SUPER–ALFVÉNIC BEAM–PLASMA INSTABILITIES IN SOLAR FLARES

Frank Verheest
Instituut voor theoretische mechanika, Rijksuniversiteit Gent
Krijgslaan 281
B–9000 Gent, Belgium

ABSTRACT. *Special type III radio bursts with significantly lower drifts have been attributed to proton beams at the flare site. These low energy beams (some MeV) are nevertheless super-Alfvénic. Recent reports deal with the stability of such beams against* resonant *scattering by waves below the proton gyrofrequency. Here a different mechanism is proposed, namely* nonresonant *Alfvén instabilities, which may have higher growth rates and can thus dominate. A selfconsistent multispecies plasma description of such instabilities, developed for solar wind and cometary plasmas, is applied to slowly drifting type III radio bursts. The instability growth rate is fairly insensitive to the beam velocity but decreases with the ratio of beam to ambient plasma densities. Even low beam densities can trigger these nonresonant and hence generic instabilities. Moreover, growth rates are sizeable fractions of the wave frequency, whereas resonant instabilities would give much lower values at low beam densities.*

1. Introduction

Conventional MHD describes magnetized plasmas as one single fluid. In many astrophysical applications, however, it is essential to distinguish between the different species making up a plasma, especially if there are different flow velocities in equilibrium. One of these cases is where a plasma or a beam flows through another (ambient) plasma, as in the case of type III radio bursts (caused by electron beams propagation in the solar corona) or of the recently reported class of special type III radio bursts with a significantly lower drift than normal and which are attributed to proton beams at the flare site (Simnett 1986, Benz & Simnett 1986). These are beams of fairly low energy protons (with energies of a couple of MeV) which are nevertheless super-Alfvénic. Similar situations are encountered in the solar wind flow around comets, and an extensive literature has attested the occurrence there of different types of low-frequency instabilities, from an observational, analytical and numerical point of view (see *e.g.* Sagdeev *et al.* 1987, Lee 1988). Recently Tamres *et al.* (1989) have reported on the stability of such super-Alfvénic proton beams in solar flare loops against *resonant* scattering by waves in the flare loop coronal plasma at frequencies below the proton gyrofrequency. We would propose here a different mechanism, namely that such beams may also be unstable to *nonresonant* Alfvén instabilities, as we know from cometary plasma physics that under certain conditions these may have the highest growth rate (Goldstein & Wong 1987) and thus are more likely to dominate. We will draw on the selfconsistent description of such instabilities developed for solar wind and cometary plasmas (Verheest 1977,1987, Lakhina & Verheest 1988, Verheest 1989) to illustrate their possible relevance for the said slowly drifting type III radio bursts. In the next section the

E. R. Priest and V. Krishan (eds.), Basic Plasma Processes on the Sun, 383–389.

theoretical framework is recalled, and then in section 3 applied to the special type III radio bursts under consideration.

2. Theoretical framework for nonresonant Alfvén beam modes

We start the selfconsistent multispecies plasma description from a set of transport equations for a number of cold plasma species with number densities n_s and fluid velocities $\mathbf{u}_s$ (Verheest 1977):

$$\begin{aligned} \frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{u}_s) &= 0, \\ \frac{\partial}{\partial t}\mathbf{u}_s + \mathbf{u}_s \cdot \nabla \mathbf{u}_s &= \frac{q_s}{m_s}(\mathbf{E} + \mathbf{u}_s \times \mathbf{B}). \end{aligned} \tag{1}$$

Pressure effects have been left out of the description, as these are not of importance for the kind of low-frequency waves we will consider. $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields. Their changes and the collisionless coupling between the different plasma species obey Maxwell's equations, of which we only need the following:

$$\begin{aligned} \nabla \times \mathbf{E} + \frac{\partial}{\partial t}\mathbf{B} &= \mathbf{0}, \\ c^2 \nabla \times \mathbf{B} - \frac{\partial}{\partial t}\mathbf{E} &= \frac{1}{\varepsilon_0} \sum_s n_s q_s \mathbf{u}_s. \end{aligned} \tag{2}$$

The summation over s in the expression for the current is over all plasma constituents.

