JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Memoir of the late Benjamin Gompertz, F.R.S., F.R.A.S., &c. By MARCUS N. ADLER, M.A., *Fellow of the Institute of Actuaries and of the Statistical Society.*

[Read before the Institute, 18th December, 1865.]

IN the demise of Benjamin Gompertz the scientific world has recently had to deplore the loss of a man, to whose labours the student of both pure and applied mathematics, but especially the actuary, is deeply indebted. The death of this distinguished *savant*, who may be considered as having been, in his later years, a connecting link between the past and present generation, should not be passed over in silence. It is a duty we owe as much to the man as to society, to trace, whilst his memory is still fresh, the chief events of his life, and to notice his principal contributions to science.

Benjamin Gompertz, a member of the Jewish community, was descended from a family which long held a distinguished position in Holland. His grandfather on the mother's side, Benjamin Cohen by name, was on intimate terms with William, Prince of Orange; and it is related that during a revolution in the middle of the last century the Dutch Stadtholder found a ready asylum at the magnificent mansion his friend had erected near a place called Amersfort. Cohen zealously patronized the arts and sciences, and was himself a mathematician of some pretension.

A curious translation of Euclid into Hebrew, and a little treatise on dialling and gnomonic projection, which were composed for Cohen's use, may even now be seen in some of our libraries.

The father of Benjamin Gompertz was a successful diamond merchant, a pursuit in which the Dutch chiefly seem to excel. Even at the present day the art of diamond cutting in Europe seems to be confined to Holland; and a visit to any of the factories there may convince the reader that the management of such an establishment requires abilities of no mean order.

The brothers of Mr. Benjamin Gompertz had little taste for a commercial life, but shared his predilection for literary pursuits. Barnet, the eldest, was an eminent amateur musician.

His brother Isaac wrote poems, which at the time were much admired.*

Another brother, Ephraim, who is still living, is an original mathematician. It is only to be regretted that so few of his investigations have been published.† Three able pamphlets written by him many years ago, on the gold question, are deserving of attention even at the present day, when we find that such discordant opinions are held on the subject by our leading thinkers.‡

Lewis, the youngest brother, was the founder of the Animals' Friend Society, and entertained some strange ideas on the relation of the brute creation to man. He published several works,|| in which he endeavoured to prove that it was not only unlawful to kill an animal, but to turn it to any use that was not directly beneficial to the animal itself. Accordingly he did not object to drink milk, or to wear cloth, because milking the cow, when not robbing the calf thereby, or shearing the sheep, was for the good of those animals; but to eat meat or to drive in a coach was contrary to his doctrine. He always maintained, and this long before railways or steam ploughs were introduced, that machinery might be made perfectly subservient to our wants. He certainly lived to see the verification of a portion of his theory. We are assured that in his will he left handsome annuities for the maintenance of some of his pet animals. It may be added, that he was of a very inventive turn of mind, and the author of several contributions to mechanical science.

* *Vide* the first number of *The Liberal*, edited by Leigh Hunt. Among the chief poems of Isaac Gompertz may be specified "The Modern Antique," "Time, or Light and Shade," "Devon," &c.

† *Vide* T. Baker's *Formulæ, Rules, and Examples.* Weale, 1862.

‡ *Theoretical Discourse on the Nature and Property of Money.* Richardson, 1820. *An Essay on Currency*; 1829. *An Attempt at an Analysis of the Currency and the Merits of the Bank of England*; 1830.

|| *Moral Inquiries on the Situation of Man and Brutes.* London, 1824. Also Fragment in defence of animals.

https://doi.org/10.1017/S2046166600002889 Published online by Cambridge University Press

Benjamin Gompertz was born on the 5th of March, 1779, at Bury Street, in the city of London. He did not enjoy the benefit of a collegiate education, and on the whole he may be considered as self-taught.

So great was his thirst for knowledge when a mere boy, that frequently, when his parents had removed all candles to prevent him from injuring his health by studying too late at night, he stole out into the garden, and pursued his investigations by moonlight. He was wonderfully familiar with the writings of the English and French mathematicians of the last century; but Newton, Maclaurin, and Emerson, were his favourite authors. The admiration he entertained for Newton was so great, that he clung to his Fluxional language and notation long after these had been abandoned by other English writers. It may be interesting to know the grounds upon which he based his great preference for the fluxional to what he calls "the furtive" notation used on the continent, and now much used in England.*

"I call the differential notation furtive, on I think a moral ground, and also on the ground of its introducing an interruption and an inconvenience in practice. The moral ground is, that it appears to give Leibnitz a greater claim to originality, to the prejudice of Newton, than I think he is justly entitled to. And the other ground is, it steals from the alphabet a letter—and one which it is most convenient to retain, in order to keep up the regular order of notation—to use it for a purpose of different intent to that for which it was originally used, and may introduce confusion. And with respect to the superior advantage of the fluxional calculus over the differential calculus, I observe that if x and y be rectangular co-ordinates of a curve in a plane, and z the length of the curve from a given point in it to the point of which x and y are co-ordinates, the fluxional calculus gives $\dot{z}=\sqrt{\dot{x}^2+\dot{y}^2}$, which is strictly true, and may be proved to be so without the introduction of infinitely small quantities: but the differential calculus gives $dz=\sqrt{\overline{dx}|^2+\overline{dy}|^2}$, true only on consideration of infinitely small quantities; and even with that consideration it cannot be proved luminously to be true, because $\sqrt{\overline{dx}|^2+\overline{dy}|^2}$ only expresses the length of the chord of an infinitely small arc, and not of the arc itself, as they have no part common with each other, but at the points of intersection. . . . . . And similar observations may be made with respect to the luminous character of the fluxional analysis when compared with the differential analysis, in the application of it to physical subjects. Thus if f be a force acting on a body, $\dot{v}$ the velocity which is generated by its momentary impulse—that is to say, its single impulse, by which is meant the finite space the body would describe in consequence of it in the finite time $\dot{t}$, but not the variable portions of space it would describe if that force were considered to be an infinitely small action, as it were continually active during

