

# TARGET PLANE CONFIDENCE BOUNDARIES: MATHEMATICS OF THE 1997 XF11 SCARE

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The uncertainty of the close approach distance of a Potentially Hazardous Object (PHO), either an asteroid or a comet, can be represented on the Modified Target Plane (MTP), a modification of the one used by Öpik. The MTP is orthogonal to the geocentric velocity at the closest approach along the nominal orbit, solution of the least square fit to the observations. The confidence regions of this solution in the 6-D space of orbital elements (for an epoch close to the observations) are well approximated by a family of concentric ellipsoids, if the observed arc is not too short. In the linear approximation these ellipsoids are mapped on the MTP into concentric ellipses, which can be computed by solving for the state transition matrix.

For a PHO observed at only one opposition, with a close approach expected after many revolutions, the ellipses on the MTP become extremely elongated and the linear approximation may fail. In this case the confidence boundaries on the MTP, i.e. the nonlinear images of the confidence ellipsoids, may not be well approximated by the ellipses. The Monte Carlo method (Muinonen and Bowell, 1993) can be used to find nonlinear confidence regions, but the computational load is very heavy: to estimate a low probability event the number of test cases must be larger than the inverse of the probability. We propose a new method to compute semilinear confidence boundaries on the MTP (Milani and Valsecchi, 1998), based on the theory developed to compute confidence boundaries for predicted observations (Milani, 1999). This method is a good compromise between reliability and computational load, and can be used for real time risk assessment.

We apply this technique to the case of asteroid *1997 XF<sub>11</sub>*, which appeared, with the observations available as of March 1998, to be on an orbit with a near miss to the Earth in 2028. Although the least squares solution had a close approach at 1/8 of the lunar distance, the linear confidence regions corresponding to acceptable observational errors are very elongated ellipses not including collision. The semilinear confidence boundaries differ in a significant way from the linear ellipses but this happens only far from the Earth: an impact by asteroid *1997 XF<sub>11</sub>* was not compatible with the observations from the discovery to March 4, 1998, unless it could be admitted that the RMS observation error was about 4 arcsec. Anyway the use of the 1990 pre-discovery observations has confirmed the impossibility of a 2028 impact; with the additional data, the semilinear confidence regions are

reduced to a much smaller subset of those estimated previously; in the linear approximation, the MTP prediction with more data is not a subset of the one with less data. Even worse, in a simulated example of Earth impacting asteroid (Bowell and Muinonen, 1992), the semilinear confidence boundary can have a completely different shape from the linear ellipse, and indeed for orbits determined with only few weeks of data the semilinear confidence boundary includes possible collisions, while the linear one does not.

We conclude that the procedure to exclude the possibility of an impact involves the following steps: (1) find the least squares fit to the existing observations, discard the outliers and select weights; (2) propagate the orbit for the time span to be monitored, and detect the dangerous close approaches; (3) given a close approach and its MTP, compute the linear confidence ellipse and analyse its size and orientation; (4) if the linear ellipse gets close to the Earth, especially when this occurs far from the nominal orbit, use the semilinear approximation; (5) if the semilinear approximation indicates the possibility of an impact resort to a fully nonlinear exploration by the Monte Carlo method (Muinonen, 1993). The algorithms described in this paper are implemented in the free software package OrbFit; thus, the next time the problem of a worrisome close approach occurs, we are ready, as well as anybody else: see <ftp://copernico.dm.unipi.it/pub/orbfit/>.

### References

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