For the sake of simplicity, a homogeneous equilibrium is now assumed, with equilibrium densities N_s and bulk species velocities $\mathbf{u}_{s0} = U_s \mathbf{B}_0 / B_0$ (possibly zero for some species, depending upon the reference frame chosen). This implies that a local picture of the wave phenomena is taken. In a selfconsistent description bulk motion perpendicular to $\mathbf{B}_0$ is the same for all plasma species, namely $\mathbf{E}_0 \times \mathbf{B}_0 / B_0^2$, and can be eliminated if need be by going to a Hoffman–Teller frame, where the motional electric field $\mathbf{E}_0$ vanishes. Also, $\mathbf{E}_0 \times \mathbf{B}_0 / B_0^2$ does not influence the stability criteria to be derived furtheron and having the beams parallel to the flare magnetic field is quite consistent with the flux tube configuration of the solar flare plasma. In such a truly selfconsistent approach there is no net charge nor current in equilibrium:

$$\sum_s N_s q_s = 0 \quad , \quad \sum_s N_s q_s U_s = 0. \tag{3}$$

Due to their high mobility, the electrons are assumed to adjust themselves so as to maintain the required charge and current neutrality. In other words, they provide the necessary return current for any ion beam present in the system, much more easily than in the reverse situation where electron beams are thought of as being of primary importance.

We shall need several global quantities for the combined plasma as a whole. First of all, there is the global bulk velocity:

$$\overline{U} = \sum_s N_s m_s U_s / \sum_s N_s m_s. \tag{4}$$

As in a first approximation there are two distinct streaming velocities to be considered, say U_A for the speed of the ambient subplasma and U_B for the beam velocity of the drifting subplasma constituents, we can rewrite $\overline{U}$ as

$$\overline{U} = \frac{(\sum_A N_s m_s) U_A + (\sum_B N_s m_s) U_B}{(\sum_A N_s m_s) + (\sum_B N_s m_s)} = \frac{U_A + \sigma U_B}{1+\sigma}, \tag{5}$$

if σ stands for the ratio of the mass densities of drifting to ambient subplasmas. If both subplasmas are taken as hydrogen plasmas, then σ is but the ratio of number densities N_B/N_A.

Similarly, we define V_A as the Alfvén velocity of the whole plasma through

$$V_A^2 = \frac{B_0^2}{\mu_0 \sum_s N_s m_s} = \frac{V_{AA}^2}{1+\sigma}, \tag{6}$$

with V_{AA} the Alfvén velocity in the ambient subplasma. Also, the normalized kinetic energy in the relative parallel motions can be expressed as

$$W^2 = \frac{\sum_s N_s m_s (U_s - \overline{U})^2}{\sum_s N_s m_s} = \frac{\sigma (U_A - U_B)^2}{(1+\sigma)^2}. \tag{7}$$

Due to the requirements (3) of charge and current neutrality in equilibrium, the term in W^2 will only be nonzero in general if there are at least three species present (such as electrons and two kinds of ions), and if these species move at differing speeds, however moderate the differences. If all plasma constituents have the same drift velocities, then one can simply use results from stationary plasmas, taken over to a drifting frame. However, with ions or subplasmas moving at different speeds, there is no longer a natural frame of reference.

The low-frequency, long-wavelength waves which we will now consider are such that

$$\omega \ll |\Omega_s| \quad , \quad |k_\parallel U_s| \ll |\Omega_s|. \tag{8}$$

Here $\Omega_s = q_s B_0/m_s$ are the gyrofrequencies, defined inclusive of the charge of the particles under consideration. The conditions (8) mean essentially that for each species the appropriate Doppler-shifted frequency $\tilde{\omega}_s = \omega - kU_s$ is small in magnitude compared to the gyrofrequency Ω_s. Then the dispersion laws for parallel and oblique propagation of such Alfvén waves are (Verheest 1977)

$$(\omega - k_\parallel \overline{U})^2 = k^2 V_A^2 - k_\parallel^2 W^2 \tag{9}$$

for the compressional or fast mode and

$$(\omega - k_\parallel \overline{U})^2 \simeq k_\parallel^2 (V_A^2 - W^2) \tag{10}$$

for the shear or slow mode. The last dispersion law is not valid close to perpendicular propagation, although the range to be excluded is very small, typically a couple of degrees around 90° (Verheest 1977). It is worth pointing out that the same dispersion laws can be obtained by starting from a full kinetic treatment instead of using the simpler cold fluid approach.

The Alfvén waves found sofar can go unstable provided

$$V_A^2 < W^2. \tag{11}$$

If this is the case, both Alfvén waves are unstable, the shear mode described by (10) at almost all directions of propagation, the fast mode described by (9) only for those directions of wave propagation obeying

https://doi.org/10.1017/S007418090008829X Published online by Cambridge University Press

$$\tan^2\vartheta = \frac{k_\perp^2}{k_\parallel^2} < \frac{W^2}{V_A^2} - 1, \tag{12}$$

where ϑ stands for the angle between the wavevector $\mathbf{k}$ and the external magnetic field $\mathbf{B}_0$. Looking at the criterion (11) or the dispersion laws (9–10), one sees that the W^2 term is destabilizing, while one can interpret V_A^2 as a confining effect of the equilibrium magnetic field, perpendicular to the field. Hence mass motions perpendicular to the magnetic field stabilize the waves, whereas similar effects parallel to the magnetic field tend to destabilize the Alfvén waves. The balance determines the final outcome of stability or instability, possibly enhanced by anisotropic pressure effects, had we included these, as in the familiar firehose instability.

