P 43. (Corrected.) Let G be a group generated by P and Q, and let H be the cyclic subgroup generated by P. If P and Q satisfy the relations $P^2 QP = Q^2$ and $Q^2 PQ^{-4} = P^k$ for some k, then the index of H in G is 1 or 7.

N. S. Mendelsohn

P 44. Show that

$$\pi^2 = 10 - \sum_{n=1}^{\infty} \frac{1}{n^3(n+1)^3}.$$

E. L. Whitney

P 45. Show that

$$\sum_{i=0}^{n} \binom{n+1}{i} \int_0^1 \binom{t}{i+2} dt = 0$$

for $n = 1, 3, 5, \ldots$, where $\binom{t}{k} = t(t-1)(t-2) \ldots (t-k+1)/k!$.

B. Wolk

P 46. Given infinitely many points in the plane such that

(a) the distance between any two of them is greater than 1,

(b) for infinitely many n, there are more than cn^2 points in the circle $|z| < n$.

Show that for any $\varepsilon > 0$ there is a line which comes closer than ε to infinitely many of the points.

P. Erdös

SOLUTIONS

P 10. (a) Prove that every set of six points in the plane can be colored in three colors in such a way that no two points unit distance apart have the same color.

(b) Show that in (a) six cannot be replaced by seven.

L. Moser and W. Moser

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