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A RELATION BETWEEN ORDER AND DEFECTS OF MEROMORPHIC MAPPINGS OF C^n INTO $P^N(C)$

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1. Introduction

Let f be a meromorphic mapping of the n-dimensional complex plane C^n into the N-dimensional complex projective space $P^N(C)$. We denote by $T(r, f)$ the characteristic function of f and by $N(r, f^*H)$ the counting function for a hyperplane $H \subset P^N(C)$.[1] The purpose of this paper is to establish the following results.

THEOREM 1. *Let $f: C^n \to P^N(C)$ be a meromorphic mapping of finite order ρ which is not a positive integer. Then for any $N + 1$ hyperplanes $H_\mu \subset P^N(C)$, $\mu = 0, 1, \cdots, N$, in general position*

$$(1.1)\qquad K(f) = \varlimsup_{r\to\infty} \frac{\sum_{\mu=0}^{N} N(r, f^*H_\mu)}{T(r, f)} \geqq k(\rho)\,,$$

where $k(\rho)$ is a positive constant depending only on ρ and satisfies

$$(1.2)\qquad k(\rho) \geqq \frac{2\Gamma^4(3/4)\,|\sin \pi\rho|}{\pi^2\rho + \Gamma^4(3/4)\,|\sin \pi\rho|}\,.$$ [2]

In case $0 \leqq \rho < 1$, we shall also obtain

THEOREM 2. *The positive constant $k(\rho)$ in* (1.1) *satisfies*

$$(1.3)\qquad k(\rho) \geqq 1 - \rho \quad for \quad 0 \leqq \rho < 1\,.$$

Remark. When ρ takes values near 0, the evaluation (1.3) is better than (1.2). On the other hand (1.2) is better than (1.3) when ρ is close to 1.

From these theorems we have readily

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1) Throughout the present paper we only consider hyperplanes H such that f^*H do not contain the origin.

2) As usual, $\Gamma(\cdot)$ stands for the gamma-function.

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COROLLARY. *If a meromorphic mapping* $f: \boldsymbol{C}^n \to \boldsymbol{P}^N(\boldsymbol{C})$ *admits* $N+1$ *hyperplanes in general position whose defects are equal to one, then the order of* f *is infinite or a positive integer.*

In case $n = N = 1$, the existence of the positive lower bound $k(\rho)$ in (1.1) was first proved by R. Nevanlinna [7, Chap. III] and he posed the problem to determine the best possible value of $k(\rho)$. In the same case Theorem 1 was proved by Edrei-Fuchs [1] and they determined the correct value of $k(\rho)$ for $0 \leqq \rho < 1$ in [2]. In case $n = 1$ and $N \geqq 1$, Toda [10] obtained the evalution (1.2) and moreover Ozawa [8] obtained the correct value of $k(\rho)$ for $\rho < 1$.

One notes that $k(\rho)$ may be determined independently of the dimension n.

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2. Notation

Let $(z_1, \cdots, z_n)$ be the natural coordinate system in $\boldsymbol{C}^n$ and set

$$\|z\|^2 = \sum_{\mu=1}^{n} z_\mu \bar{z}_\mu\,, \qquad B(r) = \{\|z\| < r\}\,,$$

$$A(r) = A \cap B(r) \quad \text{for a subset} \quad A \subset \boldsymbol{C}^n\,,$$

$$d^c = \frac{i}{4\pi}(\bar{\partial} - \partial)\,,$$

$$\chi = (dd^c \log \|z\|^2)^{n-1}\,, \qquad \eta = d^c \log \|z\|^2 \wedge \chi\,.$$

For a positive divisor D on $\boldsymbol{C}^n$ not containing the origin, set

$$n(t, D) = \int_{D(t)} \chi, \qquad N(r, D) = \int_0^r \frac{n(t, D)}{t} dt\,.$$

