The Mathematical Gazette

A JOURNAL OF THE MATHEMATICAL ASSOCIATION

Vol. 78 July 1994 No. 482

Editorial

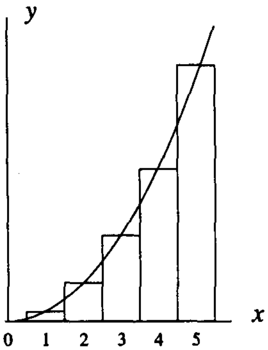
What is your reaction to the following approach to the $\Sigma\, n^2$ formula? We can approximate the sum

$$\sum_{i=1}^{n} i^2$$

by the integral

$$\int_{0.5}^{n+0.5} x^2\,dx,$$

as shown in the diagram and table below.



n	$\sum_{i=1}^{n} i^2$	$\int_{0.5}^{n+0.5} x^2\,dx$	error
1	1	1.083	0.083
2	5	5.167	0.167
3	14	14.25	0.250
4	30	30.33	0.333
5	55	55.42	0.417
6	91	91.50	0.500

It can be seen that the error is given by $n/12$, and the formula is thus

$$\int_{0.5}^{n+0.5} x^2\,dx - \frac{n}{12} = \tfrac{1}{6}n(n+1)(2n+1).$$

This innocent piece of work (which developed quite naturally at the blackboard with the help of 4Ma, and in particular Oliver Twinch, who noticed that I'd forgotten to take off 0.5^3 in the calculations, and Mark Powys and Roddy Black, who noticed that the error "seemed to be proportional to n") in retrospect struck me as being philosophically very unsound, and undermining to my students concepts of calculus. Teachers of calculus would have these worries every day if they weren't hardened.

https://doi.org/10.2307/3618566 Published online by Cambridge University Press

It has a number of superficial advantages, whose value in an immediate sense to my students is quite considerable. It uses the idea of approximating a discrete distribution by a continuous one, and indicates the value of using a continuity correction. In three months time I can revisit this work when we start approximating the binomial distribution with the normal distribution. It is quite neat on the algebraic side, and 4Ma enjoyed it, especially when they saw what was going on before I did. It is my view that this piece of work is far more worthwhile than, for example, teaching students to jump through the following hoop:

Prove by induction that $1^2 + 2^2 + 3^2 + \ldots + n^2 = 1/6\, n(n+1)(2n+1)$.

This boring, futile, incomprehensible and damaging exercise might give some students the impression that they are learning a rigorous subject, but it strikes me now as being quite horribly pointless. If this sort of thing is what rigorous mathematics has to be at school then I want no part of it.

So what is wrong with the approach above? It is historically absurd to deduce the sum result from the integral, because the integral result was deduced from the sum result, and was well known in Europe in 1650, and much earlier in Kerala. Logically it reinforces the false image of integration as existing independently of summation, that is as being the more fundamental concept. School calculus, with its heavy stress on formal methods, and misuse of the term "integration" for the procedure of finding an anti-derivative, leaves students almost completely in the dark about the reality, which is that, although the fundamental theorem of calculus may be a fine tool for the practical evaluation of areas, it does not provide a proper concept of area.

Teachers in the UK are used to living in a logical and intellectual vacuum. With the door shut and the students in their seats we perform in our own little theatre. Every teacher knows that it hardly matters what you say if the way you say it isn't right [one of my colleagues takes an extreme view annunciated as Billington's Law: At the beginning of each year you walk into a new class and you've got 5 seconds. If you don't get it right in those 5 seconds, 10% of the class will ignore you for the rest of the year!]. When philosophical issues like those discussed above surface it's perhaps best to push them back under again and get on with the job.

The winner of the March 1994 Barcode competition was Susanna Hogan, who decoded the cover as 3 141592 65358 2.

https://doi.org/10.2307/3618566 Published online by Cambridge University Press