An easy geometrical representation of the Sextic Covariant of a Binary Quartic.

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The following is an easy method of representing geometrically the Sextic Covariant of a Binary Quartic which I have not seen given elsewhere. It was suggested to me by my work on Bi-Circular Quartics. The usual method of representing this Covariant is to regard the Quartic as four points on a conic, when the Sextic is represented by the intersections of the conic and the sides of the Harmonic Triangle of the Quadrangle whose vertices are the points of the given Quartic. The present method has the advantage of regarding the given Binary Quartic as a Tetrad of points on a straight line.

Let the given Quartic be geometrically represented on a straight line by the four points A, B, C, D. Then by a known property the Sextic Covariant is represented by the Double Points of the three Involutions defined by these points taken in pairs, viz. :—

A, B : C, D

A, C : D, B

A, D : B, C.

It is also to be noted that if PQRS be the vertices of a Quadrangle in which PQ and RS meet in A, and PR and QS meet in B, then A and B are Conjugate Points with respect to any conic through P, Q, R, S.

For by Desargues' Theorem the line AB is cut in involution by all conics through PQRS, and the Double-Points of this involution are plainly A and B, since AB cuts the two conics of the system AP, AS and BP, BS in coincident points at A and B.

Let now X and Y be two points not on the given line ABCD.

Plainly $X[ABCD] = Y[ABCD]$.

Hence by the ordinary properties of cross-ratios,

$$X[ABCD] = Y[BADC] \quad (1)$$
$$X[ABCD] = Y[CDAB] \quad (2)$$
$$X[ABCD] = Y[DCBA] \quad (3)$$

Let the conics defined by the correspondences (1), (2), (3) be denoted by S_1, S_2, S_3 respectively.

Then plainly by what has been said above

A, B and C, D are each pairs of Conjugate Points with respect to S_1.

A, C ,, B, D ,, ,, ,, ,, ,, ,, ,, ,, ,, S_2.

A D ,, B, C ,, ,, ,, ,, ,, ,, ,, ,, ,, S_3.

Hence the points of intersection of S_1, S_2, S_3 with the given line ABCD form the Sextic Covariant of A, B, C, D.

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