In case $n = 1$, $n(t, D)$ is the number of elements of D in $B(t)$ with counting multiplicities. Let L denote the hyperplane bundle over $\boldsymbol{P}^n(\boldsymbol{C})$ and ω the positive definite curvature form of L arising from an hermitian metric h in L. For a meromorphic mapping $f: \boldsymbol{C}^n \to \boldsymbol{P}^N(\boldsymbol{C})$ which is holomorphic at the origin, the characteristic function is defined by

$$T(r, f) = \int_0^r \frac{dt}{t} \int_{B(t)} f^*\omega \wedge \chi\,.$$

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It is noted that the pull-back form $f^*\omega$ is a differential form with coefficients belonging to L^1_{loc} which is closed and positive in the sense of currents (cf. Lelong [6]) and that $T(r, f)$ is independent of the curvature form ω of L, up to an $O(1)$-term (cf. Griffiths-King [3]).

Let $S(r)$ be a real, non-negative and increasing function of $r \geqq 0$. Then $\overline{\lim\limits_{r\to\infty}} \log S(r)/\log r$ is called the order of $S(r)$. In particular the order of $T(r, f)$ ($N(r, D)$ resp.) is called the order of f (D resp.). Let U be an open set in $\boldsymbol{P}^N(\boldsymbol{C})$ such that $L|_U \cong U \times \boldsymbol{C}$. Then the restriction $\sigma|_U$ of a global holomorphic section $\sigma \in H^0(\boldsymbol{P}^N(\boldsymbol{C}), L)$ is naturally regarded as a holomorphic function in U and similarly $h|_U$ as a positive smooth function in U. The length of σ is defined by

$$|\sigma| = \left(\frac{|\sigma|_U|^2}{h|_U}\right)^{1/2} \qquad \text{in } U\,,$$

which is independent of the local trivialization, $L|_U \cong U \times \boldsymbol{C}$. For a hyperplane H in $\boldsymbol{P}^N(\boldsymbol{C})$, choose always a global section $\sigma \in H^0(\boldsymbol{P}^N(\boldsymbol{C}), L)$ so that the divisor (σ) is equal to H and $|\sigma| \leqq 1$, and set

$$m(r, H) = \int_{\partial B(r)} \log \frac{1}{f^*|\sigma|}\eta\,.$$

Now the following is well-known (Nevanlinna's first main theorem):

$$T(r, f) = N(r, f^*H) + m(r, H) + \log f^*|\sigma|(0) \tag{2.1}$$

provided that $f^*H \not\ni 0$.

In case $N = 1$, f is a meromorphic function in $\boldsymbol{C}^n$. Let $(f)_0$ and $(f)_\infty$ denote respectively the divisors of zeros and poles of f and suppose that $(f)_0 \cup (f)_\infty \not\ni 0$. Then (2.1) yields that

$$\begin{aligned} T(r, f) &= N(r, (f)_\infty) + \int_{\partial B(r)} \log^+ |f|\eta + O(1) \\ &= N(r, (f)_0) + \int_{\partial B(r)} \log^+ \frac{1}{|f|}\eta + O(1), \end{aligned} \tag{2.2}$$

where $\log^+ s = \max\{0, \log s\}$ for $s \geqq 0$. We return to the general case, $N \geqq 1$. We set for a hyperplane H

$$\delta(H, f) = 1 - \overline{\lim_{r\to\infty}} \frac{N(r, f^*H)}{T(r, f)}$$

which is called the defect of H.

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3. An estimate for canonical functions

For an entire function F in C^n, we set

$$M(r, F) = \max_{\|z\|=r} |F(z)| .$$

LEMMA 1. *Let F be an entire function. Then for $r < R$*

$$T(r, F) + O(1) \leqq \log M(r, F) \leqq \frac{1 - (r/R)^2}{(1 - r/R)^{2n}}\{T(R, F) + O(1)\} . \tag{3.1}$$

Proof. The first inequality follows from (2.2). We prove the second. Let $Aut(B(R))$ denote the group of holomorphic automorphisms of $B(R)$. For $z_0 \in B(R)$, there is an element $\gamma(\cdot, z_0) \in Aut(B(R))$ with $\gamma(z_0, z_0) = 0$. We define

$$\begin{aligned} \phi(z, z_0) &= dd^c \log \|\gamma(z, z_0)\|^2 , \\ \chi(z, z_0) &= \phi(z, z_0)^{n-1} \\ \eta(z, z_0) &= d^c \log \|\gamma(z, z_0)\|^2 \wedge \chi(z, z_0) . \end{aligned}$$

