Correspondence

DEAR EDITOR,

Following the publication of my article [1] in the November 1995 issue of the *Gazette,* I received from Professor C. J. Bouwkamp of the Technological University in Eindhoven a copy of his 1965 paper [2] on the same problem, that of evaluating the slowly converging infinite product $\prod \cos\frac{\pi}{n}$. This provides yet another reference to this intriguing and difficult problem, to add to those given in [1]. Prof. Bouwkamp gives two methods. The first one transforms the product into a sum by the same method that I used, but without use of Bernoulli numbers. It is particularly interesting that he uses the same technique that I did – that of isolating a single term and evaluating it exactly to improve the convergence of his series. Nevertheless he requires 14 terms to give 16 decimal places of the product, whereas I require only 10 terms. (The second method in [2] is quite different and less efficient: it amounts to approximating $\cos(\pi/n)$ for large n, by a simple function $g(n)$ such that the product $R = \prod g(n)$ can be evaluated explicitly and the product $Q = \prod [g(n)/\cos(\pi/n)]$ converges fast: we can then evaluate our product as R/Q.)

Besides drawing the attention of your readers to the new reference, my purpose in writing is to say that the refining method in my article [1] can be carried a step further. With this extra refinement, we get no fewer than 24 correct decimal places by using 10 terms (the correct value comes from [3]): 0.114942044853296200701040. In fact with a calculator only 3 terms are now needed, with the additional refinement, to obtain 9 places of decimals, instead of the 5 terms used in [1].

The new refinement amounts to isolating two more terms from the Euler sum used to evaluate the infinite product. Isolating a single term produced a summand of $\ln\frac{1}{6}$ (equation 10 in [1]) and a residual sum which converges very rapidly. Isolating two more terms produces

$$\ln \prod_{n=3}^{\infty} \cos\frac{\pi}{n} = \ln\left(\frac{1}{6}\cdot\frac{\sin\frac{2\pi}{3}}{\frac{2\pi}{3}}\cdot\frac{9}{5}\cdot\frac{9}{8}\cdot\frac{\sin\frac{2\pi}{5}}{\frac{2\pi}{5}}\cdot\frac{25}{21}\cdot\frac{25}{24}\right)$$
$$-\sum_{r=1}^{\infty}\frac{2^{2r}}{r}\left((-1)^{r-1}\frac{B_{2r}2^{2r-1}\pi^{2r}}{(2r)!} - 1 - \frac{1}{2^{2r}}\right)$$
$$\times\left[(-1)^{r-1}\frac{B_{2r}2^{2r-1}\pi^{2r}}{(2r)!}\left(1-\frac{1}{2^{2r}}\right) - \left(1+\frac{1}{3^{2r}}+\frac{1}{5^{2r}}\right)\right].$$

The same method applied to the formula of [2] (equation 6 on p. 42) gives, for the *reciprocal P* of our product

$$\ln(P) = -\ln\left(\frac{1}{6}\cdot\frac{\sin\frac{2\pi}{3}}{\frac{2\pi}{3}}\cdot\frac{9}{5}\cdot\frac{9}{8}\cdot\frac{\sin\frac{2\pi}{5}}{\frac{2\pi}{5}}\cdot\frac{25}{21}\cdot\frac{25}{24}\right)$$
$$+\sum_{k=1}^{\infty}\frac{2^{2k}}{k}\left[\zeta(2k) - 1 - \frac{1}{2^{2k}}\right]\left[\lambda(2k) - 1 - \frac{1}{3^{2k}} - \frac{1}{5^{2k}}\right],$$

where ζ is the usual Riemann zeta function and

$$\lambda(2k) = \left(1 - \frac{1}{2^{2k}}\right)\zeta(2k).$$

This yields a more accurate result than the formula actually given in [2].

It is not clear that isolating further terms from Euler's sum is profitable for finding accurate estimates of the infinite product with few additions of terms. It would be interesting to know whether there is a general theory to call on here.

I am grateful to Dr A. M. Cohen of Cardiff for his help; also to Dr Peter Giblin for guidance in the presentation of this material.

