MAGNETIC HELICITY OF OSCILLATING CORONAL LOOPS

V.Krishan[1] and E.R.Priest[2]

[1] Indian Institute of Astrophysics, Bangalore 560034, India

[2] Univ. of St. Andrews, Mathematical Institute, St.Andrews KY 169 SS, Fife, Scotland

The dynamics of the velocity and magnetic field in a coronal loop is studied using ideal MHD equations and the Chandrasekhar-Kendall representation. The complete dynamics is described by a set of infinite, coupled nonlinear ordinary differential equations which are first-order in time for the expansion coefficients of the velocity and magnetic field. Here, the coronal loop plasma is represented by a superposition of the three [n = 0 = m; n = m = 1 and n = m = 1] lowest-order C-K functions. This system, when perturbed linearly from its equilibrium state exhibits sinusoidal oscillations. The frequency S of these oscillations is given by (Krishan et al 1988):

$$S^2 = A\eta_a^2 + B\eta_b^2 + C\eta_c^2 \qquad (1)$$

where A, B and C are constants and η's are the equilibrium amplitudes of velocity field which are also equal to magnetic field amplitudes. The three quadratic invariants of this system are

$$\text{the total energy } E = 2\left[\lambda_a^2\eta_a^2 + \lambda_b^2\eta_b^2 + \lambda_c^2\eta_c^2\right]$$

$$\text{the magnetic Helicity } H_m = \lambda_a\eta_a^2 + \lambda_b\eta_b^2 + \lambda_c\eta_c^2 \qquad (2)$$

and the cross helicity H_c becomes equal to the total energy under the conditions of equilibrium $\vec{V} = \vec{B}$, which is an aligned Alfvenic state. λ's are the characteristic wave-vectors of the three modes. From equations (1) and (2), we found that the frequency S can be expressed in a very simple form as

$$S^2 = \frac{A H_m}{\lambda_a} \qquad (3)$$

Here λ_a, where $a \equiv (0,0)$ mode, can be expressed in terms of the ratio of poloidal ψ_p to toroidal ψ_t magnetic flux; λ_b and λ_c are numerical values

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related to zeros of Bessel functions (Montgomery et al (1978)) obtained from

$$(\psi_t/\psi_p) = -\frac{R}{L}\frac{|\lambda_a|}{\lambda_a}\frac{J_0'(\lambda_a R)}{J_0(\lambda_a R)}$$

$$\frac{2\pi R}{L}\gamma_b\, J_1'(\gamma_b R) + \lambda_b\, J_1(\gamma_b R) = 0$$

and $$\frac{2\pi R}{L}\gamma_c\, J_{-1}'(\gamma_c R) + \lambda_c\, J_{-1}(\gamma_c R) = 0$$

where $\lambda = \left[\gamma^2 + (2\pi R)^2/L^2\right]^{1/2}$ and (L,R) are the length and radius of the cylindrical plasma loop. By measuring the periods of oscillating loop prominences often observed in coronograph movies, one has now a way of estimating magnetic helicity which eludes any direct measurement.

References

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