Multiple Periodic Variable Stars
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CEPHEIDS

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1. Introduction

The scope of this review will be to summarize our knowledge of Cepheids at the present time concentrating primarily on the observational results obtained but at the same time relating these to the relevant results of pulsation and evolution theory. Of the many previous reviews of pulsating stars we would mention especially Payne-Gaposchkin and Gaposchkin (1938), Ledoux and Walraven (1958), Christy (1966a) and Cox (1974) as providing an excellent background to the theoretical and observational aspects of the subject. Because of these reviews we will concentrate on the more recent results obtained in connection with Cepheids. At the end a special section is devoted to the double-mode Cepheids as they are particularly relevant to this Colloquium.

2. Population I Cepheids

The position of population I or classical Cepheids on the Hertzsprung-Russell (H-R) diagram in relation to other types of intrinsic variable stars is shown in Fig. 1. Population I Cepheids occupy a narrow near vertical strip on the H-R diagram covering the range of absolute magnitude $-2 > M_V > -6$. The stars which occupy this instability strip typically have periods in the range $1 < P < 50$ days, although a few longer period Cepheids do exist.

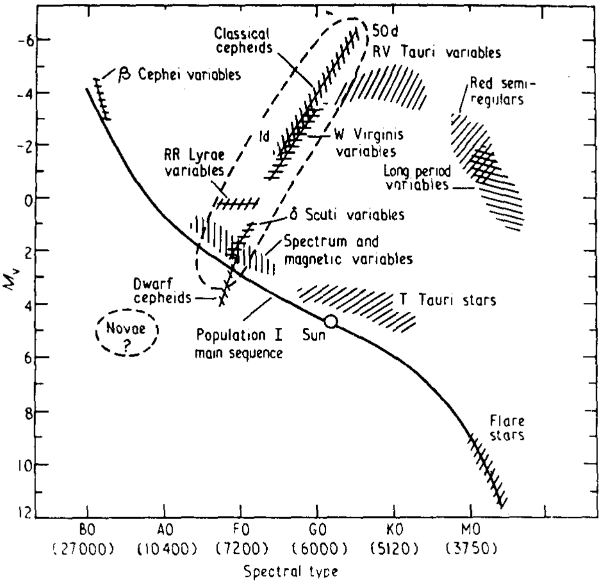

Figure 1. Location of various types of intrinsic variables on the H-R diagram (Cox, 1974).

The evolutionary history of Population I Cepheids is well understood from the extensive series of evolutionary calculations that have been carried out (see, e.g. Iben, 1967). Population I models of mass 4-9 $M_\odot$ as they ascend the red giant branch exhibit loops to the blue at almost constant luminosity on the H-R diagram. These loops can intersect the observed instability strip and this intersection occurs when the models are in the core helium burning stage.

Detailed agreement has been obtained between the theoretical and observed number of Cepheids as a function of absolute magnitude and the period-frequency histogram (Iben 1966, Hofmeister 1967). Differences in the observed short period cut-off between the Cepheids in the Galaxy, the Small Magellanic Cloud (SMC) and the Large Magellanic Cloud (LMC) have led a number of authors to propose that these differences arise as a consequence of the evolutionary loops penetrating the strip to differing degrees in the separate galaxies. The calculations of Robertson (1971) support this suggestion and indicate that both helium content, Y, and heavy element content, Z, affect the loops significantly. The extent of these loops to the blue is sensitive to many factors and extreme care is required in calculating the evolutionary model in order to produce reliable results (Lauterborn, Refsdal and Weigert 1971, Robertson 1971).

Extensive series of linearized, non-adiabatic pulsation calculations have shown the existence of a blue edge to the Population I instability strip which agrees well with the observed blue edge (Iben and Tuggle 1972, King et al. 1973). This theoretical blue edge is sensitive both to the helium abundance and to the mass-luminosity (M-L) relationship employed (Fig. 2). A comparison of the blue edges determined by linear and non-linear computations has shown that the blue edges defined by the linear, non-adiabatic calculations are valid as no case has yet been found of hard, self-excited pulsations (King et al. 1973). Thus provided an accurate transformation can be made from the observed quantities (M_V, B-V) to the theoretical parameters ($L/L_\odot$, T_e) then this dependence in principle provides a powerful way of determining the helium abundance (assuming some M-L relationship).

The existence of the red edge to the instability strip, however, has not been found theoretically although the suggestion that it is caused by the sudden onset of deep convection (Baker and Kippenhahn 1965) is borne out by recent calculations (Tuggle and Iben 1973).

Theoretical calculations of the periods of pulsating models have all shown that the period is primarily determined by the radius and mass of the model and is almost independent of the luminosity, effective temperature or chemical composition of the model (Christy 1966a, Stobie 1969b, Cox et al. 1972a). The period is little affected by non-linearities in the motion and can be accurately determined by calculating models in the linear, adiabatic approximation (Cogan 1970). The existence of this period-radius-mass (P-R-M) relationship together with an M-L relationship from the results of evolutionary models automatically implies that Cepheids will show a P-L-T_e relationship. It is to be