Yours sincerely,

E. STEPHENS

12 Carlton Avenue, Ramsgate, Kent CT11 9BP

References

1. E. Stephens, Slowly convergent infinite products, *Math. Gaz.* **79** (1995) pp. 561–565.
2. C. J. Bouwkamp, An infinite product, *Indagationes Mathematicae* **27** (1) (1965) pp. 40–46.
3. H. P. Robinson, *Popular Computing* **91** (1980) p. 19.

DEAR EDITOR,

I think I detect an error in the article 'What is centrifugal force?' by Janet Jagger and Kevin Lord (*Math. Gaz.* **79** (November 1995) pp. 484–488).

Einstein's law of equivalence implies it is impossible to show, using any physical test within the neighbourhood of a body, whether a force acting on that body is due to a gravitational field or is generated by assuming a non-inertial frame. As a consequence, if centrifugal force is fictitious, then so is the force of gravity. If we extend the dialogue given (*q.v.*) to include the ghost of Einstein, we might get:

> *Newton's perception from the surface of the Earth:* I feel my feet pressed against the ground. A force must be pushing me downwards. This is the gravitational attraction between the mass of my body and that of the Earth, given by my formula Gm_1m_2/r^2.
>
> *Einstein's perception (from some imaginary viewpoint outside the universe):* Newton is deflected from his course along a geodesic in space-time, which would take him towards the centre of the Earth; so he is accelerated away from the centre of the Earth, and by his second law (an acceptable approximation at these low speeds) there is a force in the direction of this acceleration. In reality, the ground is pushing on Newton. Newton's Third Law states that he must exert an equal and opposite force on it and his brain tells him that he is actively

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> pushing (rather than passively reacting). This is the imagined gravitational force. Poor Aristotle is deflected in two orthogonal directions. He feels the resultant of two forces, that of gravity, plus a centrifugal force. Though they would both disappear in an inertial frame of reference, both are real in Aristotle's frame of reference, relative to the car's interior. If the car's windows were blacked out, Aristotle would not be able to distinguish these forces, and could not know whether the car he was sitting in was resting on a slope (on a planet slightly larger than the Earth), travelling in a circle on the Earth's surface (as in fact it is), or even (though he may find the prospect alarming!), being swung around an imaginary pivot in interstellar space, on the end of a line collinear with his apparent weight.

Alternatively, if it is acceptable to say (as do Jagger and Lord) that the weight is a real force, then it must be consistent to admit the existence of a centrifugal force, each of these forces being the result of using a non-inertial frame, as we are entitled to do.

One may not expect sixth-formers to understand Riemannian geometry, but I can see no reason why they should not all benefit from a discussion of the above effects. Otherwise, some who may have already heard of Einstein's principle of equivalence may be puzzled by the apparent contradiction with what the teachers are saying, and some may go on to study General Relativity armed with a misconception.

Yours sincerely,

DAVID A. JOY

65 Great Central Road, Loughborough, Leicestershire LE11 1RW

DEAR EDITOR,

Thank you for sending me the Centenary number of the *Gazette*. I have received a second copy, which I have given to the Girton College Library. I hope this is in order.

The Biographical Officer of St John's College has given me some information about T. A. A. Broadbent. He was born in Consett, County Durham, and attended the local school. His father was a grocer at 7 Front Street, Consett. St John's list him as Senior Wrangler for 1924, but 'Senior' should be deleted, because in 1924 the list of names was in alphabetical order. The last *Senior* Wrangler was P. J. Daniell in 1909. E. H. Neville was second and Mordell third. I would add that Broadbent married W. V. D. Hodge's sister.

Owing to my delay in sending my biographical details, the piece about me on p. 40 of the centenary issue may have been composed in haste. May I make a correction? You have that *Methods of mathematical physics* 'stayed in print for 46 years'. The paperback is still in print after all but 50 years. Recently Cambridge University Press sent a handsomely leather-bound copy to the University of Saskatchewan which, at the instigation of the geophysicist Dr Mary Fowler (granddaughter of R. H. Fowler, and a former pupil of mine), gave me an honorary DSc last October *in absentia* because at 92 I did not feel like undertaking the journey. Mary and her husband Professor Euan Nisbet spent quite a time at Saskatoon but they are now at Royal Holloway and Bedford.