Since the isotropy subgroup of $Aut(B(R))$ at the origin consists of unitary transformations of the coordinates, these differential forms are independent of the choice of $\gamma(\cdot, z_0)$. Note that $\chi(z, 0) = \chi(z)$ and $\eta(z, 0) = \eta(z)$. Since $\log |F \circ \gamma(\cdot, z_0)^{-1}|$ is plurisubharmonic in a neighborhood of $\overline{B(R)}$,

$$\begin{aligned} \log |F(z_0)| &= \log |F \circ \gamma(0, z_0)^{-1}| \\ &\leqq \int_{\partial B(R)} \log |F \circ \gamma(z, z_0)^{-1}| \eta(z) = \int_{\partial B(R)} \log |F(z)| \eta(z, z_0) \\ &\leqq \int_{\partial B(R)} \log^+ |F(z)| \eta(z, z_0) . \end{aligned} \tag{3.2}$$

Let $\log M(r, F) = \log |F(z_0)|$ with $z_0 \in \partial B(r)$. By a unitary transformation of the coordinates, we can carry z_0 to $(r, 0, \cdots, 0)$. Therefore we may assume that $z_0 = (r, 0, \cdots, 0)$. Let us take $\gamma(z, z_0)$ as follows:

$$\gamma(z, z_0) = \frac{R}{R - (r/R)z_1}(z_1 - r, \sqrt{1 - (r/R)^2}\, z_2, \cdots, \sqrt{1 - (r/R)^2}\, z_n) .$$

By an elementary calculation we have

$$\phi(z, z_0) \leqq \frac{1}{(1 - r/R)^2}\phi(z, 0) ,$$

$$d^c \log \|\gamma(z, z_0)\|^2 = \frac{R^2 - r^2}{|R - (r/R)z_1|^2} d^c \log \|\gamma(z, 0)\|^2$$

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and so $\eta(z, z_0) \leqq \{1 - (r/R)^2\}\eta(z)/(1 - r/R)^{2n}$. Combining this with (3.2) and (2.2), we obtain the required inequality. Q.E.D.

Let ℓ be a complex line in C^n through the origin and $F_\ell(u)$ denote the restriction of F on ℓ. From Lemma 1 it follows that for every ℓ,

$$\text{order of } F_\ell(u) \leqq \text{order of } F(z) . \tag{3.3}$$

Let D be a positive divisor on C^n not containing the origin and suppose that for an integer q

$$\int^\infty \frac{1}{t^{q+1}} dn(t, D) < \infty . \tag{3.4}$$

Then according to Lelong [5, Theorem 5] (see also Stoll [9]), there exists an entire function F such that $(F) = D$, $F(0) = 1$, all the partial derivatives of $\log F$ of order $\leqq q$ vanish at the origin, the order of F is not greater than $\max\{q, \text{order of } D\}$ and

$$\begin{aligned} \log|F(z)| \leqq A(n, q)\Big\{\|z\|^q \int_0^{\|z\|} \frac{n(t, D)}{t^{q+1}} dt \\ + \|z\|^{q+1} \int_{\|z\|}^\infty \frac{n(t, D)}{t^{q+2}} dt\Big\} , \end{aligned} \tag{3.5}$$

where $A(n, q)$ is a constant depending only on n and q. Such a function F is called the canonical function of genus q associated with the divisor D.