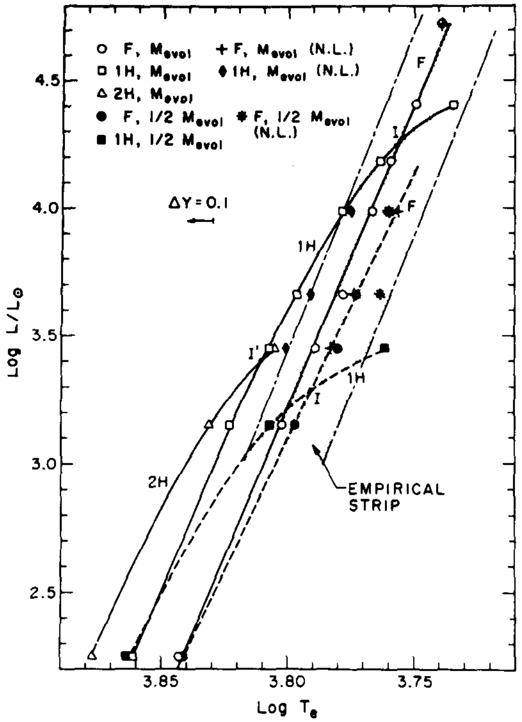

Figure 2. Blue instability edges on an H-R diagram. Solid lines, $M = M_{evol}$; dashed lines, $M = 0.5M_{evol}$; dashed-dot lines, boundaries of empirical Population I Cepheid strip. All lines in the figure refer to linear, non-adiabatic results and the symbols refer to non-linear, non-adiabatic results. F, 1H, 2H refer to pulsational instability in fundamental mode, first overtone and second overtone respectively. The horizontal arrow shows the estimated shift of the F blue edges that would be brought about by an increase in Y (helium mass fraction) of 0.1 (King et al. 1973).

noted, however, that this P-L-T_e relationship is not independent of chemical composition since the theoretical M-L relationship of Cepheids is dependent upon the chemical composition (Robertson 1971).

The P-L-T_e relationship has found strong confirmation from the observations in the existence of a period-luminosity-colour (P-L-C)

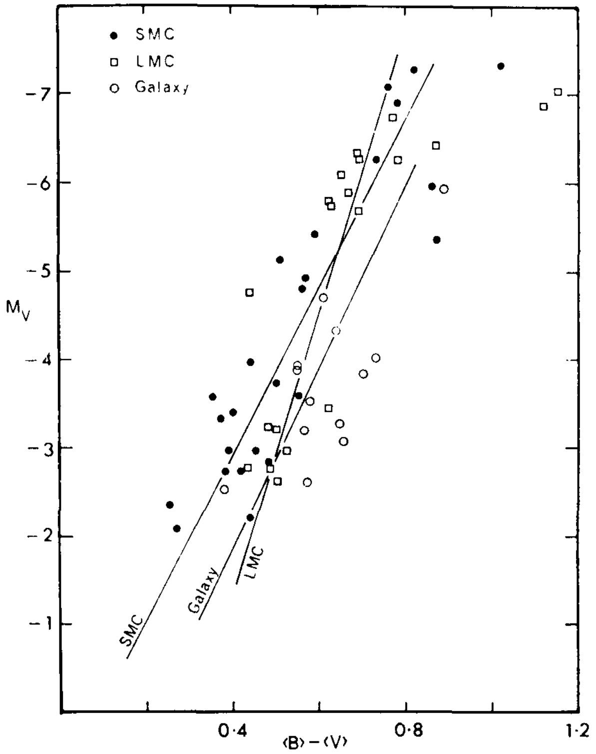

Figure 3. A composite H-R diagram for Population I Cepheids in the SMC, the LMC and our Galaxy (Gascoigne 1969).

relationship. This P-L-C relationship has been determined in our Galaxy from the thirteen calibrating Cepheids (Sandage and Tammann 1969) and also in the SMC and LMC (Gascoigne 1969, van Genderen 1969 and Butler 1975a, b). The relative positions on the H-R diagram of Population I Cepheids belonging to the Galaxy, the SMC and the LMC are shown in Fig. 3. It appears that there are significant differences at the short period end of the instability strip in that the SMC Cepheids are systematically bluer by $0^{m}.1$ in B-V relative to the LMC and Galactic Cepheids (Gascoigne 1969).

The finite width of the instability strip requires that a third parameter is included in the P-L-C relationship in order to reduce the intrinsic scatter present in the P-L and P-C relationships. Because of the importance of Cepheids as distance indicators it is vital to calibrate the P-L-C relationship as accurately as possible. Sandage and Tammann (1971) have combined the data on Population I Cepheids in different galactic systems to produce a composite and universal P-L-C relationship. On the other hand Gascoigne (1974) has shown that the P-L-C relationship is in fact sensitive to the metal abundance, Z, and he obtains a difference in distance modulus from the SMC of $0^m.41$ depending upon whether the SMC is assumed to have Z = 0.02 or Z = 0.005. Iben and Tuggle (1975) have also presented convincing arguments to show that the P-L-C relationship is not unique from galaxy to galaxy. The basic reason is that if the heavy element abundance varies between different galactic systems this affects the P-L-C relationship by changing both the M-L relationship of Cepheids and the $C-T_e$ transformation (Bell and Parsons 1972). Thus strong doubts have been cast on the applicability of a universal P-L-C relation.

Sandage and Tammann (1971) by combining the data from different galactic systems have studied the amplitudes of Cepheids as a function of position in the instability strip. They find that in the period range $0.40 < \log P < 0.86$ the amplitude decreases monotonically from the blue to the red sides of the strip. Since the amplitude (A) of a variable is found observationally to be a function of its colour and period this means that a P-L-A relationship instead of a P-L-C relationship can be derived for Cepheids in the period range $0.40 < \log P < 0.86$. Subsequent confirmation of this work has been obtained by Kelsall (1972) and Butler (1975c). Madore (1975a, b), however, from a photoelectric study of galactic and extragalactic Cepheids with $P > 11$ days finds that the amplitudes increase monotonically from the blue to the red sides of the strip in the opposite sense to that described by Sandage and Tammann. Madore also notes that the longer period Cepheids are subject to more reddening (presumably circumstellar) than the shorter period Cepheids. A similar effect has been found by Feast (1974) and he suggests that if this effect is taken into account there is no need for the composite P-L relationship derived by Sandage and Tammann (1968) to flatten out the long period end.

