THE APPLICATION OF FAINT MINOR PLANET DYNAMICS TO THE PROBLEMS OF IMPROVING THE FUNDAMENTAL REFERENCE SYSTEM

P. D. Hemenway and R. L. Duncombe
The University of Texas
Austin, Texas

ABSTRACT

The underlying method of forming the adopted Fundamental Reference System is the compilation of observations referred to some (independent) well defined coordinate systems. If the observations themselves are used to define the independent coordinate system, they may be used to help establish the Fundamental System. If the observations are differential, they may be used to refer star positions to the Fundamental System and/or smooth systematic irregularities within the Fundamental System.

We are presently studying the possible use of faint (16th mag.) minor planets as "test particles" to establish a dynamically based reference frame. Several methods of increased observational accuracy lend themselves to the determination of such a system. The present studies are directed toward determining a) the attainable accuracies for various observation types and b) the effects that varying the distribution of observation types would have on the internal consistency of such a system.

The study also includes analyzing the application of the very high projected accuracy of the Space Telescope to the problem of relating faint reference frames (e.g. the above system, the Hipparcos system, and the VLBI system) to each other and to the extant Fundamental System.

Close cooperation between this project and other ongoing programs will optimize the usefulness of the results.

Finally, some results to date are presented and some overall advantages of such a dynamically based reference frame are discussed.

The basic goal of fundamental astrometric observations is the formation of a reference frame by which the kinematic properties of celestial bodies may be determined. The ability of dynamical theories to account for the observations may then be tested by comparing predictions

E. M. Gaposchkin and B. Kołaczek (eds.), Reference Coordinate Systems for Earth Dynamics, 277–282.

https://doi.org/10.1017/S0252921100081392 Published online by Cambridge University Press

with observations. The quality of the comparison depends on three factors:

1. The accuracy of the observations.

2. The sufficiency of the theory.

3. The correctness of the reference frame to which both theory and observation are referred.

Since most dynamical theories are based on Newtonian mechanics, as modified by Einstein, the coordinate system one attempts to use should be as close to inertial as possible. Therefore, the dynamics of moving bodies tend to be used in the definition of the coordinate system itself.

Because astrometric observations have been tied to the Earth as an observing platform, the dynamics of the Earth have played an important role in the formation of the adopted Fundamental Reference System. The most accurate astrometric observations (radio interferometer observations, for example) must either solve for, or account for, the variation of station coordinates as a function of time with respect to a celestial coordinate system. The most accurate geodetic observations (observations of satellite motion, for example) have divorced themselves from measurements with respect to a stellar reference frame altogether; but their need to understand the variation of station coordinates with respect to a dynamically determined reference frame becomes paramount. The small variations now observed in the motion of the Earth and in the relative motion of individual stations makes the determination of even a consistent reference frame a very difficult task. Applying adopted geophysical definitions to forming a celestial coordinate system has become the full-time occupation of many astronomers throughout the world.

Aside from the vagaries of the Earth's motion, high precision optical observations of star images with respect to the reference frame are difficult. Observations of the Sun and planets define the astronomical coordinate system, and they are even harder to relate to the system because of extended amorphous images, and the Sun distorts any instrument used to observe it.

Minor planets afford the possibility of improving the coordinate system through more accurate observations. We are studying the feasibility and scientific validity of using accurate ground-based and space observations of a selected set of small minor planets to obtain corrections to the fundamental celestial coordinate system and provide a zero circle (the ecliptic) for a system of positions and proper motions. Many attempts to use minor planets for fundamental purposes have been undertaken in the past. The most recent discussions are Branham (1979, 1980), which illustrate accuracies of, and problems with, classical techniques applied to bright minor planets.

Two observing techniques hold the promise of increased accuracy in a coordinate system incorporating faint minor planet observations. The position of an image on a photographic plate with respect to a set of faint background star images can be measured with very high relative accuracy, e.g. 0.4 microns, plate to plate for high-information content emulsions such as IIIaJ, (Chiu, 1977). Long focus telescopes (reflectors as well as refractors) have the potential to provide accurate relative positions of minor planets with respect to background stars with unknown absolute positions.

The usual equations of condition for reducing observations of the minor planets for corrections to their orbital elements are:

$$
\begin{aligned}
(\alpha_0 - \alpha_c) &= \sum_{j=1}^{6} \frac{\partial\alpha}{\partial E_j} \Delta E_j \\
(\delta_0 - \delta_c) &= \sum_{j=1}^{6} \frac{\partial\delta}{\partial E_j} \Delta E_j
\end{aligned}
\qquad (1)
$$

where (α,δ) are the coordinates of the minor planet at the time of observation. The subscripts refer to the observed (0) and computed (c) positions, and E_j the j^{th} orbital element.

