The Fundamental Formula for the Area of a Triangle in Analytical Geometry.

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(*Read and Received 8th March 1912*).

The algebraic methods employed are usually defective in explaining the sign to be attached to an area; while the trigonometrical method with the transformation of a trigonometrical formula has always seemed a little far fetched so early in analytical geometry.

The following algebraic demonstration is direct enough, and successfully suggests the sign to be attached.

Case I. Let G_1, G_2 be two points $(1, y_1)$ and $(1, y_2)$ on the line $x = 1$.

Then the area of $OG_1G_2 = \frac{1}{2}G_1G_2 = \frac{1}{2}(y_2 - y_1)$ in magnitude ***and sign.***

Case II. Let P_1, P_2 be any two points (x_1, y_1), (x_2, y_2), and let OP_1 and OP_2 cut the line $x = 1$ in G_1 and G_2, so that G_1 and G_2 are given by $\left(1, \frac{y_1}{x_1}\right)$ and $\left(1, \frac{y_2}{x_2}\right)$, while

$$\frac{OG_1}{OP_1} = \frac{1}{x_1}, \quad \frac{OG_2}{OP_2} = \frac{1}{x_2}.$$

Hence

$$\begin{aligned} \triangle OP_1P_2 &= x_1x_2 \,.\, \triangle OG_1G_2 \\ &= \tfrac{1}{2}(x_1y_2 - x_2y_1). \end{aligned}$$

There remains the question of sign. When x_1 and x_2 are both like in sign, OG_1G_2 and OP_1P_2 are of like sense of rotation and x_1x_2 is positive. (Fig. 1).

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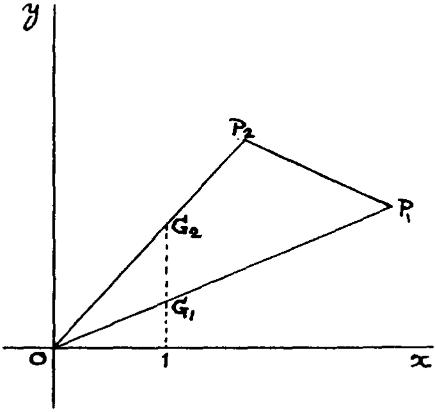

Fig. 1.

If x_1, say, is positive, and x_2 negative, then OG_1G_2 and OP_1P_2 are of opposite sense of rotation, and x_1x_2 is negative. (Fig. 2).

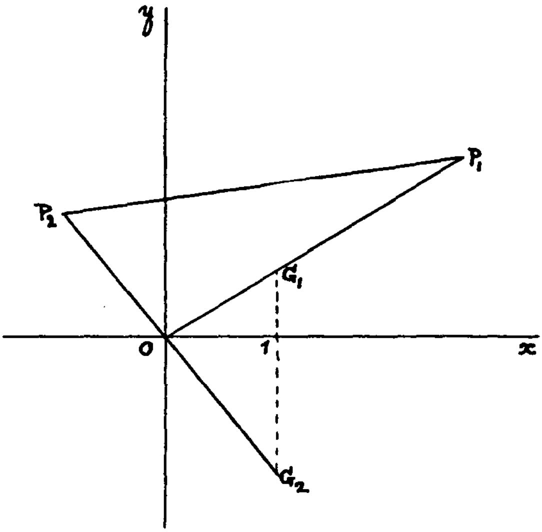

Fig. 2.

Hence in all cases the usual law for the sign of the area follows.

This is extended to the triangle $P_1P_2P_3$, and to polygons, in the usual manner.

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