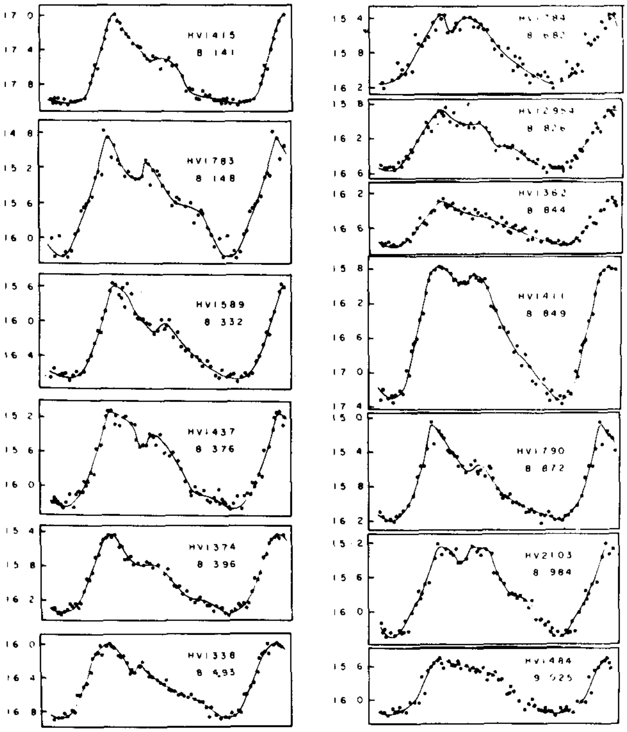

Figure 4. A selection of light curves showing conspicuous bumps amongst Population I Cepheids in the Small Magellanic Cloud (Payne-Gaposchkin and Gaposchkin 1966).

The systematics of Population I Cepheid light curve shapes have been known for a long time. The famous Hertzsprung progression of light curve shape with period may be described briefly as follows. The secondary bump in Cepheid light curves first becomes visible at periods near 7 days on the descending branch of the light curve. As the period is increased the phase of the bump decreases until it coincides with maximum light for periods near 10 days. For periods greater than 10 days the bump generally appears on the ascending branch of the light curve until for periods greater than about 15 days the bump disappears. A subsection of the Hertzsprung progression is illustrated in Fig. 4. This progression has been found in all galactic systems where Population I Cepheids have been observed and the relationship between the phase of the bump and the period is apparently identical from one galaxy to another (van Genderen 1970). The Hertzsprung progression has been found in the non-linear non-adiabatic models of pulsating stars and the phase of the bump varies systematically with period in exactly the manner predicted by the observations (Christy 1968, Stobie 1969a, b). The phase of this bump together with the period of a variable star provides a potential method of deriving the mass and radius of the star (Fricke et al. 1972). Masses and radii derived in this way, however, are systematically too low compared with masses derived from evolutionary theory and there is some uncertainty at present attached to the theoretical calibration of this relation (Stobie 1974).

The light and radial velocity curves of Population I Cepheids are known to be almost mirror images of one another (Cox 1974). This

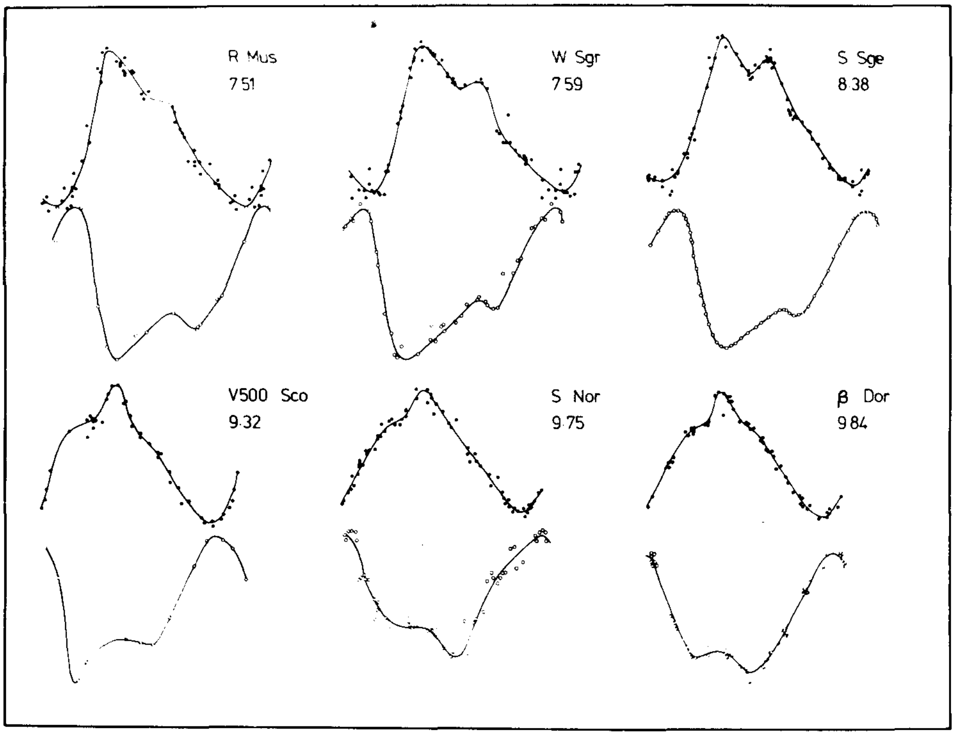

Figure 5. Light and radial velocity curves of Galactic Cepheids.