In forming a reference frame, zone corrections to an initial coordinate system as well as corrections to the Earth's orbital parameters are included on the right hand side. The least accurately known entries in equations (1) are α_0 and δ_0, the observed position. All the other terms are numbers computed using the initial conditions and an assumed model of the solar system. The differential corrections are obtained relative to the initial model. The accuracy of a "true" position is limited by the accuracy with which the transformation to the initial reference frame may be made. This accuracy is presently on the order of ±0.2 arcseconds rms (±0.15 arcseconds rms for transit circle observations, ±0.2 to ±0.3 arcseconds for photographic transfers to the FK4).

By combining observations of more than one minor planet with respect to the same set of faint background stars, the absolute positions of the background stars may be eliminated. If two minor planets pass through the same star field at different times t_1 and t_2 (see figure 1) the equations of condition become:

$$
\alpha_0^{(1)}(t_1) - \alpha_c^{(1)}(t_1) = \sum \frac{\partial\alpha^{(1)}(t_1)}{\partial E_j^{(1)}} \Delta E_j^{(1)} \qquad (2)
$$

https://doi.org/10.1017/S0252921100081392 Published online by Cambridge University Press

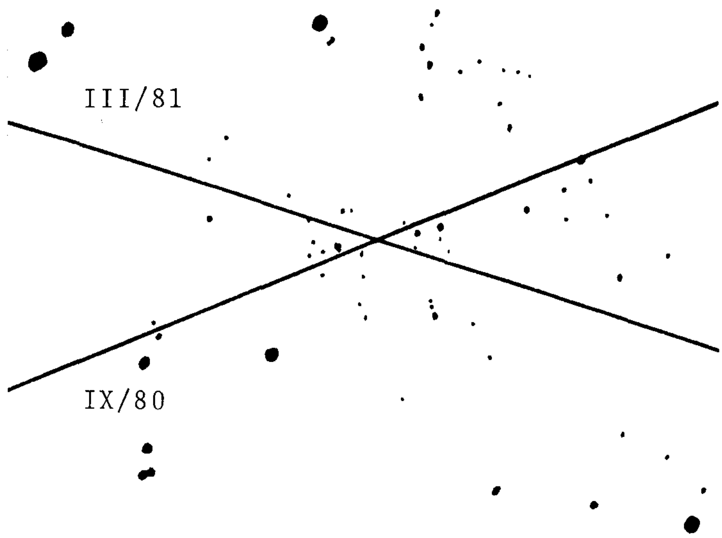

Figure 1. Paths of two minor planets through the same star field.

$$\alpha_0^{(2)}(t_2) - \alpha_c^{(2)}(t_2) = \sum \frac{\partial\alpha^{(2)}(t_2)}{\partial E_j^{(2)}} \Delta E_j^{(2)} \tag{2}$$

The difference becomes:

$$\alpha_0^{(2)}(t_2) - \alpha_0^{(1)}(t_1) = \alpha_c^{(2)}(t_2) - \alpha_c^{(1)}(t_1) + \left(\sum_2 - \sum_1\right) \tag{3}$$

We may represent the right ascension of the field center of a plate of a long focus telescope by α_{fc} and the offset of the minor planet image from that position by $\Delta\alpha_0$. Then:

$$\alpha_0^{(2)}(t_2) = \alpha_{fc}^{(2)} + \Delta\alpha_0^{(2)}(t_2) \tag{4}$$

From a central overlap plate solution,

$$\alpha^{(1)}{}_{fc} = \alpha^{(2)}{}_{fc} \quad ,$$

so we have

$$\alpha_0^{(2)} - \alpha_0^{(1)} = \Delta\alpha_0^{(2)} - \Delta\alpha_0^{(1)}$$

which depends only on the long focus plate reductions. An equation of condition among the orbital elements of two minor planets (or one minor planet at different times) results:

$$\Delta\alpha_0^{(2)}(t_2) - \Delta\alpha_0^{(1)}(t_1) = \alpha_c^{(2)}(t_2) - \alpha_c^{(1)}(t_1) + \sum_{j=1}^{6} \frac{\partial\alpha^{(2)}(t_2)}{\partial E_j^{(2)}} \Delta E_j^{(2)}$$

$$- \sum_{j=1}^{6} \frac{\partial\alpha^{(1)}(t_1)}{\partial E_j^{(1)}} \Delta E_j^{(1)} \qquad (5)$$

This equation of condition has two remarkable properties: 1) It is independent of the systematic errors of the assumed reference frame; and 2) It provides a direct mathematical relation between the orbital elements of two minor planets. Furthermore, it is an equation identical in form to the classical equation of condition, so that the methods of solution apply.