We can rewrite the instability criterion (11) as

$$\frac{V_{AA}^2}{1+\sigma} < \frac{\sigma U_B^2}{(1+\sigma)^2}, \tag{13}$$

if we place ourselves for simplicity in a frame in which the ambient subplasma is at rest, with $U_A = 0$. With the help of the Alfvénic Mach number $M = U_B/V_{AA}$ we cast (13) as

$$\frac{1}{M^2-1} \leq \sigma. \tag{14}$$

There is thus a threshold $\sigma_{\rm thr} = 1/(M^2-1)$ for the mass density ratio of both subplasmas, depending upon M. As in any case one needs $M > 1$, the relative streaming between both subplasmas has to be super-Alfvénic. In more complicated situations, one would have to define super-Alfvénic by (11).

We find so-called *purely growing modes*, but then seen as such in a frame drifting with the mean bulk of the combined plasma, first of all for the shear Alfvén mode with dispersion law (10):

$$\begin{aligned} \operatorname{Re}\omega &= k_\parallel \overline{U} = k_\parallel \frac{\sigma U_B}{1+\sigma}, \\ \operatorname{Im}\omega &= k_\parallel\sqrt{W^2 - V_A^2} = \frac{k_\parallel V_{AA}}{1+\sigma}\sqrt{\sigma M^2 - \sigma - 1} = \frac{\sqrt{\sigma M^2 - \sigma - 1}}{\sigma M}\operatorname{Re}\omega. \end{aligned} \tag{15}$$

This instability, and thus the growth rate, is nondispersive (Verheest 1987, Lakhina and Verheest 1988), in contrast to less selfconsistent treatments of the subject (Sagdeev *et al.* 1986 and Lakhina 1987). Furthermore, the parallel phase velocity of these modes is

$$\frac{\operatorname{Re}\omega}{k_\parallel} = \overline{U} = \frac{\sigma M}{1+\sigma}V_{AA}, \tag{16}$$

and hence can be very different from the usual V_A, depending in which reference frame one wants to specify the different drift velocities and the global bulk speed $\overline{U}$. Here a frame was chosen in which one of the subplasmas is at rest, as said already. At low beam densities the phase velocity could be quite smaller than V_{AA}. This has to be contrasted to the case of cometary plasma physics, where in a cometocentric frame the beam is represented by the solar wind, a high speed and high density plasma (compared with the plasma created from cometary material by different ionization processes), and where consequently the phase velocity can be about 5 times as large as V_A in the undisturbed solar wind.

If we now come to the fast Alfvén mode, with dispersion law (9), we find similarly that in the unstable regime its growth rate obeys

$$\mathrm{Im}\,\omega = \sqrt{k_{\parallel}^2 W^2 - k^2 V_A^2} = \frac{\sqrt{\sigma M^2 \cos^2 \vartheta - \sigma - 1}}{\sigma M \cos \vartheta} \mathrm{Re}\,\omega, \tag{17}$$

while $\mathrm{Re}\,\omega$ is given by the expression in (15).

3. Application to solar flare plasmas

For simplicity we will take the beams *and* the waves to propagate parallel to the solar flare magnetic field, so that we need only consider one type of Alfvén mode with its associated conditions. With proton beam energies of the order of a couple of MeV and $m_p V_A^2/2$ typically in the range of 10 keV (with thus $V_A \simeq 1{,}400$ km/s), we find that $M \simeq 10$, and hence the threshold number density ratio is about 1 %. A proton beam of 1 MeV gives beam velocities of the order 14,000 km/s, which only requires 500 eV electrons for the necessary return current. Any beam density above this low threshold (compared to the ambient plasma density) will excite the nonresonant Alfvén instabilities. We give in the following Table some values for the growth rates in three different cases, that of a low-density beam ($\sigma \simeq 0.1$), the intermediate case with $\sigma \simeq 1$ and that of a high density beam ($\sigma \simeq 10$).

TABLE. Values for σ, V_A/V_{AA}, $\mathrm{Re}\,\omega/k_{\parallel}V_{AA}$ and $\mathrm{Im}\,\omega/\mathrm{Re}\,\omega$ for two different choices of M

σ	$\frac{V_A}{V_{AA}} = \frac{1}{\sqrt{1+\sigma}}$	$\frac{\mathrm{Re}\,\omega}{k_{\parallel}V_{AA}} = \frac{\sigma M}{1+\sigma}$	$\frac{\mathrm{Im}\,\omega}{\mathrm{Re}\,\omega} = \frac{\sqrt{\sigma M^2 - \sigma - 1}}{\sigma M}$
$M = 5$	$\sigma_{\mathrm{thr}} = 0.04$		
0.1	0.95	0.45	2.37
1.0	0.71	2.50	0.96
10.0	0.30	4.55	0.31
$M = 10$	$\sigma_{\mathrm{thr}} = 0.01$		
0.1	0.95	0.91	2.98
1.0	0.71	5.00	0.99
10.0	0.30	9.09	0.31