is not true, however, in the case of Cepheids with a secondary bump in their light curve (Fricke et al. 1972). A study of six Population I Cepheids with accurate light and radial velocity curves (Stobie, unpublished) has shown that there is a systematic delay of 1.26 days between the appearance of the bump on the light curve and the appearance of the bump on the velocity curve (Fig. 5). This delay is quantitatively what would be expected if the bump corresponds to a pulse travelling with the local speed of sound (Christy 1968) which first encounters the photosphere and subsequently reaches the line forming region of the atmosphere.

The question of the mode of a pulsating star is one of fundamental importance in any application of the P-L or P-L-C relationships. Unfortunately, unlike the RR Lyrae stars, the Population I Cepheids do not form a clear cut distinction in a period-amplitude plot into first overtone and fundamental mode pulsators. Theory predicts that the first overtone Cepheid pulsators will occur at the blue side and short period end of the instability strip (Stobie 1969b, King et al. 1973). Observational evidence in favour of this is the study by the Gaposchkins of Cepheids in the SMC (Payne-Gaposchkin and Gaposchkin 1966). They isolated a group of Cepheids with sinusoidal, low amplitude light curves (which we tentatively identify with first overtone pulsators). This group of Cepheids occurs in the period range $1 < P < 3$ days and systematically lies about $0^{m}.5$ above the mean P-L relation. Nikolov and Tsvetkov (1972) in a study of the pulsation amplitude of Galactic Cepheid variables isolated a group of low amplitude variables in a $\log \Delta R - \log P$ plot. Although Nikolov and Tsvetkov do not claim these variables are first overtone pulsators we would consider them likely candidates.

Theoretically the question as to which mode a star will pulsate in has been clarified recently although a complete physical understanding of the process is not at hand (Iben 1971, Cox et al. 1972b, Tuggle and Iben 1973 and Stellingwerf 1975a, b). Although the majority of these results has been obtained in applications to RR Lyrae variables we expect that similar results will be obtained in Cepheid variables. Van Albada and Baker (1973) proposed that in RR Lyrae stars there exists a finite transition region between fundamental and first overtone pulsators in which a star will persist pulsating in the mode with which it entered the transition region. Thus the pulsation mode with which stars are observed in this transition region depends upon their evolutionary history. Subsequent non-linear pulsation calculations studying the modal stability of models (Stellingwerf 1975a, b) have confirmed this prediction for RR Lyrae models and extended the result to low mass Cepheid models. The results (Fig. 6) show that a strip of width $\Delta T_e \sim 200 - 300^{\circ}$K appears to exist in which a model can pulsate in either fundamental or first overtone pulsation.

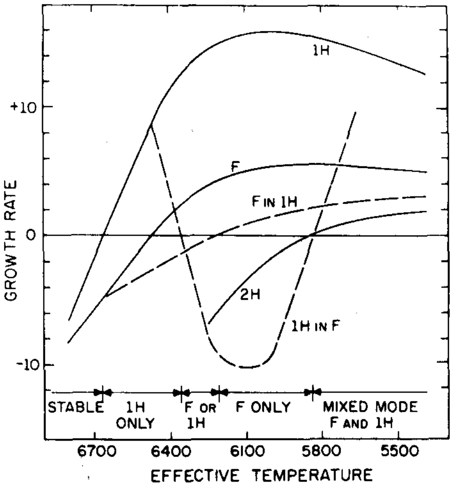

Figure 6. Composite growth rate diagram for models with $M/M_{\odot} = 0.6$, $P_0 = 2.6$ days. Solid lines: linear growth rates; dashed lines: approximate non-linear switching rates ("F in 1 H" is switching rate of 1 H towards F). The various regions of different non-linear behaviour are indicated below the curves (Stellingwerf 1975b).

Apart from a subset of Cepheids known as double-mode Cepheids the majority of Population I Cepheids appear to have very stable light curves (Asteriadis et al. 1974). Exceptions to this rule are two long period Cepheids, ℓ Car (Feinstein and Muzzio 1969) and IU Cyg (Tammann 1969). Most Population I Cepheids also appear to have stable periods. The few Cepheids which have significant changes in their period show a general tendency in that the greater the period of the star the greater the period changes observed (Parenago 1958). These period changes have been further studied by Balazs-Detre and Detre (1965) and in no case can the period changes be assigned unambiguously to expected evolutionary changes in the star. Rather the (O-C) diagram is characteristic of random period fluctuations.

Recently evidence has been advanced for the existence of non-pulsating stars in the Population I instability strip (Fernie and Hube 1971, Schmidt 1972). It seems unlikely that the observational errors can be sufficiently large to exclude these stars from the instability strip and they consequently present a problem to the theoretician. Cox et al. (1973) have proposed that these stars may be deficient in helium, at least in the region of the helium ionization zones.

