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Wild knots and arcs in 3-space

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The first four chapters of the thesis are concerned with the development of invariants of local embedding type for certain restricted classes of wild arcs. More particularly, let M be a 3-manifold and k an arc in M with an isolated wild point $w \in \mathrm{Int}M$ (that is, w has a neighbourhood in M which contains no other wild points of k). Let P_r denote the penetration index of k at w, relative to neighbourhoods of w which are "balls with r solid handles". Then if $P_1 \le 2$ and $P_0 > 3P_1$, we show in Chapter I that the cofinality class of the knot types of the solid tori occurring in a "k-sequence" is an invariant of the non-oriented local arc type of k at w.

If M is orientable and k is oriented, $P_1 \le 2$ and $P_0 > 3P_1$ as before, we can associate another infinite sequence with k if k satisfies certain rather mild geometric conditions, and we show that the cofinality class of this sequence is an invariant of the oriented local arc type of k at w.

Examples are given in Chapter IV to show how these invariants may be used to distinguish arcs in R^3 or S^3, whose only wild point is an endpoint (such arcs have been called "nearly polyhedral"). In particular, we exhibit for each odd integer $n \ge 5$ an uncountable family A_n of non-invertible nearly polyhedral arcs with penetration indices $P_0 = n$ and $P_1 = 1$ at the point w.

Chapter V relates the geometric invariant of enclosure genus of a wild knot with the algebraic invariant of its nullity. If M is the 3-sphere

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and k is a knot with one wild point, then the nullity of the Alexander module of $\pi_1(S^3-k)$ is bounded above by the enclosure genus of k. This result is the best possible, in that for each integer n knots k_n and k'_n are exhibited, both with $P_0 = 2n$ at their respective wild points, such that k_n has enclosure genus and nullity both equal to n, while k'_n has enclosure genus n and nullity 1.

The technique of "cutting and pasting", used to obtain the results of the first four chapters, is a modification of the technique used by N.F. Smythe ("Isotopy invariants of links", Ph.D. thesis, Princeton University, 1965). The results of Chapter V rest on the fact that the Jacobian module functor preserves colimits. Thus, if $\{G_i\}$ is a family of groups with colimit G, and such that for each i there exists an epimorphism α_i from G_i to a fixed group H, then there exists an epimorphism $\alpha : G \to H$ such that the Jacobian JH-module $M[G, \alpha]$ is the colimit of the JH-modules $M[G_i, \alpha_i]$.

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