

The specific topics are as follows: After introductory chapters on the classical calculus of variations and on the properties of harmonic functions and related inequalities, the author discusses the Dirichlet problem in the context of the classes  $H_p^m$  and  $H_{p_0}^m$  of  $L^p$  functions having distributional derivatives of order  $m$ , and introduces quasi-potentials. He then discusses existence theorems for variational quasi-linear elliptic systems using lower semi-continuity results of Serrin. There follows a long chapter on the differentiability of weak solutions, including the De Giorgi - Nash - Moser results and the theory of Ladyshenskaya and Uraltseva. Next the theory of coercive boundary value problems for general elliptic systems is studied, with special reference to differentiability and analyticity for non-linear systems. The author then devotes two chapters to the theory of harmonic integrals, in the real and complex domains respectively. In the real case, the variational techniques of Morrey and Eells are applied to compact manifolds and manifolds with boundary. In the complex case, a study is made of the work of Kohn on the  $\bar{\partial}$ -Neumann problem. There follows a chapter on parametric integrals (integrals invariant under diffeomorphisms) with reference to Plateau's problem in the two dimensional case. The volume concludes with a chapter on recent work on higher dimensional plateau problems.

As the foregoing summary suggests, this book is a work of pure mathematics, with primary emphasis upon analytical techniques. A prospective reader should know the elements of functional analysis and elliptic differential equations, and should be prepared for frequent references to other sources which the author must necessarily make on account of limitations of space. The book is carefully written at a high level of technique and sophistication. It will undoubtedly be a most valuable source of information and technique for both students and research workers.

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Introduction to measure and probability by J.F.C. Kingman and S.J. Taylor. Cambridge, England, 1966. x + 401 pages.

This book falls naturally into two parts, though no formal division is made. The first nine chapters give a reasonably complete and self-contained account of the theory of measure and integration. The remaining six chapters introduce the concepts of probability and apply some of the measure theory in this field.

Both parts of the book are thorough and wide-ranging. The treatment is clear, occasionally over-condensed but always elegant and compact. It is so self-contained that there is even a chapter on set theory and a chapter on point set topology. Surely all readers mature enough to deal with the later parts of the book must inevitably have absorbed the basic concept of set theory a long time before. The same

point could be argued about point set topology, but in this case the author's chapter 2 is refreshing and vigorous enough to justify inclusion.

Chapters 3 to 9 form a well-written account of standard measure and integration theory. The concept of measure is taken as basic, and integration is introduced by means of simple functions. The concept of linear functional on the space of continuous functions is discussed later and its connection with the earlier ideas is established. In addition to routine material such as  $L_p$ -spaces, Fubini's theorem and the Radon-Nikodym theorem, the authors also include the mean ergodic theorem and prove the result often quoted but seldom seen in standard texts, that a function is Riemann integrable if and only if its set of discontinuities has measure zero. Thus the area covered is wide, by no means exclusively geared to the applications in probability. A disappointing exception, for this reviewer, is the rather sketchy account, confined to metric groups, of the Haar measure. Uniqueness apart from a constant multiple is not proved, and the wording of Ex. 7, page 260, though formally accurate, might mislead an incautious reader by giving him the impression that no uniqueness theorem for non-compact groups is known.

The probability half of the book again covers a variety of topics in some depth. The introductory chapter 10 - "What is probability?" - contains a good discussion of the relationship between measure and probability. It is made quite clear what assumptions are involved, and that it would be unreasonable to expect a proof of them. However, a thorough discussion of elementary examples makes the assumptions very plausible for beginners. The discussion of statistical independence and product measures is particularly lucid.

The final chapters - eleven to nineteen - headed, in order, "Random Variables", "Characteristic Functions", "Independence", "Finite Collections of Random Variables" and "Stochastic Processes" provide a sound groundwork in Probability Theory. Where space forbids a complete account, as with the law of the iterated logarithm, suitable references are given.

Altogether, this is an excellent book, particularly suited for graduate students or for undergraduates in their senior year.

A. M. Macbeath

Elementary Differential Geometry by Barret O'Neill. Academic Press, 1966.

Geometry has been advancing very rapidly in research level; by contrast the traditional undergraduate course has changed very little in the last few decades. There should be general agreement that the undergraduate course needs to be brought up to date, and this book may be said to be one that meets well this demand in presenting both materials and the mathematical style. After introducing the notations and symbols