3. Population II Cepheids

The evolutionary status of Population II Cepheids has clarified considerably in the last ten years. Stars on the horizontal branch are believed to be in the stage of core helium burning. As the helium in the core is exhausted the stars evolve to higher luminosities and eventually approach the asymptotic red giant branch. Depending upon their initial position on the horizontal branch and helium content the stars can subsequently evolve through the instability strip in a suprahorizontal branch stage (Fig. 7). These suprahorizontal branch stars explain the existence of the short period group of Population II Cepheids in the period range 1 - 2 days observed in globular clusters and in the field. As the star ascends the asymptotic red giant branch it is subject to thermal pulses during the helium shell burning stage. These pulses cause it to execute loops in the H-R diagram which may intersect the instability strip (Schwarzschild and Härm 1970). This stage of evolution is thought to correspond to the longer period group of Population II

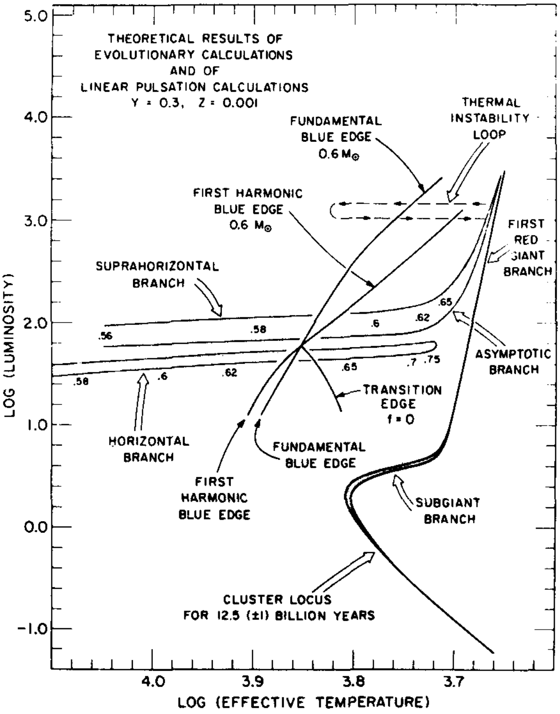

Figure 7. Theoretical H-R diagram showing results of evolution and linearized pulsation calculations for Population II stars having Y = 0.3, Z = 0.001. The numbers along the horizontal and suprahorizontal branches give the stellar masses, in solar units, and the mean locations of stars of the respective masses (Iben and Huchra 1971).

Cepheids in the period range 8 - 25 days.

Masses of Population II Cepheids have been derived from the P-R-M relationship (Böhm-Vitense et al. 1974). The results give $M/M_{\odot} \sim 0.55$ consistent with the pulsation masses derived for RR Lyrae stars. These masses imply that the stars have at some stage (presumably before the horizontal branch phase) lost mass as their main sequence progenitors have masses nearer $M/M_{\odot} \sim 0.8$ (Iben 1972).

The helium content of Population II Cepheids can be determined from the agreement of the observed and theoretical blue edges.

Demers and Harris (1974) find that a value of Y ~ 0.3 is predicted by their observations. This is consistent with the helium content predicted for RR Lyrae variables (Christy 1966b) and also with the helium content of globular cluster stars determined by the ratio of the number of horizontal branch stars to the number of red giant stars with magnitude brighter than the mean magnitude of the horizontal branch (Iben 1972).

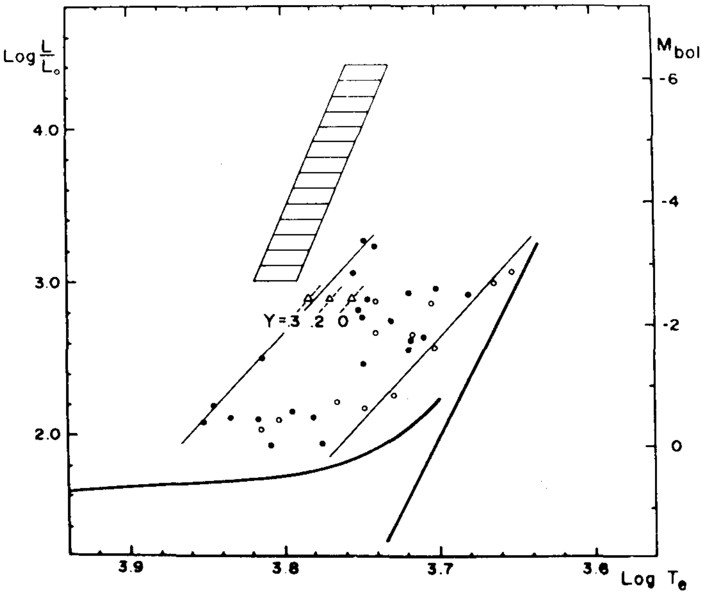

Figure 8. H-R diagram for Population II Cepheids. Hatched region is Population I instability strip. The theoretical blue edges for three different helium contents are shown (Demers and Harris 1974).

The location of Population II Cepheids in the H-R diagram has been studied by Demers and Harris (1974). They find that the Population II Cepheid instability strip is considerably broader and flatter than the Population I Cepheid instability strip (Fig. 8). P-L relationships for Population II Cepheids have been derived by a number of authors (Fernie 1964, Kwee 1968b and Demers and Wehlau 1971). The slope of the relationship is shallower than the slope of the corresponding Population I Cepheid relationship. This difference is a consequence of the Population II variables all having a similar mass independent of their luminosity. Caputo and Castellani (1972) have investigated P-L, P-C relationships and amplitude dependences for globular cluster Cepheids. They find correlations between the amplitude, colour and luminosity of a variable in the strip in the sense that for $\log P < 1$ largest amplitude occurs for the brightest and bluest variables whereas for $\log P > 1$ the correlations are reversed.