With best wishes to you and for the future of the *Gazette*,

Yours sincerely,

BERTHA JEFFREYS

160 Huntingdon Road, Cambridge CB3 0LB

DEAR EDITOR,

The following question was set in the London Board Statistics Module T 1 for January 1996:

> 'There are 8 competitors in the final of a 100m race. The first 3 competitors to complete the course will all receive medals.
>
> (a) Calculate the number of different possible groups of medal winners.
>
> Two of the competitors represent the Arrows athletics club, the other six represent different clubs.
>
> (b) Find the probability that the Arrows win at least one medal.'

Part (a) is easily answered as ${}^8C_3 = 56$, but part (b) is more problematic as we are not told whether the possible finishing combinations are all equally likely.

The name of the running club of the two competitors in question could well indicate they are certainties for medal placings!

It is interesting to compare such questions with those set in mechanics where the wording is always carefully chosen to make sure there are no ambiguities – hence our 'frictionless' pulleys and 'elastic' strings – but there do not seem to be any corresponding conventions yet in statistics examinations. Now the omission of the word 'fair' when referred to a coin or a die (although it should of course be included) would not cause too many problems, simply because most coins and dice *are* fair, but in a race it is most improbable that all competitors would be equally likely to win! Horse-racing bookmakers certainly do not operate under that assumption!

Both the Chief Examiner and University Reviser have said that there is nothing wrong with the question as written. In fact it was not a good paper generally, since the Chief Examiner did admit that one of the other questions he set was outside the syllabus and it was ignored in the marking.

Yours sincerely,

ALAN J. W. THORN

19 Speedwell Close, Merrow Park, Guildford GU4 7HE

DEAR EDITOR,

My thanks to Tom Roper for his enthusiastic discourse on 'The mathematics of bowls' [1]. I did receive the definitive account of Maurice Brearley and Beverley Bolt [2] in Australia in which it is established that for level, constant retardation greens the angle of delivery of the bowl is the same for any length of jack. Other mathematical bowlers have also offered evidence and then practical advice. Has my bowling improved? Well – yes – I have confidence in theory being put into practice.

Yours sincerely,

JOHN BRANFIELD

87, Bramcote Drive, Beeston, Nottingham NG9 1DU.

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1. T. Roper, The mathematics of bowls, *Math. Gaz.* **80** (July 1996) pp. 298-307.
2. M. N. Brearley and B. A. Bolt, The dynamics of a bowl, *Quarterley Journal of Mechanics and Applied Mathematics* **11** (1958) pp. 351-363.

Paul Erdös (26 March 1913 - 20 September 1996)

Already many short obituaries for Paul Erdös have been published in newspapers, and soon there will be biographies in the journals of learned societies. Indeed, in the not too distant future, there will be whole books on the man and his mathematics. For now, let us offer *Gazette* readers a first few words of appreciation.

As a human activity, mathematics requires its share of legendary players, and it is a privilege to have been around when Erdös was a living legend. No matter how we loved hearing or reading about his idiosyncratic wisdom and eccentricities, Erdös will be remembered first and foremost as a setter and solver of problems in mathematics. From his wide ranging knowledge, he conjured up problems which were usually interesting and frequently significant. Besides having a huge arsenal of technical 'weapons' with which to attack such problems himself, he also knew to whom such problems should be given. If his friends could not solve them soon enough for his taste, he would offer money to encourage their continued efforts. He loved beauty in mathematics, and he used to say that God has a book which contains the best proof of every theorem, so that his highest praise for an elegant solution was that it was 'from the book'.

We should also add that, however unworldly in his personal conduct, he was not unaware of current affairs and worldly troubles. Indeed, the many of us who had the good fortune to have met him will long remember his care, concern and kindnesses towards others. Farewell our good friend. Now you can read the book in peace.

PETER SHIU

Department of Mathematical Sciences, Loughborough University, Leicestershire LE11 3TU

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