a positive answer to this question for an arbitrary group Γ. I also prove that a 4-set generator always exists off of a (topologically) meager set, thus answering a question of A. Kechris from the mid-90s. Lastly, I show that any aperiodic action of Γ admits a Γ-equivariant Borel map to the aperiodic part of the 2-shift.

Part 2: We investigate pairs of countable Borel equivalence relations $E \subseteq F$, where E is of finite index in F. Our main focus is the well-known problem of whether the treeability of E implies that of F: we provide various reformulations of it and reduce it to one natural universal example. In the measure-theoretic context, assuming that F is ergodic, we characterize the case when E is normal. Finally, in the ergodic case, we characterize the equivalence relations that arise from almost free actions of virtually free groups.

Part 3: We consider natural complexity measures for recursive programs from given primitives and derive inequalities between them, answering a question asked by Yiannis Moschovakis.

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JOSEPH ZIELINSKI, ***Compact Structures in Descriptive Classification Theory***, University of Illinois at Chicago, 2016. Supervised by Christian Rosendal. MSC: 03E15. Keywords: Borel reducibility, descriptive classification theory, compact metrizable structures, homeomorphism of compact metric spaces.

Abstract

An equivalence relation, E, on a Polish topological space, X, is Borel reducible to another, F, on Y, when there is a Borel-measurable function from X to Y assigning F-classes as complete invariants for E. Descriptive classification theory is the programme whose aim is to assess the complexity of naturally arising isomorphism relations by identifying their positions in the Borel reducibility preorder.

This thesis considers the class of compact metrizable structures: compact Polish spaces equipped with closed relations. The natural isomorphism relation in this setting is homeomorphic isomorphism, whereby two such structures are isomorphic when there is a homeomorphism between them that preserves this relational structure. In work carried out jointly with C. Rosendal, we observe that the relation of homeomorphic isomorphism between compact metrizable structures in any countable relational language is Borel reducible to the orbit equivalence relation of an action of a Polish group. We then illustrate how this can be used to bound the complexity of other equivalence relations (e.g., topological group isomorphism between locally compact or Roelcke precompact Polish groups) by encoding the objects into compact structures.

Among the orbit equivalence relations of Polish group actions there are some of greatest complexity, in the sense that every other orbit equivalence relation is Borel reducible to them. Such relations were first exhibited by H. Becker and A. S. Kechris, arising from a universal Polish group acting by shift on the space of its closed subsets. Subsequently, it was shown by S. Gao and A. S. Kechris, and by J. D. Clemens, that the isometry relation between separable complete metric spaces has this same complexity. From this, M. Sabok showed that the same is true of the isomorphism relation between separable C*-algebras.

Continuing this line of research, we establish the following:

THEOREM. *The homeomorphism relation between compact metric spaces is Borel bi-reducible with the complete orbit equivalence relation for Polish group actions.*

This is done in two stages: in the first, the objects of Sabok's construction are encoded into compact metrizable structures; in the second, it is shown that the additional relational structure can be eliminated by encoding it into the topology of another compact space. We also present an improvement of this, from joint work with C. Rosendal. Using similar techniques to those described above, we replace the first stage with a short proof that the complete orbit equivalence relation of Becker and Kechris is Borel reducible to the homeomorphic

isomorphism relation for a class of compact metrizable structures. This provides a more direct proof of the theorem above and allows one to view the earlier results of Sabok and of Clemens, Gao, and Kechris as consequences of it.

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ANTON BOBKOV, ***Computations of Vapnik–Chervonenkis Density in Various Model-Theoretic Structures***, University of California, Los Angeles, 2017. Supervised by Matthias Aschenbrenner. MSC: Primary 03C, Secondary 05C. Keywords: VC-density, trees, Shelah–Spencer graphs, superflat graphs, p-adic numbers.

Abstract

Aschenbrenner et al. have studied Vapnik–Chervonenkis density (VC-density) in the model-theoretic context. We investigate it further by computing it in some common structures: trees, Shelah–Spencer graphs, and an additive reduct of the field of p-adic numbers. In the theory of infinite trees we establish an optimal bound on the VC-density function. This generalizes a result of Simon showing that trees are dp-minimal. In Shelah–Spencer graphs we provide an upper bound on a formula-by-formula basis and show that there isn't a uniform lower bound, forcing the VC-density function to be infinite. In addition we show that Shelah–Spencer graphs do not have a finite dp-rank, so they are not dp-minimal. There is a linear bound for the VC-density function in the field of p-adic numbers, but it is not known to be optimal. We investigate a certain P-minimal additive reduct of the field of p-adic numbers and use a cell decomposition result of Leenknegt to compute an optimal bound for that structure. Finally, following the results of Podewski and Ziegler we show that superflat graphs are dp-minimal.

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ATHIPAT THAMRONGTHANYALAK, ***Extensions and Smooth Approximations of Definable Functions in O-minimal Structures***, University of California, Los Angeles, 2013. Supervised by Matthias Aschenbrenner. MSC: Primary 03C64, Secondary 14P10, 32B20. Keywords: o-minimal structures, Whitney Extension Theorem.

Abstract

A jet of order m on a closed set $E \subseteq \mathbb{R}^n$ is an indexed family $(f_\alpha)_{\alpha\in\Lambda}$, where $\Lambda = \{(\alpha_1,\dots,\alpha_n) \in \mathbb{N}^n : \sum_{i=1}^n \alpha_i \leq m\}$. In 1934, H. Whitney proved Whitney's Extension Theorem, which gives a necessary and sufficient condition on the existence of C^m-extensions of a jet of order m on a closed subset of $\mathbb{R}^n$. In the same year, he asked how one can determine whether a real-valued function on a closed subset of $\mathbb{R}^n$ is the restriction of a C^m-function on $\mathbb{R}^n$ and gave an answer to the case $n = 1$. Later, the case $m = 1$ was proved by G. Glaeser using the concept of "iterated paratangent bundles". A complete answer to Whitney's Extension Problem was provided much later in early 2000s by C. Fefferman.

In the first part of this thesis, we study the above questions in an o-minimal expansion of a real closed field. We prove a definable version of Whitney's Extension Theorem. In addition, we solve the C^1 case of Whitney's Extension Problem in o-minimal context.

In the rest of this thesis, we discuss the following question: Suppose R is a real closed field and U is an open subset of R^n. If $f: U \to R$ is continuous, definable in an o-minimal expansion of R, and $\varepsilon \in R^{>0}$, is there a definable C^m-function $g: U \to R$ such that $|g(x) - f(x)| < \varepsilon$ for all $x \in U$? We gave a positive answer to this question. This result was inspired by a series of articles by A. Fischer.