Relatively few pulsation calculations have been carried out relevant to Population II Cepheids. Some linear, non-adiabatic calculations have been reported by Cox and King (1972) and by Cox et al. (1973). Non-linear, non-adiabatic models of W Virginis have been constructed by Christy (1966c) and Davis (1972). The model by Davis showed a stillstand on the descending branch of the light curve which he attributed to the formation of a shock at the photosphere.

The light curves of field Population II Cepheids have been studied by Kwee (1968a). He finds that bumps or shoulders are a common feature in the light curves. In particular in the group of longer period Cepheids, $13 < P < 19$ days, a shoulder on the descending branch of the light curve is a frequent occurrence (cf stillstand in theoretical light curve of Davis 1972). The short period group of Cepheids, $1 < P < 3$ days, also show conspicuous bumps in their light curves. Theoretically bumps at this period are expected from the same mechanism that causes bumps in the Population I Cepheid phenomenon (Christy 1970). It has been proposed that a Hertzsprung progression amongst Population II Cepheids may exist similar to that observed in Population I Cepheids and observational evidence has been presented in support of this (Stobie 1973, Mandel 1971).

There is little data on the stability of the light curves of Population II Cepheids from epoch to epoch. One extreme example of an unstable light curve is RU Cam (Broglia and Guerrero 1973). In general Population II Cepheids appear to be much more subject to abrupt period changes than Population I Cepheids, the frequency and amplitude of the period changes being an order of magnitude greater (Parenago

1958, Balazs-Detre and Detre 1965 and Kwee 1967).

4. Double-mode Cepheids

Oosterhoff (1957a) was first to recognize the existence of a distinct class of Cepheid variable characterized by an abnormally large scatter in their photoelectric light curves. Subsequent analysis of the photoelectric observations revealed that the observations could be satisfactorily accounted for by the light curve being modulated with beat period, P_b. Interpreting this variation as being caused by the superposition of two component periodicities of period P_0, P_1 we have the relation

$$\frac{1}{P_1} - \frac{1}{P_0} = \frac{1}{P_b} \tag{1}$$

Note that there is an ambiguity in equation (1) in attempting to determine the secondary period if the only information available is the primary period and the beat period. The value of secondary period depends upon whether the primary period is assigned to P_0 or to P_1. This ambiguity, however, can be resolved if, after the primary period has been found, the mean curve corresponding to this primary period is subtracted from the data and the residuals then searched for the second period. The presence of two periodicities has also been found in the radial velocity observations (Oosterhoff 1957b).

The number of double-mode Cepheids whose light curves have been analysed for their component periodicities totals eight. The light curve characteristics of seven of these are listed in Table 1. The one Cepheid omitted is V439 Oph (Gusev 1967) on the basis that photoelectric observations by Sturch (1966) show no scatter in excess of observational error. The primary periods of the Cepheids lie in the range 2 to 4 days and the period ratios in the narrow range 0.703 $P_1/P_0 < 0.711$. The amplitudes ΔV, ΔB of the two periodicities have been calculated according to the method of Stobie (1970). It is apparent that in all cases (except AX Vel) the amplitude of the longer period exceeds the amplitude of the shorter period.

Table 1

Periods and amplitudes of double-mode Cepheids

Star	P_0	P_1	P_1/P_0	ΔV_0	ΔV_1	ΔB_0	ΔB_1	Reference
Y Car	3.6398	2.5590	0.7031	0.58	0.29	0.78	0.36	Stobie 1972
TU Cas	2.1393	1.5183	0.7097	0.66	0.31	0.99	0.46	Oosterhoff 1957b
BK Cen	3.1739	2.2366	0.7047	0.52	0.20	0.81	0.28	Leotta-Janin 1967
VX Pup	3.0117	2.136	0.709	0.46	0.33	0.52	0.49	Stobie 1970
U TrA	2.5684	1.8249	0.7105	0.47	0.25	0.79	0.35	Oosterhoff 1957a, Jansen 1962
AP Vel	3.1278	2.1993	0.7031	0.55	0.41	0.79	0.60	Oosterhoff 1964
AX Vel	3.6731	2.5928	0.7059	0.22	0.33	0.28	0.49	Stobie and Hawarden 1972

The two periods present P_0, P_1 have been identified with the periods of the fundamental and first overtone radial modes of a pulsating star. This is primarily because the period ratio corresponds very closely to the value expected theoretically. Note that the values of P_0, P_1 derived for VX Pup in Table 1 are uncertain. Fourier analysis (Stobie, unpublished) of the observations of VX Pup (Mitchell et al. 1964, Takase 1969) has shown that there are a number of possible solutions to the periods present in this star. Further photoelectric observations are required before a definitive solution can be given.

Unlike the case of some delta Scuti stars where the periods present at one epoch of observation may not predict the behaviour of the star at another epoch it appears that the periods present in a double-mode Cepheid are stable (e.g. results of Jansen 1962). Although the superposition of the two mean curves corresponding to the periods P_0, P_1 explains the main characteristics of the light variation, it does not precisely reproduce the observed light curve because non-linearities are present in the interaction of one pulsation mode with another. Non-linearities in the superposition of the modes are clearly seen in the observations of U TrA (Oosterhoff 1957a) since the amplitude variation at maximum light is greater than the amplitude variation at minimum light.

