THE ROLE OF MAGNETIC FIELDS IN THE FORMATION OF STARS*

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ABSTRACT

We review the role of the interstellar magnetic field: (i) in the formation of interstellar clouds; (ii) in determining critical states for gravitational collapse; (iii) in affecting the collapse and fragmentation of interstellar clouds; and (iv) in resolving the "angular momentum problem" during star formation. Finally, we review the manner in which the field decouples from the matter via ambipolar diffusion; new time-dependent solutions are discussed.

1. INTRODUCTION

This review is written specifically for nonspecialists in the field of interstellar magnetohydrodynamics as it relates to star formation. The fundamental problems of star formation resolved or posed by the presence of interstellar magnetic fields, and the significant results of theoretical investigations on the role of magnetic fields in the formation of stars (and planetary systems) are described physically, with only occasional reference to the underlying mathematical formalism. We restrict our attention to the early stages of star formation. (The effects of magnetic fields on stellar structure and later stages of stellar evolution are reviewed by Mestel in this volume.) More specifically, we review some key physical processes expected to take place during (or to determine) the formation and dynamical contraction of interstellar clouds out of the mean number density of the interstellar medium ($n \sim 1\ cm^{-3}$), which is permeated by a mean magnetic field $B \sim 3$ microgauss. The relevance of the relatively diffuse stages of contraction of a cloud with a given mass stems from our current theoretical understanding that physical quantities, such as angular momentum and

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magnetic flux, present at these stages, may (i) determine to a large extent the subsequent dynamical evolution of a cloud; and (ii) reach residual (or, terminal) values during these stages. Thus, stellar rotation (referring both to single stars and multiple stellar systems) and stellar magnetic fields, at least at the onset of the Hayashi phase, may be obtainable from theoretical calculations of cloud formation and collapse.

The theoretical problems remaining unsolved on the role of magnetic fields in star formation are of such fundamental nature that analytical work is not only possible but also essential for (i) isolating the relevant physical processes; (ii) specifying the minimum set of parameters necessary for a unique determination of a solution; and (iii) obtaining a formal solution and interpreting it physically. If the qualitative nature of the conclusions is expected to change once a simplifying assumption is relaxed, such a calculation is merely an intellectual exercise. On the other hand, if relaxation of a simplifying assumption results only in a quantitative change in the conclusions, the calculation can be regarded as trustworthy, and numerical computations can be undertaken to improve quantitatively the analytical results under a more realistic set of assumptions. Some of the fundamental questions relating to magnetic fields that a theory of star formation must answer are the following.

1) Does the interstellar magnetic field play any role at all in interstellar gas dynamics in general and star formation in particular?

2) Can the field support a dense cloud against self-gravity and, if so, for how long?

3) Does it affect fragmentation and, if so, how?

4) Can the magnetic field resolve "the angular momentum problem" during star formation?

5) If it is dynamically important in the first place, does it ever become insignificant? At which stage, and by what means does this transition occur?

6) Does a cloud's magnetic energy increase or decrease upon contraction? If magnetic energy is released at some stage, in what form does it appear?

The unsettled observational questions are at least as significant as the theoretical ones. Although the evidence that a magnetic field of a few microgauss permeates the interstellar medium is firm, there is only meager observational evidence on the topology and correlation of the field strength and the gas density in diffuse H I and dense molecular clouds. The latter kind of information is essential if observations are to provide checks and input to theoretical calculations.

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In §II we review the observational evidence for the interstellar magnetic field. This account is not complete in that it does not necessarily refer to the latest observations (e.g., see review by Chaisson and Vrba 1978). The emphasis is on the physics on which each observational method is based and on the difficulties in interpreting the observational results. In subsequent sections we review the progress made to date on the six fundamental issues raised above and on other matters concerning the role of magnetic fields in star formation.

2. OBSERVATIONAL EVIDENCE FOR THE INTERSTELLAR MAGNETIC FIELD

Detection of the interstellar magnetic field is based either on the fact that the field can be instrumental in the production of electromagnetic radiation or on the fact that it can modify radiation propagating through the space in which the field exists.

2.1 Synchrotron Radiation

Synchrotron radiation is produced by highly relativistic electrons gyrating in a magnetic field. It is emitted in a small solid angle about the instantaneous electron velocity, so that the line of sight must lie in the plane of the electron's orbit if the radiation is to be observed at all. The radiation from an ensemble of electrons is characterized by a power-law spectrum and by a high degree of linear polarization, with the electric field normal to the plane defined by the magnetic field and the line of sight (Ginzburg and Syrovatskii 1965; also, Bless 1968). With independent evidence for the existence of 1 GeV cosmic-ray electrons (see review by Meyer 1969), the synchrotron mechanism accounts for a major fraction of the background radio continuum emission in the Galaxy (e.g., see Spitzer 1968 or 1978 and references therein).

