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A computational study of several problems in stochastic modelling

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This thesis investigates the computational aspects of two different approaches to the study of mathematical models consisting of systems of stochastic differential equations. These two approaches, presented as alternatives to the simple but usually expensive method of Monte Carlo simulation, are:

(i) calculation of the appropriate probability distribution of the state variables by solution of the partial differential equation of a related diffusion model; and

(ii) solution of systems of deterministic ordinary differential equations for the moments of the state variables.

We investigate assumptions for the use of diffusion models and show how a diffusion model can be used in the study of the Interior First Passage Problem, wherein we are interested in the first passage time for a process to reach some point or sub-region within the region on which the process is defined. Since analytical solutions are not usually available for the partial differential equations associated with diffusion models we look in detail at finite difference methods of solution. We demonstrate the relationship between a finite difference solution and a random walk solution for a simple problem. A particular finite difference scheme is defined for which sufficient conditions are established for the convergence of several iterative methods of solution of the finite difference equations. In addition we discuss stability of the finite difference scheme. We demonstrate the use of a diffusion model in a study of the

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question of the settlement of Polynesia. The aim of the study is to calculate probabilities of ocean-drift voyages to and between various island groups in Polynesia, in order to test Heyerdahl's hypothesis of the settlement of Polynesia from the Americas.

In some cases of continuous stochastic systems it might be sufficient or more appropriate to solve for moments of the state variables. However, except for models consisting of completely linear systems of stochastic differential equations, the moment equations usually form an infinite coupled system in which equations for moments of any given order involve moments of higher orders. Then to facilitate solution it is necessary to approximate the infinite system with a finite closed system of equations. We investigate one method of achieving this which involves the use of quasi-moments, which are the expectations of multi-dimensional Hermite polynomials in the state variables. This method is shown to be satisfactory in theory but unattractive in practice due to the considerable algebraic manipulation involved in deriving the moment equations and the quasi-moment hierarchy truncation approximations. Therefore we describe and demonstrate an algorithm, written in the Snobol4 programming language, which acts as a preprocessor to the continuous systems simulation language ACSL. This enables ACSL to be extended to allow for the definition of stochastic differential equations, from which the preprocessor automatically generates an ACSL program for the moment equations, involving hierarchy truncation approximations wherever necessary.