Table 2 gives a list of known and suspected double-mode Cepheids. Because of the observed period range of known double-mode Cepheids (from 2 to 4 days) we have restricted the search for further candidates to the period range 1 to 5 days. Only Cepheids with photoelectric observations have been considered. The data are taken from the catalogue of Schaltenbrand and Tammann (1971). The standard deviations σ_B, σ_V show that these stars have a much larger scatter about their mean light curve than would be expected for a single-mode Cepheid. There are a number of reasons as to why a Cepheid might *have a spuriously large scatter in its light curve (wrong period,* variable period, systematic differences between photometric systems of observers). In drawing up the list of suspected double-mode Cepheids we have attempted as far as possible to exclude variables subject to the

Table 2

Data on known and suspected double-mode Cepheids
in the period range $1 < P < 5$ days

Star	P	⟨V⟩	⟨B-V⟩	⟨U-B⟩	σ_B	σ_V
Y Car	3.64	8.13	0.64	0.38	0.186	0.136
DY Car	4.67	11.36	1.08	-	0.074	0.050
EY Car	2.88	10.34	0.85	0.58	0.072	0.065
FZ Car(1)	3.58	11.85	1.18	0.82	-	-
GZ Car	4.16	10.22	1.00	-	0.067	0.051
TU Cas	2.14	7.72	0.61	0.39	0.131	0.094
UZ Cen	3.33	8.77	0.79	0.52	0.082	0.064
BK Cen	3.17	9.99	0.90	0.61	0.108	0.071
TZ Mus	4.94	11.66	1.35	0.99	0.144	0.180
VX Pup	3.01	8.18	0.61	0.42	0.140	0.089
U TrA	2.57	7.93	0.64	0.39	0.100	0.071
AP Vel	3.13	10.03	1.07	-	0.141	0.098
AX Vel	2.59	8.23	0.72	0.48	0.089	0.071

Note: (1) This star is not in Schaltenbrand and Tammann's (1971) catalogue because the observations are poorly distributed in phase. The scatter, however, is still considerable and the star is included as requiring further observation.

above spurious effects. It should be noted that there are a number of variables mentioned in Kukarkin et al. (1969) as showing variations in their light curve but these have not been included in Table 2 unless the photoelectric observations also showed an unambiguous scatter.

The variables listed in Table 2 form a significant fraction of the short period Cepheids with photoelectric observations in Schaltenbrand and Tammann's catalogue as is shown in Table 3.

Table 3

Fraction of double-mode Cepheids as a function of period

Period range (days)	Number of Cepheids			Fraction of double-mode
	type I	type II	double-mode	
1.0 - 2.0	2	9	0	0.00
2.0 - 3.0	3	3	4	0.40
3.0 - 4.0	20	4	6	0.20
4.0 - 5.0	42	2	3	0.07

Although there may be some selection effects at work there is no strong reason to suppose that these numbers are not representative of the relative fraction of double-mode Cepheids in the solar neighbourhood. Thus the double-mode Cepheids form a significant subset of solar neighbourhood Cepheids in the period interval 2 to 4 days.

All the variables in Table 2 have been classified as type I Cepheids although TU Cas has been classified both as a type I (Mianes 1963) and a type II Cepheid (Petit 1968). None of the variables lies more than 250 parsecs from the galactic plane (assuming luminosities consistent with Population I Cepheids). Thus the properties of double-mode Cepheids are consistent with them being members of the young disc or Population I Cepheids.

In view of the relative fraction of double-mode Cepheids in the solar neighbourhood it is at first sight somewhat surprising that these variables have not been found in external galaxies (especially the SMC where Cepheids in the period range 2 to 4 days are abundant). The scatter in the light curves of some known double-mode Cepheids is sufficiently high (a range of $0\overset{m}{.}5$ at maximum light) that there should be no difficulty detecting these stars photographically if they exist. Wesselink and Shuttleworth (1965) comment on a few SMC Cepheids with periods near 3 days which exhibited considerable irregularities in their light curve. Stobie (unpublished) has Fourier analysed the observations by Butler (1975a, b) of 20 Cepheids in the LMC and SMC for which it had proved difficult to determine the primary period present. In no case was there any evidence that the difficulty in determining the period or the existence of a large scatter in the light curve was caused by the presence of two periods.

The importance of the double-mode Cepheids lies in the fact that with two periods present, which we identify with the fundamental and first overtone modes of radial pulsation, there is the possibility of deriving the masses and radii of the stars solely from a knowledge of these two periods (Petersen 1973). The ratio P_1/P_0 also determines the pulsation constant Q_0 and hence the mean density of the star, giving values which are almost model independent (Fitch 1970). It is well known that the period of a variable star is determined principally by its radius and mass. The period of the linear, adiabatic model also gives values very close to the period derived from the non-linear, non-adiabatic model. The recent calculations by Stellingwerf (1975b) have shown that in a theoretical model of a double-mode Cepheid the interaction of one mode with another does not change the periods derived from the single-mode calculations by more than 0.5 percent. Thus if we know the theoretical P_0-R-M and P_1-R-M relationships we can in principle calculate R and M. The results (Fig. 9) show that the masses and radii of double-mode Cepheids lie in the range $0.7 < M/M_\odot < 1.7$, $14 < R/R_\odot < 23$ (Petersen 1973, Stellingwerf 1975b).

