

The limit-point and limit-circle classification of the Sturm-Liouville operator $(py)'+qy$

Ian Walker Knowles

The formal Sturm-Liouville operator

$$\tau y(t) = (p(t)y'(t))' + q(t)y(t)$$

defined on the real interval I can be classified according to the behaviour of the solutions of $\tau y = 0$ at the end-points of I . In particular, we say that τ is of limit-circle type at the end-point b of I if and only if every such solution is square integrable over some neighbourhood of b , and of limit-point type at b if this is not the case. The aim of this thesis is to present a detailed account of this topic.

In chapter one we gather some background information on the various differential operators which can be formed from τ by defining a suitable domain on which τ may act; included here is the relation between the deficiency indices of τ , self-adjointness, and limit-point type. The second and third chapters, which deal with the number of zeros of solutions of $\tau y = 0$, form the basis of much of the new work presented in chapter four.

Chapter four itself is devoted to assembling all of the known conditions on p, q which are sufficient for τ to be of limit-point type at ∞ . The overriding aim in this section is to reduce the many diverse criteria which have been discovered over the years, to a few very powerful ones, from which the rest can then be derived; included here are

Received 2 February 1973. Thesis submitted to the Flinders University of South Australia, November 1972. Degree approved, January 1974.
Supervisor: Professor D.B. Sears.

generalizations of the well-known criteria of Brinck [1], Dunford and Schwartz [2], p. 1410, Hartman [3], Ismagilov [4], and Sears [5]. For the sake of completeness, all of the available limit-circle results have also been included.

The thesis concludes with a comparison of the effectiveness of several criteria, and some interesting applications of the theory.

References

- [1] Inge Brinck, "Self-adjointness and spectra of Sturm-Liouville operators", *Math. Scand.* 7 (1959), 219-239.
- [2] Nelson Dunford and Jacob T. Schwartz, *Linear operators*, Part II (Interscience [John Wiley & Sons], New York, London, 1963).
- [3] Philip Hartman, "The number of L^2 -solutions of $x'' + q(t)x = 0$ ", *Amer. J. Math.* 73 (1951), 635-645.
- [4] Р.С. Исмагилов [R.S. Ismagilov], "Об условиях самосопряженности дифференциальных операторов высшего порядка" [Conditions for self-adjointness of differential equations of higher order], *Dokl. Akad. Nauk SSSR* 142 (1962), 1239-1242.
- [5] D.B. Sears, "Note on the uniqueness of the Green's functions associated with certain differential equations", *Canad. J. Math.* 2 (1950), 314-325.