Let D be a positive divisor on C^n not containing the origin, whose order is less than $q + 1$. Then (3.4) is satisfied. Let F be the canonical function of genus q associated with D, ℓ a complex line in C^n through the origin and suppose that $F_\ell(u)$ does not vanish for all $u \in \ell \cong C$. Then by (3.3), $F_\ell(u) = e^{P(u)}$ where $P(u)$ is a polynomial of degree $\leqq q$. Since all the derivatives of $\log F$ of order $\leqq q$ vanish at the origin and $F(0) = 1$, $P(u) \equiv 0$ and then $F_\ell(u) \equiv 1$. Regarding ℓ as a point of $P^{n-1}(C)$ in the natural manner, we see

LEMMA 2. *Let* $E = \{\ell \in P^{n-1}(C) : \ell \cdot D = \phi\}$, ($\ell \cdot D$ = *intersection of* ℓ *and* D *with counting multiplicities*). *Then* E *is an analytic subset and for* $\ell \in E$, $F_\ell \equiv 1$ *and for* $\ell \notin E$, F_ℓ *coincides with the Weierstrass product of genus* q *associated with* $\ell \cdot D$.

Remark. It follows from (3.3) that $\int^\infty dn(t, \ell \cdot D)/t^{q+1} < \infty$.

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Proof. The first two assertions follow immediately from the above arguments. We show the last. Let $\Pi(u)$ denote the Weierstrass product of genus q associated with $\ell \cdot D$. Noting that the orders of $\Pi(u)$ and $F_\ell(u)$ are less than $q+1$, we have

$$F_\ell(u) = e^{P(u)}\Pi(u) ,$$

where $P(u)$ is a polynomial of degree $\leqq q$. For the same reason as above, $P(u) \equiv 0$. Q.E.D.

Let us set

$$\phi(t) = \frac{1}{2\pi}\int_0^{2\pi} \frac{d\theta}{|te^{i\theta} - 1|} .$$

Then by Edrei-Fuchs [1, p. 303] we have for $0 < \beta < 1$

$$\int_0^\infty \phi(t)t^{\beta-1}dt \leqq \frac{\pi^2}{\Gamma^4(3/4)\sin(\pi\beta)} . \tag{3.6}$$

LEMMA 3. *The above canonical function F satisfies*

$$\int_{\partial B(r)} \log^+ |F|\eta \leqq \frac{1}{2}\int_0^r \frac{n(t, D)}{t}\, dt + \frac{r^q}{2}\int_0^\infty n(t, D)t^{-q-1}\phi\left(\frac{t}{r}\right)dt . \tag{3.7}$$

Furthermore in case $q = 0$ we have

$$\int_{\partial B(r)} \log^+ |F|\eta \leqq \int_0^r \frac{n(t, D)}{t}dt + r\int_r^\infty \frac{n(t, D)}{t^2}dt . \tag{3.8}$$

Proof. First we show (3.7). From Lemma 2 and Edrei-Fuchs [1, p. 302] we obtain for $u \in \ell \in \boldsymbol{P}^{n-1}(\boldsymbol{C})$ with $||u|| = r$

$$\frac{1}{2\pi}\int_0^{2\pi} \log^+ |F_\ell(ue^{i\theta})|\, d\theta + \frac{1}{2\pi}\int_0^{2\pi} \log^+ \frac{1}{|F_\ell(ue^{i\theta})|}d\theta$$
$$\leqq r^q \int_0^\infty \frac{n(t, \ell \cdot D)}{t^{q+1}}\phi\left(\frac{t}{r}\right)dt .$$

From Nevanlinna's first main theorem and $F_\ell(0) = 1$ it follows that

$$\frac{1}{\pi}\int_0^{2\pi} \log^+ |F_\ell(ue^{i\theta})|\, d\theta \leqq N(r, \ell \cdot D) + r^q \int_0^\infty \frac{n(t, \ell \cdot D)}{t^{q+1}}\phi\left(\frac{t}{r}\right)dt . \tag{3.9}$$

Letting $\lambda(\ell)$ denote the standard volume form on $\boldsymbol{P}^{n-1}(\boldsymbol{C})$ defined by χ, we have

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$$(3.10)\qquad \int_{\partial B(r)} \log^+ |F| \eta = \int_{\ell \in P^{n-1}(C)} \lambda(\ell) \frac{1}{2\pi} \int_0^{2\pi} \log^+ |F_\ell(ze^{i\theta})| \, d\theta ,$$

where $z \in \ell$ and $\|z\| = r$. Since $n(t, D) = \int n(t, \ell \cdot D)\lambda(\ell)$ by definition, using Fubini's theorem we get (3.7) from Lemma 2, (3.9) and (3.10).