To deduce the magnitude of the field, one usually introduces a number of dubious assumptions, the most common of which is energy equipartition between magnetic fields and cosmic-ray protons. [At a given energy per particle, the number of cosmic-ray electrons is only about 2% that of protons (Earl 1961; Meyer and Vogt 1961).] Additional uncertainties enter in estimating the size of the emitting region. However, even if this size is known, further assumptions concerning its internal structure are necessary for estimating the strength of the magnetic field. This is so because the measured intensity of radiation at some frequency is proportional to the line integral (along the line of sight) of the product of the number density of relativistic electrons and a power (usually around 1.8) of the perpendicular (to the line of sight) component of the magnetic field. Large-scale magnetic fields ranging from 10 to 50 μgauss have been deduced (Woltjer 1965; Davis and Berge 1968). Daniel and Stephens (1970) used the fluxes of cosmic-ray electrons and synchrotron radiation observed at the earth to estimate an energy spectrum for electrons with energies $\lesssim$ 5 GeV

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(because the observed one has been modulated by the solar wind) and to show that this spectrum joins smoothly with the observed spectrum above 5 GeV (which does not suffer solar modulation) only if the magnetic field is in the range 6 - 9 μgauss. They assumed, however, that the region of emission was homogeneous. If the cosmic-ray density is fairly uniform, regions of strong fields are overweighted, and the mean background interstellar field may actually be weaker than the one deduced by Daniel and Stephens.

2.2 Polarization of Starlight

The observation that light from distant stars is partially polarized (Hall 1949; Hiltner 1949), and the correlation of the degree of polarization with interstellar reddening have been attributed to the dynamical alignment of elongated dust grains by the interstellar magnetic field (Davis and Greenstein 1951; Davis 1958; Miller 1962). The grains are presumed to be paramagnetic and to have a complex index of refraction. Jones and Spitzer (1967) used statistical arguments to arrive at the same conclusions.

In the absence of a magnetic field, a prolate grain in kinetic equilibrium with the surrounding gas will have equal rotational kinetic energies about each of its principal axes. The angular momentum about each principal axis is thereby proportional to the square root of the moment of inertia about that axis. A grain, therefore, tends to rotate mainly about an axis perpendicular to the axis of symmetry. In the presence of a magnetic field, dissipation of angular momentum due to magnetic torques will tend to align the axis of rotation with the direction of the field. Thus, the axis of symmetry (major axis) of the prolate grains will tend to be perpendicular to the magnetic field. [It should be emphasized, however, that alignment itself with respect to the field does not necessarily depend on the presence of dissipative torques (see Spitzer 1978, pp. 183-190).] It is essential in these considerations that the grain temperature be different from (in fact, less than) the gas temperature, so that the system will not be in thermodynamic equilibrium, which would destroy the alignment through collisions with gas atoms. The magnetic field needed to sufficiently orient the grains is of the order of 10 μgauss although a weaker field (1 μgauss) would do if the grains were ferromagnetic (Jones and Spitzer 1967). The value 10 μgauss is larger by a factor of 3 - 4 than the mean field strength obtained by reliable Faraday rotation measurements (see §2.3 below).

As starlight propagates through interstellar space, the component of the electric field perpendicular to the major axis of each grain (and, therefore, nearly parallel to the magnetic field) is less efficiently absorbed by these particles. Consequently, a map of the observed polarization vectors will also reveal the topology of the interstellar magnetic field (more precisely, of its perpendicular component) between the observed stars and the earth. The field lines, as unveiled by polarization measurements, exhibit an orderly large-scale behavior,

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with prominent arches over distances of a few hundred parsecs (Mathewson and Ford 1970; Davis and Berge 1968). The large-scale structure of the field correlates strongly with that of atomic hydrogen (Heiles and Jenkins 1976). Clouds and cloud complexes lie in valleys of the field lines, and arches of matter rising high above the Galactic plane coincide with the magnetic arches revealed by starlight polarization measurements.

