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Planetesimal Formation by Gravitational Instability — The Goldreich-Ward Hypothesis Revisited

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Abstract. We consider the formation of planetesimals via gravitational instability. While minimum solar nebula (MSN) models are known to be gravitationally stable, we find that sufficiently metal enriched and/or colder discs can yield planetesimals by the Goldreich-Ward mechanism (GWM). This is because the shear between gas and solids, previously believed to render the GWM ineffective, can only stir a finite amount of solids.

1. Background

The planetesimal hypothesis for the formation of rocky planets, and perhaps the cores of gas giants, is widely accepted despite the controversy surrounding planetesimal formation mechanisms. In the GWM, solids settle to a thin midplane layer which undergoes gravitational fragmentation (Goldreich & Ward 1973; Safronov 1969). For a MSN disk (Hayashi 1981), the GWM yields roughly comet-sized bodies. A layer of solids is deemed unstable when the density exceeds a modified Roche limit: $\rho > .62 M_*/\varpi^3$, where ϖ is the cylindrical radius, and $M_\star$ is the mass of the central stellar object, here a solar mass (Sekiya 1983).[1]

The main objection to the GWM has been that midplane turbulence keeps the solids well-stirred. Radial pressure gradients cause gas-dominated and particle-dominated regions to rotate at different rates, yielding a vertical shear in stratified disks: $\Delta v_\phi = \eta v_K$, where:

$$\eta \equiv -(\partial P/\partial \varpi)/(2\rho_g \varpi \Omega_K^2) \sim (c_g/v_K)^2, \tag{1}$$

is a non-dimensional measure of pressure support, v_K is the Keplerian speed, ρ_g is the midplane gas density (valid throughout the thin particle layer), and c_g is the isothermal sound speed. For the MSN, $\Delta v_\phi \simeq 50\,\mathrm{m\,s^{-1}}$. Wiedenschilling (1980) noted that turbulent motions of this order imply gravitational stability since Toomre's Q criterion requires a velocity dispersion $< 7\ \mathrm{cm\,s^{-1}}$ for instability. Numerical simulations by Cuzzi, Dobrovolskis, & Champney (1993) use a semi-empirical mixing length formulation to support the view that an MSN disc with solar abundances is not subject to the GWM, leading to the invocation of unproved sticking mechanisms.

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[1]Following Sekiya (1983) we use the total density of solids and gas in the instability criterion. Most workers use only the particulate density, ρ_p. The choice is whether to allow the self-gravity of gas (usually small) to aid in the onset of instability.

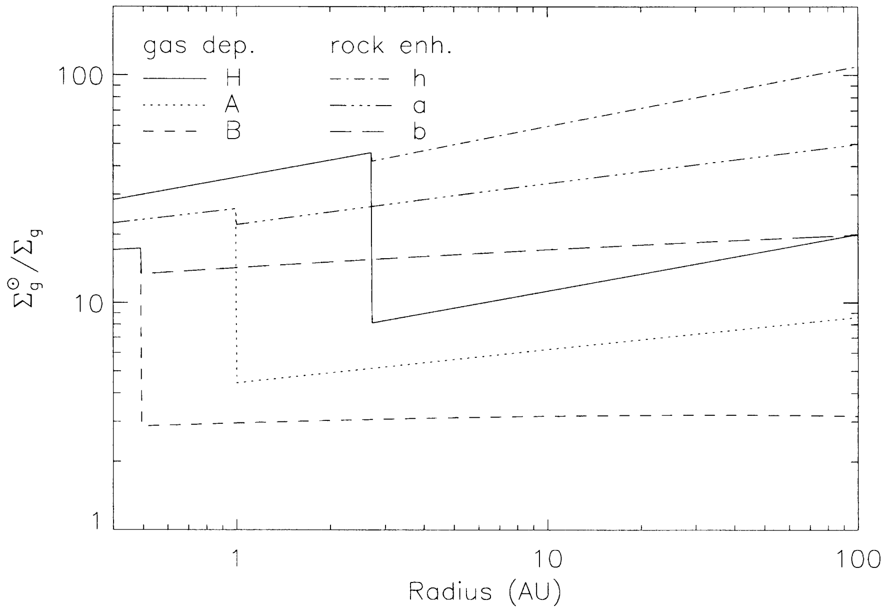

Figure 1. Metallicity enhancements required for marginal gravitational instability as a function of radius for various models. Smaller enhancements are required for instability in low temperature models, especially outside the iceline.