In case $q = 0$ we have by Lemma 2 and Hayman [4, p. 28]

$$\log |F_\ell(u)| \leqq \int_0^r \frac{n(t, \ell \cdot D)}{t} dt + r \int_r^\infty \frac{n(t, \ell \cdot D)}{t^2} dt$$

for $u \in \ell \in P^{n-1}(C)$ with $||u|| = r$. Then the rest of the proof is similar to the above. Q.E.D.

4. Representation of meromorphic mappings

In this section let us fix a homogeneous coordinate system $(w_0; \cdots; w_N)$ in $P^N(C)$. Then we may take

$$(4.1)\qquad \begin{aligned} h &= \sum_{\mu=0}^N |w_\mu|^2/|w_\nu|^2 \qquad \text{if } w_\nu \neq 0 , \\ \omega &= dd^c \log \left(\sum_{\mu=0}^N |w_\mu|^2 \right) . \end{aligned}$$

A meromorphic mapping $f\colon C^n \to P^N(C)$ is represented as

$$(4.2)\qquad f = (f_0; \cdots; f_N) ,$$

where f_μ are entire functions and codim $\{f_0 = \cdots = f_N = 0\} \geqq 2$. If $f = ('f_0; \cdots; 'f_N)$ is another representation of f, then there is an entire function g such that $'f_\mu = e^g f_\mu$ for all μ. By (4.1) and (4.2) we have

$$(4.3)\qquad T(r, f) = \int_{\partial B(r)} \log \left(\sum_{\mu=0}^N |f_\mu|^2 \right)^{1/2} \eta - \log \left(\sum_{\mu=0}^N |f_\mu(0)|^2 \right)^{1/2}$$

provided that $\sum_{\mu=0}^N |f_\mu(0)|^2 \neq 0$, i.e., f is holomorphic at the origin.

LEMMA 4. *Let $f\colon C^n \to P^N(C)$ be a meromorphic mapping of order $< q + 1$ and suppose that $f^*\{w_\mu = 0\}$, $\mu = 0, \cdots, N$ do not contain the origin. Then f is represented as*

$$f = (F_0; F_1 e^{P_1}; \cdots; F_N e^{P_N}) ,$$

where each F_μ is the canonical function of genus q associated with $f^*\{w_\mu = 0\}$ if $f^*\{w_\mu = 0\} \neq \phi$, or $\equiv 1$ if $f^*\{w_\mu = 0\} = \phi$ and P_μ are polynomials in $z_1, \cdots, z_n$ of degree $\leqq q$.

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Proof. By the assumption and (2.1) the orders of $f^*\{w_\mu = 0\}$ are less than $q+1$. Thus we may take the canonical functions F_μ of genus q associated with $f^*\{w_\mu = 0\}$ (if $f^*\{w_\mu = 0\} = \phi$, we take $F_\mu \equiv 1$). f is represented as

$$f = (F_0\,;\, F_1 e^{g_1}\,;\, \cdots\,;\, F_N e^{g_N})\,, \tag{4.4}$$

where g_μ are entire functions. Hence it suffices to show that the order of e^{g_μ}, say e^{g_1}, is less than $q+1$. From (4.4), (4.1) and (2.1) it follows that

$$\int_{\partial B(r)} \log^+ \left|\frac{F_1}{F_0} e^{g_1}\right| \eta \leqq T(r, f) + O(1)\,. \tag{4.5}$$

Noting that $\log^+ ab \leqq \log^+ a + \log^+ b$, we have

$$\int_{\partial B(r)} \log^+ |e^{g_1}|\,\eta \leqq \int_{\partial B(r)} \log^+ \left|\frac{F_1}{F_2} e^{g_1}\right| \eta + \int_{\partial B(r)} \log^+ |F_0|\,\eta + \int_{\partial B(r)} \log^+ \frac{1}{|F_1|}\,\eta\,.$$

From (2.2),

$$\int_{\partial B(r)} \log^+ \frac{1}{|F_1|}\,\eta \leqq \int_{\partial B(r)} \log^+ |F_1|\,\eta + O(1)\,.$$

So we see that

$$\int_{\partial B(r)} \log^+ |e^{g_1}|\,\eta \leqq T(r, f) + T(r, F_0) + T(r, F_1) + O(1)\,.$$

As the orders of f, F_0 and F_1 are less than $q+1$, so is that of e^{g_1}.