On a smaller scale, that of individual clouds, the polarization vectors through a cloud merge smoothly with those observed in the vicinity of the cloud (Dyck and Lonsdale 1979). Polarization observations of several dark clouds by Vrba, Strom, and Strom (1976), as extended by a recent study of the R Corona Austrina dark cloud and its neighborhood by Vrba, Coyne, and Tapia (1980), support the same conclusion; the densities probed are less than about 10^3 cm^{-3}. Thus, the assumption of theoretical calculations made mainly by Mestel and his co-workers and Mouschovias and his collaborators (namely, that a cloud's magnetic field links smoothly with the field of the external medium) is beginning to find an observational foundation. The observed ordering of the field over the cloud also seems to rule out the simplifying assumption made sometimes by theorists and observers, namely, that magnetic fields in clouds are tangled up and that their effect can be represented merely by the scalar pressure $B^2/8\pi$.

In order to obtain the magnitude of the field from extinction and polarization measurements, one must know the gas temperature and density and, in addition, less certain quantities such as the shape, composition and temperature of grains. Although our understanding of the nature and evolution of interstellar grains has improved significantly (see reviews by Aannestad and Purcell 1973; Spitzer 1978), it is still premature to put much faith in field strengths deduced from polarization observations; they should only be regarded as order-of-magnitude estimates. In any case, Vrba et al. (1980) estimate a field strength of about 120 μgauss (which seems to us too high) at a density of about 350 cm^{-3}, and they find that the field scales with the gas density as $B \propto \rho^{0.38}$. Although they point out that such scaling is consistent with theoretical predictions (Mouschovias 1976b), which show an exponent 1/3 - 1/2 in the cores of dense clouds, the theoretical predictions should by no means be regarded as having been confirmed by these observations. The accurate (but somewhat conservative) conclusion is that those calculations have not been contradicted by observations yet.

2.3 Faraday Rotation

A tenuous plasma becomes optically active (or, birefringent) in the presence of a magnetic field. Faraday rotation refers to the rotation of the plane of polarization of a linearly-polarized electromagnetic wave, or to the rotation of the major axis of an elliptically-polarized wave, passing through such a medium. The angle of rotation over a distance L is given by (Spitzer 1978, p. 66)

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$$\Delta\theta = \lambda^2 R_m \equiv \lambda^2 (0.81 \int_0^L ds\, n_e\, B \cos\phi) \quad \text{rad}, \tag{1}$$

where the wavelength (λ) is measured in meters, the electron density (n_e) in cm^{-3}, the magnetic field (B) in μgauss, and the distance along the line of sight (s) in parsecs. The angle between the field $\vec{B}$ and the propagation vector $\vec{k}$ is denoted by ϕ. The sign convention is that $\Delta\theta$ is positive for right-hand rotation along the direction of propagation. The rotation measure is denoted by R_m.

Typical rotation measures for the interstellar medium fall in the range 1 - 100 rad m^{-2}. It is therefore clear that Faraday rotation is negligible for optical wavelengths. In principle, one can use optical polarization to establish a standard and then measure $\Delta\theta$ for radio waves. Unfortunately, not many radio sources emit in the optical. To obtain R_m (see discussion by Davis and Berge 1968), one must measure $\Delta\theta$ for at least two radio wavelengths. However, because of the indistinguishability of rotation angles differing by π and because the position angle of the plane of polarization at the source is not usually known, one must measure $\Delta\theta$ at several wavelengths, plot the observed position angle as a function of λ^2, and fit a straight line through the points. In principle, several points differing by multiples of π must be plotted for each observation and that set must be selected which fits a straight line best. The slope of the line gives R_m, and its extrapolation to $\lambda^2 = 0$ gives the position angle at the source.

Once the rotation measure is obtained, one may determine the mean value of the magnetic field along the line of sight to the observed radio source only if the distance to the source and the interstellar electron density are known. To obtain the latter would have been very difficult without the discovery of pulsars. Regular signals from pulsars reaching the earth exhibit a dispersion effect (i.e., a delay in the arrival time of a signal as the wavelength increases) that can be precisely measured. This is given by

$$\Delta t = \lambda^2 (4.60 \times 10^{-2} \int_0^L ds\, n_e) \quad \text{sec}, \tag{2}$$

where λ is measured in meters, s in parsecs, and n_e in cm^{-3}. The dispersion measure, $D_m \equiv \int_0^L ds\, n_e$ pc cm^{-3}, is obtained by a simple measurement and constitutes a direct measure of the column density of electrons along the line of sight. If R_m and D_m are measured for the same source, one can obtain the mean value of the magnetic field along the line of sight, $\langle B_{||} \rangle$, weighted by the electron density. (The contribution of the earth's ionosphere is properly subtracted.) Reversals in the direction of the field would produce cancellations in $\Delta\theta$, so that the measured $\langle B_{||} \rangle$ would be smaller than its value in the general interstellar medium.