* *Philosophical Transactions.* Part I, 1862, p 513

https://doi.org/10.1017/S2046166600002889 Published online by Cambridge University Press

infinitely small portions of time; the fluxional calculus gives $ft=v$, and is correctly true, however large v and t are. But the differential analysis gives $f.dt=dv$, which is correct only if dt and dv are infinitely small, and is then only to be considered so in virtue of the hypothesis that infinitely small quantities of the second and higher degree may be omitted."

We are not about to enter into that long-vexed dispute as to the relative claims of Newton and Leibnitz. Posterity has now decided that Leibnitz was an independent discoverer and the first promulgator of the calculus;* and the general adoption of his mode of analysis, as well as of his notation, shows that it has not been found so inconvenient in practice. On the other hand, we venture to think that the doctrine of fluxions has the advantage of perspicuity of reasoning and clearness of illustration. Indeed it is a matter of surprise that whereas Newton's method was the only one acknowleged in England during the last century, its use should since then have been discarded up to within the last few years. A change seems to be now creeping in, so far at least as the Newtonian mode of reasoning is concerned; and we are inclined to think that Mr. Gompertz, by his steadfast advocacy of the claims of our illustrious Newton, deserves some of the credit for the reaction which has set in. In the paper from which we have already quoted Mr. Gompertz explains his own peculiar notation of partial fluxions or differentials; he also exhibits a mode of indicating the common or Napierian logarithms or anti-logarithms by a new foot-index of crosier shape.† Again, his embodiment of zeros will be found quite a boon in calculations where large numbers or very small fractions occur. Where but a few significant figures are of importance, he puts these down followed or preceded by the actual number of cyphers placed in a ring. Thus, 898000000 or ·00000000763 he denotes by 898⑥ and ⑧763 respectively. All these matters are worthy of attention, and it is but to be regretted that they are explained in a magazine that is not easy of access to the student of elementary subjects.

Mr. Gompertz whilst still a youth, in fact as early as the year 1798, took a prominent part among the mathematicians who proposed and answered the ingenious questions contained in the *Gentleman's Mathematical Companion*. Scientific men of high repute, such as Ivory, Nicholson, Griffith Davies, &c., contributed to this publication. The reader will at once be impressed with

* *Vide* article "Commercium Epistolicum," in the *Penny Cyclopædia*.

† He uses q for the common logarithm, q for the Napierian logarithm of a number; also q for the common anti-logarithm, and q for the Napierian anti-logarithm.

the great analytical powers possessed by Mr. Gompertz from the fact that, from the years 1812 to 1822 (when his want of leisure prevented him from continuing his contributions), he gained, without a single exception, the annual prize for the best solutions of the prize questions.

We will here only advert to a problem solved by Mr. Gompertz, in the *Companion* for the year 1822, to show how practically useful some of those inquiries were.

The solidities of many solids can be found, provided the dimensions of the middle section of each particular solid is given. If A and B are the areas of the ends of the solid, C the area of a section parallel to and equidistant from the ends, and L the length or distance between the ends,

$$\text{the solidity } S=\tfrac{1}{6}\{A+B+4C\}\times L.$$

The solidities of the wedge, the prismoid, the pyramids, and their frustums, also of the whole or a segment or any portion of the whole contained between two parallel planes perpendicular to the axis of a sphere, ellipsoid, paraboloid, and of a hyperboloid, may be found by this general formula.

Mr. Gompertz was intended by his father to follow a purely mercantile profession. In compliance with this intention he made his start in life as a member of the Stock Exchange, and continued as such almost to the time that he undertook the management of an Insurance Office.

In the year 1810 he married Miss Abigail Montefiore, the sister of the well-known philanthropist Sir Moses Montefiore, Bart. Their union was blessed with one son, who died young; and two daughters, both married, who survive him.

A commercial life was not to the taste of Mr. Gompertz, and he was glad to retire when he could from the noise of 'Change Alley to the calmness and quiet of his study, or to the instructive discussions at the different learned Societies of which he was a member. In this respect he much resembled a man for whom he entertained the warmest friendship, and who also made his start in life as a member of the Stock Exchange—the late Francis Baily. Among other distinguished men with whom Mr. Gompertz was on terms of intimacy we may mention the late erudite Mark Noble, Sir John F. W. Herschel, Sir Jas. South, Mr. Babbage, &c.

