Correspondence

DEAR EDITOR,

I was pleasantly surprised to stumble across yet another example of the same discovery made independently in different parts of the globe: The divisibility test for 19 given in the November 1998 issue of the *Mathematical Gazette* by Humphreys and Macharia was also offered, as part of a more general result, by a high school student in India, Apoorva Khare, in 'Divisibility Tests' by A. Khare, *Furman University Electronic Journal of Undergraduate Mathematics*, **Volume 3,** 1997, pp 1-5. That is not to take anything away from the H-M article, which I found explained the special case more usefully.

Yours sincerely,

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DEAR EDITOR,

In a recent note entitled 'The convergence of a Lucas series' [*Math. Gaz.* **83** (July 1999) pp. 273-274], T. Koshy undertook to prove that, for integral $k \geqslant 2$, the ratio $(2k - 1)/(k^2 - k - 1)$ is integral if, and only if, $k = 2$ or $k = 3$. Koshy's approach was unnecessarily involved. Here is a more direct proof of this result.

For $k \geqslant 2$, $(2k - 1)/(k^2 - k - 1)$ is positive. Furthermore, this ratio can only be integral if $(k^2 - k - 1) \leqslant (2k - 1)$, i.e. if $k(k - 3) \leqslant 0$. The only solutions for integral $k \geqslant 2$ are then obviously $k = 2, 3$.

Yours sincerely,

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DEAR EDITOR,

With regard to the article [1] by K. Robin McLean, an interesting variation on 'Diffy' is to play 'Quiffy', in which one finds the larger quotient of each number with its successor in the cycle, ending with a cycle of ones e.g.

$$\begin{array}{cccc} 4 & 7 & 2 & 1 \\ 7/4 & 7/2 & 2 & 4 \\ 2 & 7/4 & 2 & 16/7 \\ 8/7 & 8/7 & 8/7 & 8/7 \\ 1 & 1 & 1 & 1 \end{array}$$

The reason that the process works is simple, since, if we take logarithms of each number, we are then playing 'Diffy' (unsigned differences) and reaching a cycle of zeros ($\ln 1 = 0$).

An interesting sidelight is a very simple proof that, if the cycle of four positive numbers is not equivalent to a purely increasing sequence, the process terminates after at most six steps. (Cycles are equivalent when cycled, reversed, multiplied by a constant, or raised to a power.)