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Faraday rotation measures have also been obtained and analyzed for many extragalactic radio sources (Morris and Berge 1964; Gardner and Davies 1966; Gardner, Morris and Whiteoak 1969; Wright 1973). Although these observations have the advantage that extragalactic radio sources, unlike pulsars, are distributed all over the celestial sphere, in a strict sense they only yield $<n_e B_{||}>$, rather than $<B_{||}>$ itself, since an independent determination of $<n_e>$ is not usually made.

Wright (1973) analyzed the rotation measures from 354 extragalactic radio sources, and Manchester (1974) did the same for 38 pulsars. Their results are in good agreement and indicate a large-scale magnetic field directed toward $\ell \approx 90°$ both above and below the Galactic plane. This direction of the field is in fair agreement with that determined by Appenzeller (1968) from interstellar polarization observations of stars near the south Galactic pole. He found that the mean direction of the polarization vectors was $\ell \approx 80°$. According to these workers, the local helical field, which was suggested in order to explain the starlight polarization data (Hornby 1966; Mathewson 1968; Mathewson and Nicholls 1968; Mathewson 1969), is in conflict with the Faraday rotation observations. This resolved a long-standing theoretical dilemma: a nonvanishing magnetic field in the Galactic plane, having opposite directions above and below, would imply that there exists a current sheet in the plane.

The magnitude of the field determined from Faraday rotation measurements lies in the range 1 - 3 μgauss. Superposed on the background field, both Wright and Manchester found field "irregularities" with a typical scale of a few hundred parsecs and with field strength comparable to that of the background field. The "irregularities" are reminiscent of the data of Gardner, Whiteoak and Morris (1967) on rotation measures of extragalactic radio sources, which they interpreted as the result of field lines protruding from spiral arms at least at some regions. A theoretical explanation will be provided in §4.3.

2.4 The Zeeman Effect

The splitting of the 21-cm line into three components in the presence of a weak (on laboratory standards) magnetic field allows direct observation of the interstellar field, at least in H I clouds. The frequency separation between the two shifted, or σ, components of the line depends only on the component of the magnetic field in the direction of propagation, and is given by

$$\Delta\nu \equiv \Delta\nu_{+1} - \Delta\nu_{-1} = \frac{e\ B \cos\phi}{2\ \pi\ m_e\ c} \ , \tag{3}$$

where the subscripts +1 and -1 refer to values of the azimuthal quantum number m_F', and the rest of the symbols have their conventional meaning. The split $\Delta\nu$ is equal to 2.8 Hz per μgauss for propagation along the field ($\phi = 0$). Since line widths are typically measured in kHz, observations of the Zeeman effect in hydrogen are very difficult, and

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special techniques become necessary (e.g., see Davis and Berge 1968, pp. 762-765 for an excellent discussion and for the reason why the transverse Zeeman effect is even more difficult to detect). As in the case of Faraday rotation, only the mean component of the field along the line of sight is measured. However, fields measured through Zeeman observations are indicative of conditions in the interiors of interstellar clouds, rather than being representative of the ambient interstellar field. This is due to the expected enhancement of the field upon cloud contraction (see below). Field strengths ranging from a few to about 50 μgauss have been measured in a number of normal H I clouds (Verschuur 1971 and references therein; review and evaluation by Mouschovias 1978, Fig. 1).

Zeeman observations have also been undertaken on the 1667 MHz (Turner and Verschuur 1970; Crutcher et al. 1975), 1665 MHz (Beichman and Chaisson 1974), and 1720 MHz lines of OH (Lo et al. 1975). By and large, upper limits of a few hundred microgauss are set, and some positive detections of a few milligauss have been reported in maser regions. The paradoxes raised by several observers (e.g., Beichman and Chaisson (1974), Lo et al. (1975), and Heiles (1976)) concerning theoretical expectations and observed strengths or upper limits of the interstellar magnetic field in dense clouds do not arise if the theoretical predictions of self-consistent calculations (Mouschovias 1975b; 1976a,b) on the nonhomologous contraction and equilibria of self-gravitating clouds are adopted. These calculations determined the precise value of the exponent κ in the relation between the field strength and the gas density, $B \propto \rho^{\kappa}$. A reasonable approximation of the exact result, in a cloud core, is given by

$$(B_c/B_{bk}) = (n_c/n_{bk})^{\kappa}, \qquad 1/3 \lesssim \kappa \lesssim 1/2, \tag{4}$$

where B_{bk} and n_{bk} are the "background" values of the field and the gas number density, respectively. For example, B_{bk} = 3 μgauss and n_{bk} = 1 cm^{-3} (the approximate mean interstellar particle density). Thus, milligauss fields should be found only in regions of density $\sim 10^6 - 10^9$ cm^{-3}.