The first learned Society that Mr. Gompertz joined was the Mathematical, whose members met at Crispin Street, Spitalfields. The object of this Society, founded so far back as 1717, was to introduce to the middle classes studies which till then had been confined to the libraries of the learned or the investigation of the opulent. Their valuable library and rich collection of instruments offered great facilities to the student, and a constant interchange of ideas was fostered by frequent discussions and occasional lectures. The originality displayed by Mr. Gompertz, and his thorough acquaintance with all branches of pure and applied mathematics, soon gave him a prominent place among the members; and his urbanity secured for him their good will, so that eventually he became the President of the Society. Subsequently, when the number of members had dwindled down from 81 to the square root of that number, it amalgamated with the Astronomical Society.

Mr. Gompertz communicated his first original paper of importance to the Royal Society, in 1806, through the then Astronomer Royal, Rev. Dr. Maskelyne. It treated on the application of the method of differences to a species of series whose sums were obtained, by the help of impossible quantities, by Mr. Landen in his Memoirs. The method given is simple, somewhat akin to that taught by Euler in his *Institutiones Calculi Integralis*. The Scholium is perhaps the most interesting part of the paper, and discusses those particular cases which cannot be summed by the ordinary processes pointed out by Landen or by the author himself.

The next important subject that engaged Mr. Gompertz's attention was an investigation into the theory of imaginary quantities. He was anxious to communicate his results to the Royal Society, who seemed unwilling to give insertion to them in their *Transactions*. One day, meeting Dr. Wollaston, who was Secretary to the Royal Society at that time, Mr. Gompertz asked him the reason of their refusal. "The fact is," replied Wollaston, "your papers are too profound for us, no one would be able to comprehend them." Such opinions were flattering in one respect, though rather disheartening in another.

Mr. Gompertz, was, however, not deterred, and eventually published, at his own expense, a series of original tracts on mathematical subjects. Next to the papers on his law of mortality, he considered these his most successful essays.

The first of these, treating on imaginary quantities, dedicated to his distinguished pupil Miss Lousada, was published in the year 1817. This branch of mathematics has made wonderful strides within the last thirty years; still it may be interesting to know how the matter was viewed at the time we are speaking of. In the introduction Mr. Gompertz says:—

"Immediately after the invention of methods for the discovery of the roots of an equation above the first degree, algebraic expressions must have occurred, which seemed to require the extraction of the square root of a negative quantity, the impracticability of which could not have failed to throw much embarrassment in the way of the early promoters of algebraic knowledge. Ages of study on this subject have pointed out beauties where deformity seemed only to reign, and the reiterated attempts of analysts to unriddle the secrets of science have been rewarded by the discovery that those very expressions termed impossible or imaginary quantities, which were sterile in appearance, were the most powerful instruments we could possess. Still the method of their operation is far from being universally known, and the truths of their results are by no means generally acknowledged to be legitimately obtained, and would by many be wholly rejected in the absence of other demonstrations."

After pointing out what he conceived to be the original steps which led to the introduction of imaginary quantities in algebra, and showing what an assumed law of continuity ought to lead to in the operation, he advances the following fundamental "position":—

"That wherever the operation by imaginary expressions can be used, the propriety may be explained from the capability of one arbitrary quantity or more being introduced into the expressions, which are imaginary previously to the said arbitrary quantity or quantities being introduced, so as to render them real without altering the truth they are meant to express; and that, in consequence, the operation will proceed on real quantity; the introduced arbitrary quantity or quantities necessary to render the first steps of the reasoning arguments on real quantity, vanishing at the conclusion; whence it will follow that the non-introduction of such can produce nothing wrong."

Thus, in operating with $\sqrt{-1}$, we are to operate as we would with $\sqrt{\rho-1}$, and afterwards expunge ρ; or, in short, suppose ρ to be there without inserting it. Mr. Gompertz calls expressions containing arbitrary quantities, such as those above referred to, *porismatic* expressions, and devotes a voluminous Tract, published in the year 1850, to a most lucid discussion of different porisms. He applies his analysis to a solution of a number of geometrical problems, more especially to a class of questions connected with *loci*, or, as he terms them, *local porisms*. Reverting to the subject of imaginary quantities, Mr. Gompertz successfully applied his method to the solution and interpretation of numerous questions in which the square root of a negative quantity plays an important part, such as the irreducible case of cubic equations, the exponential forms of the trigonometrical functions, the solution of linear differential equations, &c.

In the second Tract, published soon after the first, and dedicated to his brother Ephraim, he gives a geometrical interpretation to imaginary quantities, by having recourse to what he styles "functional projection." We regret we cannot give here a sufficiently elementary description of the method. It is highly ingenious, though it must be admitted it is not so simple as the geometrical interpretation generally adopted at present, for a full account of which the reader is referred to Professor De Morgan's *Double Algebra*, and sundry papers in the *Cambridge Philosophical Transactions*. However, to convince the reader of the effectiveness of Mr. Gompertz's method, we would call attention to the prize question proposed by him in the *Gentleman's Mathematical Companion* for 1820:—"The sides of a pentagon are respectively as 1, 2, 3, 4, 5, and the three angles subtending the largest side are equal, what are those angles?" This will be found no easy problem, but by the principles of functional projection the question is solved without difficulty. The angles are found to be 114° 58′ nearly.