Sekiya (1998) developed an analytic one-fluid, quasistatic equilibrium model in which small well-coupled solids such as dust grains and chondrules settle to the midplane until the Kelvin-Helmholtz instability triggers weak turbulence. Sekiya's particle density profiles, $\rho_p(z)$, are subject to the GWM only if solids are enhanced or gas depleted, in an MSN disk, by roughly an order of magnitude.

In this work we explain the role of metallicity in gravitational instability and investigate different thermal profiles.

2. Results

Sekiya's dust density profiles develop a midplane density cusp, $\rho_p(0) \rightarrow \infty$, as the surface density of solids approaches a finite limit:

$$\Sigma_{p,c} = \eta\varpi\rho_g s(\psi), \quad (2)$$

$$s(\psi) \equiv [(1+\psi)\ln[(1+\psi+\sqrt{1+2\psi})/\psi] - \sqrt{1+2\psi}], \quad (3)$$

where $\psi \equiv 4\pi G\rho_g/\Omega_K^2$ is a measure of the self-gravity of the gas. We interpret equation (2) as the maximum amount of solids which can be stirred by vertical shear. The kinetic energy (KE) of turbulent gas motions must provide enough potential energy (PE) to keep the particles lofted above the midplane. We estimate the KE per unit volume as $\rho_g(\eta v_K)^2$. The height of the particle layer is roughly, $H_p \sim \eta\varpi$, which follows from Sekiya's analysis or dimensional analysis

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of the Richardson criterion. Thus the PE per unit volume stored in the stirred solids is roughly: $\langle\rho_p\rangle(\eta v_K)^2$, where the angle brackets indicate an average over the thickness of the layer. Thus on average, $\langle\rho_p\rangle \leq \rho_g$, i.e. the gas can only stir its own mass of solids within a layer of fixed height. This allows us to estimate $\Sigma_{p,c} \sim \rho_g \eta \varpi$ which agrees with equation (2) except for $s(\psi)$, the order unity correction due to self-gravity.

We consider models in which the surface density of rock is fixed at MSN values: $\Sigma_r = 7.1(\varpi/\mathrm{AU})^{-3/2}\,\mathrm{g\,cm^{-2}}$, but the gas content is lowered from solar abundances, $\Sigma_g^{\odot} = 1700(\varpi/\mathrm{AU})^{-3/2}\,\mathrm{g\,cm^{-2}}$, to a value, Σ_g, which yields a midplane cusp and gravitational instability. (A steeper surface density profile, $\Sigma_r \propto \varpi^{-1}$, gives gravitational instability at solar abundances beyond ~ 10 AU.) When $T < 170$ K, ice freezes out in our models at a fixed (at MSN values) fraction of the gas or the rock, corresponding to the astrophysical choices of enhancing disc metallicity by the return of solids from a bipolar outflow or by depletion of gas, respectively. A system which arose from a metal rich molecular cloud (perhaps some exoplanets, but not our solar system) would behave as the later category. The gas depletion cases (indicated by capital letters) contain more ices than the rock enhancement cases (labeled with lower-case letters).

The temperature profiles of our gas disks, assumed passive and vertically isothermal, obey radial power-laws, $T = T_1(\varpi/\mathrm{AU})^{-q}$. Hayashi's original MSN (models H and h) has $T_1 = 280\,\mathrm{K}$ and $q = 1/2$. Millimeter wave continuum observations (Osterloh & Beckwith 1995) give cooler temperatures for many T Tauri discs. Thus we also investigate models A/a ($T_1 = 170\,\mathrm{K}$, $q = .63$) and B/b ($T_1 = 100\,\mathrm{K}$, $q = 3/4$). Figure 1 shows the metallicity enhancements required for marginal gravitational instability. Further enhancements are required to yield planetesimals, and the size of the resulting planetesimals depends on the amount of this excess.

The feasibility of this "saturation" mechanism for achieving gravitational instability depends on temperature; on the efficiency with which (probably stratified) discs can become enriched in metals via winds, photoionization, and outflows; and on vertical shear being the dominant source of midplane turbulence.

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References

Cuzzi, J. N., Dobrovolskis, A. R., & Champney, J. M. 1993, Icarus, 106, 102

Goldreich, P., & Ward, W. R. ApJ, 183, 105

Hayashi, C. 1981, Prog. Theor. Phys. Supp., 70, 35

Osterloh, M., & Beckwith, S. V. W. 1995, ApJ, 439, 288

Safronov, V. S. 1969, Evolution of the Protoplanetary Cloud and Formation of the Earth and the Planets. (Moscow: Nauka Press)

Sekiya, M. 1983, Progress in Theoretical Physics, 69, 1116

Sekiya, M. 1998, Icarus, 133, 298

Weidenschilling, S. J. 1980, Icarus, 44, 172