It is clear that it would be extremely difficult, if at all possible, to obtain and plot much needed isopedion (i.e., equal-magnetic-field-strength) contours with the above traditional methods of measuring magnetic fields in interstellar clouds. New ways to detect magnetic fields will have to be invented.

3. BASIS FOR THE RELEVANCE OF THE INTERSTELLAR MAGNETIC FIELD

A necessary, but not sufficient, condition for the magnetic field to be important in interstellar gas dynamics (and, therefore, in star formation) is to exert forces on (or, equivalently, to impart momentum to) the bulk of the interstellar matter which are comparable in

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magnitude with other forces exerted on the matter (e.g., by gravitational and stellar-radiation fields, and thermal, turbulent, and cosmic-ray pressure gradients). Such forces can be estimated only indirectly. One therefore settles for a comparison of (scalar) energy densities present in interstellar space. The energy density in the magnetic field, as obtained from the observed field strength discussed in the preceding section, is comparable with the other interstellar energy densities. This by itself, however, does not establish the dynamical importance of the magnetic field. The additional condition that the decay time of the field (e.g., due to the finite electrical conductivity of the interstellar gas) be long compared with other dynamical times (or typical life times of interstellar clouds) must be satisfied. A finite electrical conductivity causes a decay (ohmic dissipation) of the field in a stationary medium at a rate given by $\partial\vec{B}/\partial t = (c^2/4\pi\sigma)\nabla^2\vec{B}$, from which it follows that the characteristic time is $\tau_{ohmic} = 4\pi\sigma L^2/c^2$. The quantity $\sigma \sim 10^7\ T^{3/2}$ (Spitzer 1962) is the electrical conductivity; c is the speed of light in vacuum; and L is the scale length over which B varies significantly --it is commonly taken as the size of the system. To obtain as small a value for τ_{ohmic} as possible, we use $T = 10°K$ and $L = 0.1$ pc, which are typical for dark clouds. Then, $\tau_{ohmic} \gtrsim 10^{16}$ years >> age of the universe. Ohmic dissipation of the field can therefore be neglected.

There is an additional process which can render the field dynamically insignificant. Since the bulk of the interstellar matter is electrically neutral, it will be affected by magnetic forces only insofar as it is collisionally coupled to the ionized matter (ions, electrons, and charged grains), which is attached to the magnetic field. If such coupling is so efficient that no significant diffusion of the plasma (and the field) relative to the neutrals takes place within a dynamical time scale, the magnetic field is said to be "frozen in" the (neutral) matter. It varies in time only due to gas motions, in accordance with the ("flux freezing") equation $\partial\vec{B}/\partial t = \nabla \times (\vec{v} \times \vec{B})$; where $\vec{v}$ is the gas velocity. The full strength of the magnetic forces is transmitted to the neutrals in this case. On the other hand, if momentum exchange between ions and neutrals (the electrons contribute negligibly due to their small mass, and grains become important only for neutral densities $\gtrsim 10^9\ cm^{-3}$) is not very efficient, a relative drift velocity will be set up between the plasma and neutrals, and magnetic forces will not affect the bulk of the neutral matter. This process is referred to as "ambipolar diffusion" (Mestel and Spitzer 1956) because electrons and ions, tied to the field, diffuse together through the neutral matter (Spitzer 1978). It is commonly expected that such decoupling of the field from the neutral matter will occur only in the very dense molecular clouds in which the degree of ionization falls below 10^{-8}. We shall first review the dynamical effects of magnetic fields frozen in the matter, and then discuss our current understanding (and misunderstanding) of the process of ambipolar diffusion.