That all these labours were duly estimated by the scientific world is shown from the fact that on the 29th January, 1819, Mr. Gompertz was elected a Fellow of the Royal Society. He continued a prominent member of that illustrious body for above 46 years. Although it is rather anticipating matters, we may here state that he was elected a member of the Council on the 30th November, 1832, and served the Royal Society in that capacity for several years.

In the year 1820 falls the foundation of the Astronomical Society, which, notwithstanding the opposition it at first met with at the hands of the President of the Royal Society (who was apprehensive lest his own Institution might thereby suffer), soon took a most prominent position among the leading Societies of the kingdom. Mr. Gompertz may be considered one of its founders, or at all events its warm supporter from the day of its foundation. He was elected on the Council on the 9th of February, 1821, and he continued a committee-man for nearly 10 years. Many of the most valuable contributions to the Society were committed to his care, and not a few of those so entrusted to him he made more complete by supplying them with explanatory notes.*

His first contribution of importance which was placed before the Astronomical Society, was "On the Theory of astronomical instruments." He presents us therein with a method by which

* See Littrow's paper "On the correction of the transit instrument": *Astronomical Transactions*, vol. i., p. 275. Also Kreil's paper "On the use of the equatorial," vol. iv., p. 501.

due allowances and corrections can be made, with great accuracy, for certain defects in the instrument.

Thus, if with the intention of forming and fixing a transit instrument, we form an instrument of which the axis of motion is not scrupulously fixed perpendicularly to the meridian, nor even parallel to the horizon, nor the line of collimation of the telescope scrupulously perpendicular to the axis of motion, nor the altitude limb perpendicular to the axis of motion, the value of the portions of the altitude limb may be obtained by what he styles "inverse construction," and the theory of the instrument laid down such, that, with certain provisos, the instrument with all its defects may answer the purpose both of a well-constructed transit instrument and of a mural quadrant. A theoretical method for the accurate adjustment of Hadley's sextant and kindred instruments forms the substance of two papers and of a supplement which was read the 12th December, 1823.

Investigations on the aberration of light seem to have next invited the attention of Mr. Gompertz. A passage which occurs in a paper on this subject, read the 14th June, 1822, gives such a vivid expression of the enthusiasm with which he followed scientific inquiries, that we cannot forbear from quoting it:—"In the acquirement of the sciences there is, besides the pleasure arising from the acquirement of knowledge of practical utility, a peculiar charm bestowed by the reasoning faculty in a well-directed pursuit of facts; and though the results shown by the arguments are frequently considered the only objects of value by the unlearned, the man of absolutely scientific ardour will often, whilst he is enraptured with the argument, have not the least interest for the object for which his argument was instituted." In the course of the essay, several of the problems on aberration which are treated of in Biot's *Astronomy* are arrived at by the author in a less symbolical manner, and are made to apply not only to the solar system but also to the fixed stars.

In another paper, read before the Astronomical Society in 1824, he explained a new instrument, appropriately called, by Mr. Francis Baily, the differential sextant, for accurately measuring the angular distances of any two celestial phenomena caused by such varying circumstances as produce but small alterations, such as refraction, parallax, or aberration.

Another paper, published in the *Transactions of the Astronomical Society*, is one on the convertible pendulum, for the discovery of which the world is so much indebted to Captain Kater. The property of the pendulum, that if the centre of oscillation relative to any proposed centre of suspension be made the centre of suspension, the proposed centre of suspension will become the centre of oscillation, can only be received as true if the axes of motion or knife edges are in the two cases parallel to each other; if the supports of the axis of vibration be not horizontal, the length of the pendulum performing the vibration in a given time will be shorter than if the pendulum vibrates (in the same time) on a horizontal support in the proportion of the cosine of inclination to the horizon to the radius. Mr. Gompertz enters more particularly into the solution of this problem.

One of the most practically important objects to which Mr. Gompertz's attention was drawn about this time, was the reduction of apparent to mean places of the fixed stars. To quote the words of Sir J. F. W. Herschel, in his excellent memoir of the late Francis Baily,* "It seems almost astonishing that these computations, which lie at the root of all astronomy, and without which no result can be arrived at, and no practical observer can advance a single step, should have remained up to so late a period as the twentieth year of the nineteenth century in the loose, irregular, and troublesome state which was actually the case; and that not from their theory being ill understood, but from their practice not having been systematized. Each of the uranographical corrections had to be separately computed by its own peculiar tables, and with coefficients on whose magnitude no two astronomers agreed. The latter evil, indeed, might be tolerated at a time when the tenth of a second of space was not considered of so much consequence as at present; but the calculations were formidable and onerous in the extreme to private astronomers, whatever they might be rendered in public establishments by habit and the use of auxiliary tables. So far as the fundamental stars were concerned, the subject had for some time attracted attention, and had begun to receive its proper remedy by the publication, by Professor Schumacher, in Denmark, of their apparent places for every tenth day; and by the laudable exertions of Sir James South, in our own country, who for some years prepared and circulated similar tables for every day. But sidereal astronomy for stars, beyond the bounds of the favoured list, might be almost said to have been interdicted to the private astronomer, owing to the excessive irksomeness of the calculations."