In forming a reference frame, zone corrections to the assumed coordinate system as well as corrections to the Earth's orbital elements must be determined explicitly. Combining suitably modified observational equations of the classical type and of the crossing point type can result in the determination of such corrections. The present goal is to estimate the accuracies of various observing and reduction techniques, and to determine the accuracies of correction to an adopted reference system using such techniques.

The plan is to compare the speed and accuracy of hand measurements, simple centroiding, as well as one and two dimensional Fourier deconvolution of trailed star and minor planet images scanned automatically by a microdensitometer. The measuring (setting) error will be determined by fitting several measurements of one exposure to each other. The relative positional accuracy will be determined by fitting the short-arc motion of a minor planet to several exposures over a short time interval. Minor planet Hypatia (238) was observed at 5 hour angles throughout one night, with the 0.76 meter telescope (f/13) at McDonald Observatory. The plates have been raster-scanned and measured by hand.

Nemausa (51) has a long history of observation for coordinate system work, (Møller, 1978). We are undertaking a program with Leif Kristensen to observe Nemausa at a crossing point on September 17, 1980 and March 18, 1981. (See IAU circular #3480). Astrometric plates will be obtained at several observatories with long focus instruments. The observations will be reduced to a common frame of faint 13th to 16th magnitude, background stars. The consistency of the times of "crossing" will be compared with the ephemeris of Nemausa to determine the actual accuracy of the technique relative to Nemausa's motion. (Nemausa has one of the best known motions of all minor planets). While Nemausa is larger (150 km) than the minor planets which are being contemplated for

https://doi.org/10.1017/S0252921100081392 Published online by Cambridge University Press

this project (30 km), it will provide a good test of the method.

From the meager data, one can see that sub-1/100th arcsecond rms accuracy is a real possibility. With telescopes of 20-meter focal-length or longer, (e.g. the 2.1 meter f/13 relfector at McDonald Observatory) a few milli-arcseconds accuracy for the relative positions of two minor planet images may be possible. The estimated accuracy of a relative position with the Space Telescope is ±0.002 arcseconds/observation. The accuracy of a Hipparcos position relative to the "rigid" Hipparcos system also has that level of expectation. Thus, a combination of Hipparcos, Space Telescope, and ground based observation of minor planets could provide a stellar reference system which is inherently based on the dynamics of the solar system, and is systematically error free at a very low level.

The regular motion of minor planets across large arcs of the celestial sphere are analogous to the regular motion of the optical axis of a transit circle over large arcs due to the rotation of the Earth. For any given accuracy (say ±0.001 arcsecond) the classical methods (e.g. transit circles) would require a knowledge of the geocentric position of the instrument to a corresponding linear accuracy (3 cm at 6000 km). For the same positional accuracy, the minor planet methods would require a knowledge of the position of the instrument with respect to the minor planet to the corresponding linear accuracy (0.75 km at 1 A.U.). One kilometer accuracy is trivial for the knowledge of the station coordinates on the Earth, but may be difficult to realize from observations and dynamics of minor planets. Hence, the burden of observational and theoretical precision is shifted from the realm of geodesy to the realm of celestial mechanics. We are attempting to determine whether the new techniques will be sufficient to carry the burden. If so, we will propose to undertake a program using the techniques outlined here. The coupling of such a system to a geodetic system on the scale of centimeters is a problem for your consideration.

We are indebted to Leif Kristensen for the crossing point data on 51 Nemausa. We particularly thank Paul Herget for his active participation in this study. We appreciate the support of the National Science Foundation under Grant AST-79-16870 and the Organizing Committee of this Conference.

REFERENCES

Branham, R. L., 1979, Astron. Jour., 84, pp. 1399.
Branham, R. L., 1980, Astronomical Papers on the AENA, XXI, Part 3.
Chui, L. G., 1977, Astron. Jour. 82, pp. 842-848.
Møller, O., "The Work on a Possible Contribution to the Fundamental System of Reference by Observation of Minor Planet 51 Nemausa", in Reiz and Andersen (1978).
Reiz, A. and Andersen, T., 1978, Astronomical Papers Dedicated to Bengt Stromgren, Copenhagen University Observatory, J. J. Trykteknik a/s, Copenhagen.