As one can see from this table, the relative phase velocity at a given Alfvénic Mach number M increases with the density ratio σ, whereas the relative growth rate decreases. The latter is fairly insensitive to changes in M and is determined almost exclusively by σ. So, if the beam densities are above the low threshold to trigger an Alfvén instability, then this instability has a fairly generic character. Moreover, its growth rate is a sizeable fraction of the real wave frequency, around 1 at equal ambient and beam subplasma densities. Resonant instabilities (not discussed here, but see Tamres *et al.* 1989) would seem to have much lower growth rates, certainly for low-density beam subplasmas.

The picture emerging is that the source of free energy, namely the relative streaming between the ambient and beam subplasmas parallel to the solar flare magnetic field, can certainly drive several low-frequency waves unstable. That is, of course, as long as nothing changes in the given parameters. However, as the unstable waves grow to large amplitudes, inducing large magnetic fluctuations, they take away some or most of the free energy. Consequently the beam distribution has a tendency to relax to a stable state via the excitation of Alfvén wave turbulence followed by the scattering of the proton beam by the excited waves.

To describe this process theoretically, a form of quasilinear theory must be used. It seems to us, however, that a proper and truly selfconsistent quasilinear theory has not yet been given for a multispecies plasma. In particular the heating of the ambient subplasma or the deceleration of the beam subplasma has not been considered in any detail, including the required feedback. This question is also of importance in similar contexts, wherever one plasma flows through another, as *e.g.* the solar wind in the neighbourhood of a cometary coma. So at the moment we have to be content with hand-waving arguments to arrive at a situation where the differences in bulk speeds between the ambient and the beam subplasmas are of the order of the Alfvén velocity or less. This effectively quenches the Alfvén instabilities discussed in the present paper. In this respect, the treatment by Isenberg and Hollweg (1982) would perhaps offer the best starting point to compute in a theoretical way the final density and velocity parameters for both ambient and beam subplasmas, when the Alfvén waves have reached finite amplitudes, but also there no feedback is included as yet. As amply demonstrated, the effects discussed in the present paper are not amenable to a single fluid description, and hence a fully multispecies approach in all its details is necessary, however more cumbersome.

Acknowledgements

It is a pleasure to thank the National Fund for Scientific Research (Belgium) for a research grant, which made the stay in India possible.

References

Benz A O & Simnett G M (1986) "Solar radio signatures suggestive of proton beams", *Nature* **320**, 508–509

Isenberg P A & Hollweg J V (1982) "Finite amplitude Alfvén waves in a multi-ion plasma: Propagation, acceleration, and heating", *J. Geophys. Res.* **87**, 5023–5029

Lakhina G S (1987) "Low-frequency plasma turbulence during solar wind–comet interaction", *Astrophys. Space Sci.* **133**, 203–218

Lakhina G S & Verheest F (1988) "Alfvén wave instabilities and ring current during solar wind–comet interaction", *Astrophys. Space Sci.* **143**, 329–338

Lee M A (1988) "Ultra-low frequency waves at comets", *Proc. Chapman Conf. Plasma Waves and Instabilities in Magnetospheres and at Comets* (in press)

Sagdeev R Z, Shapiro V D, Shevchenko V I & Szegő K (1987) "Plasma phenomena around comets: Interaction with the solar wind", *Invited Papers 18th Int. Conf. Phenomena Ionized Gases* (Swansea) **2**, 134–143

Simnett G M (1986) "A dominant role for protons at the onset of solar flares", *Solar Phys.* **106**, 165–183

Tamres D H, Melrose D B & Canfield R C (1989) "On the stability of proton beams against resonant scattering by Alfvén waves in solar flare loops", *Astrophys. J.* **34**, 1284

Verheest F (1977) "Alfvén instabilities in streaming plasmas with anisotropic pressures and their relevance for the solar wind", *Astrophys. Space Sci.* **46**, 165–173

Verheest F (1987) "Alfvén wave plasma turbulence during solar wind–comet interaction", *Astrophys. Space Sci.* **138**, 209–215

Verheest F (1989) "Linear and nonlinear Alfvén waves in cometary plasmas", *Invited Papers 19th Int. Conf. Phenomena Ionized Gases* (Beograd, in press)

DISCUSSION

KUIJPERS: Can you estimate how deep a realistic flare proton beam penetrates?

VERHEEST: From first estimates the e-folding length would seem to be a small fraction of the loop length, indicating that the beam would be destroyed before it could travel down a sizeable fraction of the loop.