Messrs. Baily and Gompertz perceiving this want, proceeded to supply it. The subject was investigated generally, and a method

* *Vide* Monthly Notices of the Royal Astronomical Society for November 8, 1844.

https://doi.org/10.1017/S2046166600002889 Published online by Cambridge University Press

devised for arranging the terms of the corrections for aberration and solar and lunar precession. Some of the tables had already been computed, when they heard of Bessel's labours in the same field. Finding that that astronomer had proceeded upon a similar principle, but that besides the other corrections he had taken in that of refraction, proper motion, &c., Messrs. Baily and Gompertz willingly gave way. Certainly nothing more perfect than the *Fundamenta Astronomiæ*, by Bessel, could have been desired. However, Baily and Gompertz's work was of some use. The complete Catalogue of Stars of the Royal Astronomical Society may be looked upon as one of the united labours of these men.

Although the last published paper by Mr. Gompertz on astronomy dates so far back as the year 1829, he continued to turn his attention to this branch of science from time to time.

During the last years of his life he was engaged in investigations on meteors, shooting stars, comets, &c., but he had to lay them aside to enable him to complete a paper for the Royal Society on life contingencies.

We must now advert to the labours undertaken by the subject of our Memoir in the field of life assurance.

On the 29th June, 1820, he presented to the Royal Society his sketch of an analysis and notation applicable to the estimation of the value of life contingencies. In the introduction he forcibly dwelt upon the desirability that Assurance Companies should promulgate the information and experience they individually acquired, with a view to be guided by more truthful tables than were at that time acted upon. He expressed the hope, that actuaries of different Societies would, by their mathematical skill, collect for the common good of all, from multiplied resources, that which they could not obtain from a less general observation.

All this will, no doubt, be considered very trite at the present day; but we must recollect that the science of life insurance was at that time in its infancy, and that it required continual urging, before the Government could be induced to ascertain the mortality of its annuitants, and the authorities of the Equitable Society that of their assured.

As regards the latter establishment, it may be said, that Mr. Gompertz was the first to make known, with any degree of accuracy, the mortality that prevailed amongst its members. Mr. Morgan, in the sixth edition of Dr. Price's work on reversionary payments, compares the mortality experienced at the different decennia of life in the Equitable Society with that deduced from the Northampton tables. By the help of a lemma,* and the application of his law of mortality, to which we shall refer further on, Mr. Gompertz constructed a complete table of mortality, starting from age 10, which tallies very closely with that deduced by Mr. Griffith Davies. Now, although the observations of both these gentlemen on this head were made public about the same time, viz., the year 1825; yet, on reference to Mr. Gompertz's original manuscripts, it will be seen, that he framed the table in its more important particulars prior to October, 1820.

In Tract I. Mr. Gompertz establishes a system of notation which is as ingenious as it is exhaustive, though it must be admitted, that it lacks that compendious character which would have recommended it for more general adoption.

By means of his methods of summation he determines more accurately than by the common modes of approximation the value of incomes dependent upon a number of joint lives. He then applies the method of fluxions to obtain the probabilities of survivorships of two or more lives, assuming the functions of life to be of the same continuous character. He investigates a problem to determine what would be the law of mortality between two lives A and B, so that, should it be known that they are both extinct, there would be an equal chance which of them died first, and it was found that such relation could only exist when the decrements for each life proceed in geometrical progression. The fluxional calculus is then applied to a number of complicated contingencies and survivorships, involving two, three, and more lives, which had been previously investigated by Mr. William Morgan† and Mr. Baily.‡ A scholium is added, in which the valuation of annuities payable more than once a year, and of assurances payable at certain fixed periods after death, are dealt with. This important paper has hardly met with the amount of attention it so fully deserves. The fact can, however, easily be explained, if we but bear in mind that the *Philosophical Transactions* are not accessible to everyone, and that the notation, as well as the mode of reasoning, are of a somewhat abstruse character.

A sequel to this paper, contained in a letter to Mr. Francis Baily, and read before the Royal Society on the 16th June, 1825, was destined to play a more important part. It treats on the nature of a function expressive of the law of human mortality, and

* *Vide Philosophical Transactions* for 1825, p 526.
† *Philosophical Transactions*, 1788, 1789, 1791, 1794, and 1799.
‡ *The Doctrine of Life Annuities and Assurances*, chap. viii.

https://doi.org/10.1017/S2046166600002889 Published online by Cambridge University Press

on a new mode of determining the value of life contingencies. Mr. Gompertz having remarked that for certain intervals the law of mortality approaches nearly to a geometrical progression, considers the propriety of looking upon it as involving the actual law of nature. He establishes the law *à priori*, by supposing a person's resistance to death to decrease as his years increase, in such a manner that at the end of equally infinitely small intervals of time, he loses equally infinitely small proportions of his remaining power to oppose destruction. He further supposes that, among any given number of persons of equal vital powers, the probability of death is the same, but that it is inversely proportional to the vitality.

It is to be regretted that in the analytical investigation which is given errors have crept into the text. As Mr. Gompertz was not able to carry out his intention of publishing a list of errata of these Tracts, notwithstanding the hope he held out in his letter published in the *Assurance Magazine* (vol. ix., p. 296), an exact transcript of the demonstration contained in Mr. Gompertz's own copy is herewith given, with a view to enable those that possess the work to correct the errors contained in the proof of the fundamental proposition.