The evolutionary stage of the double-mode Cepheids is not clear at present. The masses and radii derived from the periods P_0, P_1 are inconsistent with the masses and radii expected for Population I Cepheids of similar period (i.e., $M/M_\odot \sim 4.5$, $R/R_\odot \sim 30$). Accepting the low radii of these double-mode Cepheids together with their effective temperature ($T_e \sim 6200^\circ K$) leads to a luminosity of $M_{bol} \sim -2$, which is approximately $0^m.7$ fainter than the absolute magnitude of Population I Cepheids of similar period. The spectroscopic gravities of these variables give conflicting results as Rodgers and Gingold (1973) analysing U TrA found that it had a gravity consistent with the gravity of

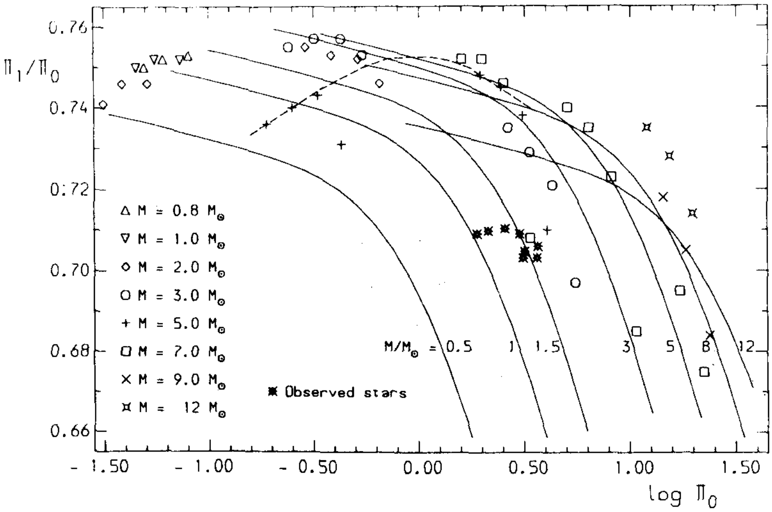

Figure 9. Period ratio as function of the period of the fundamental mode for an extreme Population I composition. Full curves are based on fitting formulae given by Cox et al. (1972a); the number at each curve gives the mass in solar units (Petersen 1973).

Population I Cepheids of similar period. On the other hand Schmidt (1974) finds that TU Cas has a higher gravity than a Population I Cepheid.

Stellingwerf (1975a, b) has presented new and exciting results on the modal behaviour of pulsating stars (Fig. 6). From an examination of the stability of the fundamental mode and first overtone limit cycle pulsations there seem to be two possibilities for the occurrence of double-mode Cepheids. One is that they have been caught in the stage of switching from one mode to another mode (F or 1 H region in Fig. 6). This was the supposition made previously (Stobie 1970, Rodgers 1970) that these stars were in a transition region between fundamental and first overtone pulsators. Stellingwerf's results, however, indicate that the timescale for switching modes is so short that there is little chance of observing a star in this stage. Instead it seems more likely that these double-mode Cepheids occur at the red edge of the instability strip where a stable mixed-mode behaviour can exist (Fig. 6). Mixed-mode models occur for temperatures, $T_e < 5850^oK$ which may be compared with the observed effective temperatures of $T_e \sim 6200^oK$ for double-mode Cepheids (Schmidt 1972, Rodgers and Gingold 1973).

It has been proposed (Petersen 1973, Stellingwerf 1975b) that the double-mode Cepheids represent a new type of variable star occurring in the instability strip intermediate in luminosity and mass between the RR Lyrae variables and the Population I Cepheids. The difficulty with this hypothesis, however, is that no evolutionary models of this mass have been found to enter the instability strip during the core helium burning stage.

We consider that it is not excluded that the double-mode Cepheids are in fact normal Population I Cepheids lying at the red edge of the instability strip. This explanation is tantamount to saying that the masses and radii derived from the P-R-M relationships are wrong. From the theoretical point of view the period of a pulsating star is one of the most accurately determined parameters in the calculations of models of pulsating stars. However, since the powers of R and M in the P_0-R-M and P_1-R-M relationships (in the region relevant to double-mode Cepheids) are similar, the calculation of R or M becomes very sensitive to small changes in the constants in these relationships (Stobie 1974). It may be that some change in the input physics to the model calculations could produce masses and radii consistent with Population I Cepheids. Petersen (1974), however, has investigated some arbitrary changes in the opacity and finds that these changes make little difference to the P-R-M relationships.

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Discussion to the paper of STOBIE

FITCH: Your earlier figures showed that 60% of all 2-4 day Cepheids are double mode variables. Does it seem reasonable that all of them should be at the red edge of the strip?

STOBIE: Yes, it just means that they don't penetrate so far into the strip from the red side.

PETERSEN: You showed a diagram from my paper on double mode Cepheids. The points representing individual models were taken from the investigation by Cogan in 1970. These results agree very well with the fitting formulae of Cox, King, and Stellingwerf from 1972. I tried to vary the chemical composition, and found that the mass values derived depend little on the composition parameters.

FITCH: You get better agreement with evolution theory if you don't use the CKS formulae. Fitch and Szeidel developed interpolation formulae by fitting to Cogan's models, and got better (i.e. higher) masses for the double mode Cepheids. However, these were still below evolutionary calculation expectations.

WAYMAN: Is it not crucial as to whether double mode Cepheids are found in the Magellanic Clouds?

STOBIE: I have looked hard amongst data for the Cloud Cepheids and have been surprised to be unable to definitely find any. They would be found relatively easily from photographic surveys whether higher or lower luminosities apply.

KWEE: Regarding galactic Population II Cepheids, you said that there exist very little data on this kind of object. Recently, I have observed 14 Population II Cepheids with periods between 1 and 3 days. The observations were made with the UBV photometer at La Silla in Chile. The resulting light curves will be published in due time and can be seen at this meeting.