{-1}[0,\infty)$.

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On the other hand, suppose $\hat{d}$ is finite and continuous. Then the sequence $\{d_n(x,y)\}$ is eventually constant since $d_n(x,y)$ is always an integer, and therefore $\hat{d}$ is an integer-valued cocycle such that $\mathcal{T} = \mathcal{T}_{\hat{d}}$ by Remark 1.10. ∎

EXAMPLE. There are $\mathbb{Z}$-analytic TAF algebras for which the function $\hat{d}$ defined in Theorem 2.2 is not continuous. The authors would like to thank Alan Hopenwasser and Allan Donsig for introducing them to the following example.

Let $\mathfrak{A}_n = \mathbf{M}_{2^n}$ with matrix units $\{e_{ij}^{(n)}\}$, and let σ_n denote the standard embedding from $\mathfrak{A}_n$ to $\mathfrak{A}_{n+1}$. For each n, let P_n be the $2^{n+1} \times 2^{n+1}$ permutation matrix formed by interchanging rows 2^n and $2^n + 1$ of the identity matrix, and define $j_n\colon \mathfrak{A}_n \hookrightarrow \mathfrak{A}_{n+1}$ by $j_n(a) = P_n\sigma_n(a)P_n$. Now let $\mathcal{T}_n$ be the set of upper triangular matrices in $\mathfrak{A}_n$ and let $\mathcal{T} = \varinjlim(\mathcal{T}_n, j_n)$. For $(x,y) \in \hat{e}_{i,i+1}^{(n)}$, define $d(x,y) = 1$ if $1 < i < 2^n - 1$ and $d(x,y) = 2$ if $i = 1, 2^n - 1$. Then d can be extended uniquely to an integer-valued cocycle such that $\mathcal{T} = \mathcal{T}_d$. However, if $x_0 = (\hat{e}_1^{(1)}, \hat{e}_1^{(2)}, \hat{e}_1^{(3)}, \ldots)$ and y_0 is the unique element of X such that $(x_0, y_0) \in \hat{e}_{12}^{(2)}$, then $\hat{d}(x_0,y_0) = 1$ but $\hat{d}(x,y) = 2$ for all $(x,y) \in \hat{e}_{12}^{(2)}$ with $x \neq x_0$. Hence, $\hat{d}$ is not continuous at (x_0, y_0).

Because $\hat{d}$ is not always continuous, we make the following definition.

DEFINITION. If $\mathcal{T} = \mathcal{T}_d$ is a $\mathbb{Z}$-analytic TAF algebra and the function $\hat{d}$ of Theorem 2.2 is continuous, then we say that $\mathcal{T}$ is *standard $\mathbb{Z}$-analytic* and that $\hat{d}$ is the *generic form* of d. Note that $\hat{d}$ is determined by the clopen subset $\hat{d}^{-1}(\{1\})$.

For the rest of Section 2, we assume that the $\mathbb{Z}$-cocycles are given in generic form, particularly in Definition 2.6 and Lemma 2.7. Also, in Theorem 2.8 and Corollary 2.9, $\mathcal{T}$ is assumed to be standard $\mathbb{Z}$-analytic, and the algebras $\mathcal{T}$ and $\mathcal{S}$ in Example 2.10 and $\mathcal{T}(\phi, x_0)$ in Theorem 2.4 are standard $\mathbb{Z}$-analytic.

In a subsequent paper [2], it is shown that the standard $\mathbb{Z}$-analytic algebras are precisely the inductive limits of direct sums of upper triangular matrix algebras generated by "standard" embeddings, *i.e.*, generalizations of $\varinjlim(\mathcal{T}_n, \sigma_n)$.

REFERENCES

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