"If L_x be the number of living at the age x, we shall have

$aL_x \times q^x x$ for the fluxion of the number of death $= -(L_x)^{\cdot}$;

$$\therefore axq^x = -\frac{\dot{L}_x}{L_x};$$

$-\dfrac{aq^x}{\text{log. of } \frac{1}{q}} =$ hyp. log of $\dfrac{L_x}{d}$, d being a constant: therefore put $a \div$ com. log. of $\dfrac{1}{q} \times$ square of hyp. log. of $10 = c$, $cq^x =$ com. log of $\dfrac{L_x}{d}$; $\therefore$ putting $10^c = g$, $L_x = d.g^{q^x}$."

This law, which it will be seen is based upon a physiological principle of high probability, leads to conclusions which agree very satisfactorily with observed facts. To use the words of Professor De Morgan,* "Had the law been propounded in the days of Newton, *vitality* would have been made a thing of, like *attraction*."

In recent times, the formula has been extensively used for purposes of interpolation and adjustment of tables of mortality;† but

* The *Athenæum*, July 22, 1865.

† *Tables of Mortality derived from the Experience of the Amicable Society*, by Thomas Galloway. *Life Contingency Tables*, by E. J. Farren. *Rates of Premium for Assurances of Military Officers in Bengal*, by Charles Jellicoe; *Assurance Mag*, vol. i., p. 166. *Mortality prevailing in Eagle Insurance Company*, by Charles Jellicoe; *Ass. Mag.*, vol. iv., p. 199. *Mortality prevailing in Royal Insurance Company*, by Percy M. Dove.

nothing will more clearly exhibit its superiority to other laws, that have been presented by different authors, than actually placing these before the reader for comparison.

Passing over D'Alembert's, D. Bernoulli's, and Duvillard's analytical investigations, which had for their chief object the determination of the mortality produced by the small-pox, we have the following formulæ; x being the age, and y the corresponding number of the living.

I. De Moivre's*:—

$$y=86-x.$$

II. Lambert's†:—

$$y=10000\left\{\frac{96-x}{x}\right\}^2-6176\left\{e^{-\frac{x}{13\,682}}-e^{-\frac{x}{2\,431}}\right\}.$$

III. Thomas Young's‡:—

$$y=368+10x-11\{156+20x-x^2\}^{\frac{3}{2}}+\frac{1}{2\cdot85+2\cdot05x^2+2\left(\frac{x}{10}\right)^6}$$

$$-5.5\left(\frac{x}{50}\right)^{10}+\frac{(5\cdot5)^2}{4000}\left(\frac{x}{50}\right)^{20}-5500\left(\frac{x}{100}\right)^{40}.$$

IV. Babbage's||:—

$$y=6199\cdot8-9\cdot29\frac{x}{1}-1\cdot5767\frac{x(x-1)}{2}.$$

V. Littrow's§:—

$$y=598\cdot1673-8\cdot4175x+\cdot2309x^2-\cdot0052x^3+\ \ldots\ .$$

VI. Moser's¶:—

$$y=1-ax^{\frac{1}{4}}+bx^{\frac{9}{4}}-cx^{\frac{17}{4}}-dx^{\frac{25}{4}}+ex^{\frac{33}{4}};$$

in the case of Brune's *Life Tables*, $a=\cdot2$, $b=\frac{\cdot7125}{10^5}$, $c=\frac{\cdot1570}{10^8}$, &c.

VII. Gompertz's original formula:—

$$y=d.g^{q^x}.$$

VIII. Makeham's** modification thereof:—

$$y=d.g^{q^x}s^x,$$

* *Annuities on Lives*, by Abraham De Moivre. London, 1727.
† Gaeta and Fontana: *Dottrina degli azzardi.* Milano, 1776.
‡ *Philosophical Transactions*, 1826, Part III.
|| See Note by Mr. Peter Gray: *Assurance Magazine*, vol. vi., p. 185.
§ Professor Littrow: *Ueber Lebensversicherungen und andere Versorgungsanstalten.* Vienna, 1832; p. 52.
¶ Dr. Ludwig Moser: *Die Gesetze der Lebensdauer.* Berlin, 1839; p. 315.
** *Assurance Magazine*, vol. viii., p. 301; and vol. xii., p. 315.

It would carry the reader too far if much commentary were added to these formulæ. They not only compare unfavourably with Gompertz's law, on account of being purely empirical, but also on account of their being (with the exception of De Moivre's law) of so unwieldy a character as to be altogether unfit for practical use. Besides, these laws will only bear comparison with individual tables of mortality.*

Mr. Gompertz, in his contribution to the Royal Society's *Transactions* of 1825, examines the coincidence of his law with several tables of the best authority, such as Deparcieu's, the Northampton, Carlisle, and Swedish tables, obtaining in each case the necessary constants, by what he terms his "Vital Rule of Three."

In the second chapter, Mr. Gompertz points out how annuities on several lives may be computed by the use of the tables called by him "accommodated ratios," wherein he assumes the number of persons living at equal intervals of successive ages to be in simple geometrical progression, the periods being taken sufficiently short to permit this assumption to be a near approximation to truth.