Q.E.D.

5. Proof of Theorem 1

First we take a homogeneous coordinate system $(w_0\,;\, w_1\,;\, \cdots\,;\, w_N)$ in $\boldsymbol{P}^N(\boldsymbol{C})$ so that $H_\mu = \{w_\mu = 0\}$. Let q denote the largest integer not exceeding ρ. By Lemma 4, f is represented as

$$f = (F_0\,;\, F_1 e^{P_1}\,;\, \cdots\,;\, F_N e^{P_N})\,.$$

By (4.3) and Lemma 4 we see that

$$\begin{aligned} T(r, f) &\leqq \sum_{\mu=0}^{N} \int_{\partial B(r)} \log^+ |F_\mu|\,\eta + \sum_{\mu=1}^{N} \int_{\partial B(r)} \log^+ |e^{P_\mu}|\,\eta + O(1) \\ &\leqq \sum_{\mu=0}^{N} \int_{\partial B(r)} \log^+ |F_\mu|\,\eta + O(r^q)\,. \end{aligned}$$

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Now we apply Lemma 3 to this. Setting $n(t)=\sum_{\mu=0}^{N} n(t, f^*H_\mu)$ and $N(r)=\int_0^r n(t)dt/t$, we get from (3.7)

$$2T(r, f) \leqq N(r) + r^q \int_0^\infty n(t)t^{-q-1}\phi\left(\frac{t}{r}\right)dt + O(r^q) .$$

Similarly to Edrei-Fuchs [1, § 4] this inequality yields

$$2 - K(f) \leqq K(f)\rho \int_0^\infty t^{\rho-q-1}\phi(t)dt .$$

From this and (3.6) we deduce that

$$K(f) \geqq \frac{2\Gamma^4(3/4)\,|\sin \pi\rho|}{\pi^2\rho + \Gamma^4(3/4)\,|\sin \pi\rho|} .$$

Hence we have (1.2). Q.E.D.

6. Proof of Theorem 2

As in the previous section, f may be represented as

$$f = (F_0\,;\, c_1F_1\,;\, \cdots\,;\, c_NF_N) ,$$

where c_μ are non-zero constants. By (4.3) we have

$$T(r, f) \leqq \sum_{\mu=0}^{N} \int_{\partial B(r)} \log^+ |F_\mu|\,\eta + O(1) .$$

Using the same notation $n(t)$ and $N(r)$ as in section 5, we have by Lemma 3

$$T(r, f) \leqq N(r) + r\int_r^\infty \frac{n(t)}{t^2}dt + O(1) .$$

In view of integration by parts this implies

$$T(r, f) \leqq r\int_r^\infty \frac{N(t)}{t^2}dt + O(1) . \tag{6.1}$$

Noting that the order of $N(r)$ is ρ, by Hayman [4, Lemma 4.7] we can take a sequence $'r \uparrow \infty$ for an arbitrarily small $\varepsilon > 0$ such that

$$N(t) \leqq \left(\frac{t}{'r}\right)^{\rho+\varepsilon} N('r) \qquad \text{for } t \geqq {'r} . \tag{6.2}$$

From (6.1) and (6.2) we get

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$$T(r, f) \leqq {}'r^{1-\rho-\varepsilon} N('r) \int_{'r}^{\infty} t^{\rho+\varepsilon-2} dt + O(1)$$
$$= \frac{N('r)}{1-\rho-\varepsilon} + O(1) .$$

Thus $K(f) \geqq 1 - \rho - \varepsilon$. Letting $\varepsilon \to 0$, we deduce (1.3). Q.E.D.

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