This important paper did certainly not meet at first with the attention it so fully merits. To Professor De Morgan no little credit is due, not only for popularizing the subject, but for finding out some interesting properties of the law.†

Messrs. Woolhouse's, Sprague's, and Makeham's labours should also here be mentioned with much praise. But to Mr. Gompertz himself we are indebted for the greatest improvements in and extension of his law. He was, however, not enabled to resume the investigation of this subject till about the year 1860. During this long interval of 35 years he was so much engaged in the management of life assurance concerns that he found little leisure for private study.

When the Guardian Assurance Office was established, Mr. Gompertz applied for the situation, but Mr. Griffith Davies was the successful candidate. So far was Mr. Gompertz from feeling piqued towards his competitor, who had been in his way on several previous occasions, that he was all the more obliging. Eventually

* Mr. T. R. Edmonds, in the *Philosophical Magazine* for January, 1866, professes to give a new expression for the law of human mortality, derived from analogy with the law of density of saturated steam. The question of the accuracy or novelty of this law does not concern us. We regret, however, that when speaking of his old formula, he omits making any mention of, much less doing justice to the subject of our Memoir. It is a noteworthy circumstance, that Mr. Edmonds, in the course of the paper, no longer insists upon the fact that life is made up of three periods—infancy, florescence, and senescence; but contends for two periods of life only—childhood and manhood.

† *Vide Penny Cyclopædia*, art. "Mortality"; *Philosophical Magazine*, Nov., 1839; *Assurance Magazine*, vol. viii., p. 181.

https://doi.org/10.1017/S2046166600002889 Published online by Cambridge University Press

he introduced him at the Royal Society, and soon after carried his election as a Fellow.

In the year 1823, when he suffered under the severe visitation of the death of his only son—a boy, then, nine years of age—his friends felt that he could only gain relief of mind by altogether devoting himself to a profession which seemed to be most congenial to his feelings. Chiefly through the endeavours of his friends, Sir Moses Montefiore, Messrs. N. M. Rothschild, and Samuel Gurney, he was appointed actuary to the Alliance British and Foreign Assurance Company, which was founded at that time.

Having had only one month allowed him to prepare all the rates and regulations of the Company, he without hesitation adopted the Carlisle table of mortality, which he found satisfactorily to agree with the Equitable Experience. The Office under his guidance made most satisfactory progress. One measure the directors of his Company were anxious to promote at the time—viz., to compete with Government as to the grant of annuities. This he opposed strenuously. The terms on which the Government were granting these annuities were based on the Northampton tables, which, though safe enough for assurances, could not but entail ruinous consequences with regard to annuities. Eventually he even convinced the able Government Actuary, Mr. John Finlaison, of the heavy loss that was caused by using these tables; and in the year 1829 those rates were superseded by the higher ones, based on the mortality experienced by Government Annuitants.

Mr. Gompertz's views on this subject are stated very clearly before a Parliamentary Committee that sat in the year 1827.*

In a sketch of Mr. Gompertz's life in the *Athenæum* of the 22nd July last, reference is made to another actuary of eminence, the late William Frend, of the Rock Office, who, though on friendly terms with Mr. Gompertz, was his opponent in more than one respect. In matters of life assurance the one may be considered the champion of the Carlisle, the other, in conjunction with Mr. Morgan, of the Northampton table of mortality. Mr. Gompertz was as staunch a Newtonian as Mr. Frend was an anti-Newtonian. Both may be called the last of their class. Again, with regard to imaginary quantities, we have seen Mr. Gompertz's labours in that field. Mr. Frend also directed his attention to that subject, but with a view to the rejection of imaginary as well as of negative quantities.†

* *Vide* Minutes of Evidence before the Select Parliamentary Committee on the laws respecting Friendly Societies, 1827.

† *Principles of Algebra.* London, 1796.

https://doi.org/10.1017/S2046166600002889 Published online by Cambridge University Press

The ability with which Mr. Gompertz conducted Office valuations, as well as other actuarial calculations, may be seen from the fact that, at the public sales of reversions, &c., his opinion ruled, more or less, the market. The mode in which he treated questions for the sale of life annuities and contingent reversionary property is interesting, but a description thereof would carry us too far.

As may be supposed, he was frequently referred to by Friendly Societies and kindred institutions for advice. For the Army Medical Board, by which he was consulted, he made elaborate computations, and revised the rates they charged in connexion with the Fund for Widows and Orphans.

We must also mention the prominent part he took in promoting the publication of the Experience Table of the 17 Assurance Companies, which appeared in the year 1843.

The transfers of life interests and reversions were becoming so frequent that he advised the establishment of a Company, whose exclusive business it should be to deal in such property. Accordingly, the National Reversionary Investment Company was founded, and he continued its actuary from its establishment to the time that ill health made it necessary for him to retire. In the year 1847 he also relinquished the actuaryship to the "Alliance."

He continued, however, sedulously to attend the meetings of the different learned Societies to which he belonged, and otherwise to promote their interest. Amongst these Societies we have to mention the Institute of Actuaries, of which he was an honorary member; the Statistical Society, in the establishment of which he took part; the Royal Literary Fund, of which he was steward on several occasions; the Society for the Diffusion of Useful Knowledge, on the Committee of which he served from the year 1828; the new London Mathematical Society, for which he was actually preparing a paper when attacked by his last illness.

During the ten years that immediately followed his retirement from active duties, we do not hear so much of his original investigations and labours; in fact he suffered so much from ill health, that his chief occupation consisted in diverting himself with mechanical contrivances, in which even as a boy he had delighted.

At length his complaint, the gout, became so serious, that he was quite unable to move from his room; and during the last six years of his life he did not so much as leave his bed-room floor. He bore this confinement with wonderful patience; and the more his bodily strength failed, the greater became the activity of his mind.

In the year 1860 he presented to the International Statistical Congress which met at London, a paper on one uniform law of mortality from birth to extreme old age; and the ideas contained therein were further elaborated in the first supplement to his papers of 1820 and 1825, that were presented to the Royal Society.

Besides many other valuable investigations, he gives a more complete formula for the law of mortality, involving several terms, which may be reduced to the form we are already acquainted with, viz., $y = dg^{q^x}$, and yet express the mortality from the commencement of life to extreme old age, if $d\ g\ q$, instead of being constant, are supposed to vary slowly.

Gompertz's improved law is presented under the following two forms—

$$y = \text{constant}.\,A^{e^x} B^{e^x_{,} x} C^{q^x} D^{P_x}, \text{ where } P_x = \theta\omega^{\pi^x \overline{x-u}};$$

$$\lambda y = C\beta^x + \underset{11}{k\epsilon^x} + k\epsilon^x - \lambda^{-1}(e^x \lambda q_0 . \overline{x - h}) + \mu\nu^x.$$

where from birth to extreme old age all the quantities but x and y are constant.

$\underset{11}{k\epsilon^x}$ commences its significance at birth, and is particularly interesting, because in the Carlisle table it shows a surprising agreement with the stated mortality of children from birth till the age of 1 year.

$k\epsilon^x$ commences significance at birth, but gradually sinks; and at age 20 and after sinks into entire insignificance.

$C\beta^x$ and $\lambda^{-1}(e^x\lambda q_0 . \overline{x-h})$ are of significance from birth to extreme old age.

$\mu\nu^x$ is insignificant till x equals about 80, and then slowly increases during the remainder of life.

These constants are obtained by a peculiar process of differencing, an extension of the so-called "vital rule of three."

Now some people think Gompertz's law untenable, because the mortality of man, or at all events of certain classes, does not increase by a steady progression, but exhibits certain climacteric departures at certain periods of life. That such is the case cannot be doubted. Similar climacteric departures are exhibited even at infancy. The mortality at birth is much greater than that at a few years later, which again increases before the period of puberty.

Now, we have just seen how Gompertz's extended law applies to those cases, and the author ventures to assert, that by applying

https://doi.org/10.1017/S2046166600002889 Published online by Cambridge University Press

Gompertz's peculiar process of differencing at certain stages of life, the law will be found to adapt itself to the exceptional mortality at those ages.

In the Supplement under consideration methods are also pointed out by which the most intricate questions of life contingencies, involving any number of lives, may be solved without difficulty.

An instructive consideration of questions of special risk, and what he terms "special influenced contingencies," which have an important bearing upon survivorships, is also given. In conclusion, he shows how his theoretical law of mortality not only applies to infancy, but holds also for sickness. He finds that the values obtained from his "vital rule of three" compare most satisfactorily with the tables deduced by Messrs. Ansell and Neison from the experience of Friendly Societies.

A second supplement, placed before the Royal Society the 12th May, 1864, contains some valuable suggestions on the Commutation Tables of Barrett; and professes to give a method by which the existing competition amongst Assurance Companies, which the author considers injurious to their interests, may be checked.

Though his mind had not been affected, his bodily state of health began at last so to give way, that he could not complete this last paper to his satisfaction. In April this year he had a stroke of paralysis, and then indeed his days were numbered. He lingered on till the 14th July, when a calm death released him from further sufferings. He was buried a few days afterwards, with every mark of respect, in the Jewish burial ground near Victoria Park.

To do full justice to Mr. Gompertz's character is no easy task. We have adverted to one or two incidents in his life, wherein he showed true nobleness of disposition and kindness of heart. But these qualities were not the results of the occasional promptings of his feelings, they characterized all his actions.

It is not too much to say, that he was never known to decline giving his assistance in the cause of charity, and this aid he offered to an extent which far exceeded his means. But true charity is not limited to the giving of mere alms, it consists chiefly in aiding by the powers of the intellect. We accordingly find him a prominent member of some of the leading Jewish charities. He also worked out a plan, which has eventually been acted upon, by which the dispensation of charity is placed in the hands of a Board of Guardians, whose duty it is to investigate the claims and keep a register of all appli-

https://doi.org/10.1017/S2046166600002889 Published online by Cambridge University Press

cants for relief, to assist them in distress, and to procure them employment.

What a kind husband, what a loving father, Mr. Gompertz was, only the immediate members of his family know: those to whom he extended his friendship were at a loss which more to admire, his social qualities or mental attainments.

May science ever find votaries as worthy and ardent as the late Benjamin Gompertz!

https://doi.org/10.1017/S2046166600002889 Published online by Cambridge University Press