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4. FORMATION OF INTERSTELLAR CLOUDS

Although the issue of how interstellar clouds form has received much attention during the last two decades and although significant physical problems (e.g., thermal instability, magnetic Rayleigh-Taylor instability) have been solved in the process, it is fair to state that cloud formation remains an outstanding problem in theoretical astrophysics. A clue on the nature of the mechanism responsible for the formation of interstellar clouds can be obtained if observations can determine whether clouds (atomic and molecular) are confined to spiral arms or whether they also exist in the interarm region. The physical conditions (density, temperature, strength of magnetic field) are so different in the two regions that a prospective cloud formation mechanism may be eliminated in one or the other region from the outset.

There is little doubt today that atomic-hydrogen clouds ($n \sim 20$ cm^{-3}, $T \sim 80°K$, $M \lesssim 10^3\ M_\odot$) are concentrated in spiral arms, mainly in cloud complexes found in "valleys" of magnetic field lines. If they existed as such in the interarm region, they would be seen in absorption against background continuum radio sources. One cannot ignore the well-known observational evidence, however, that the Magellanic Clouds (two nearby irregular galaxies; type Irr I) contain interstellar clouds (and young stars) without any evidence for spiral structure (or for much dust). Irrespective of the nature of the mechanism responsible for the formation of H I clouds, the conclusion seems to follow that physical conditions in spiral arms are much more condusive to cloud formation than conditions elsewhere. The issue of whether molecular clouds are also concentrated in spiral arms is a very controversial one. Scoville, Solomon and Sanders (1979) conclude that no such concentration exists, while Few (1979) arrives at the opposite conclusion. The difference seems to be due to ambiguities in kinematic distances when random or systematic but noncircular (galactocentric) cloud motions are present. Blitz and Shu (1980) have recently reviewed the issue and have made the following additional point. Since interstellar dust seems to be concentrated in (and, in fact, to trace) spiral arms of external galaxies, and since it is thought that a large fraction of the interstellar dust is in molecular clouds, molecular clouds themselves must be concentrated in spiral arms. It seems certain that the last word on the subject has yet to be added. It is nevertheless the case that, given the present state of the observational evidence, if cloud formation in spiral arms is understood, the bulk of the problem will have been solved.

4.1 Jeans Instability

The oldest available mechanism which may possibly account for cloud formation is the Jeans instability (Jeans 1928; Chandrasekhar and Fermi 1953) which refers to the development of self-gravitating condensations in an infinite, uniform medium, threaded by a uniform magnetic field. Only scale lengths λ longer than the Jeans wavelength,

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$\lambda_J = 1.23 \times 10^3\ (T/6000°K)^{1/2}\ n_H^{-1/2}$ parsecs, parallel to the field lines, and longer than $\lambda_J(1 + a)^{1/2}$ perpendicular to the field lines can become unstable. The quantity a is defined by $a = 0.47\ (B/3\ \mu G)^2/(T/6000°K)\ n_H$, and the hydrogen number density is measured in cm^{-3}; a 10% helium abundance has been accounted for. (The temperature 6000°K is near the equilibrium temperature of a gas with $n_H \approx 1\ cm^{-3}$, $n_e \approx 10^{-2}\ cm^{-3}$, which is heated by cosmic-ray ionization at the rate $10^{-17}\ sec^{-1}$ and cooled by e-H collisions.) The e-folding time of the instability along field lines is given by $\tau_{Jeans} = [4\pi G\rho\ (1 - h^2)]^{-1/2} = 2.3 \times 10^7\ [n_H\ (1 - h^2)]^{-1/2}$ years, where $h \equiv \lambda_J/\lambda \leq 1$. The requirement that τ_{Jeans} be less than 3×10^7 years (otherwise stars formed by the collapse of such clouds would appear too far downstream from a galactic shock, contrary to observations --see Roberts 1969) implies that $h \leq 0.64$; i.e., $\lambda \geq 1.56\ \lambda_J \approx 1.9$ kpc. To gather matter from such long distances into a region even as large as 100 pc (a large cloud indeed) within a time 30 million years, speeds of about 60 km s^{-1} are required. They much exceed typical free-fall velocities, and they are certainly not observed over such extended regions in the Galactic plane. Attempts to attribute cloud formation to a Jeans instability bear the additional burden of explaining the lack of central concentration (the signature of self-gravity) in H I clouds and complexes (and, possibly, in molecular cloud complexes as well [Blitz and Shu 1980]) and the fact that most normal H I clouds are not even self-gravitating, as evidenced by their relatively low densities and masses.

4.2 Thermal Instability

A thermal instability (Field 1965) is a second candidate mechanism for cloud formation. Spitzer (1951) first suggested that relatively cold and dense interstellar clouds are in pressure equilibrium with a hot and tenuous intercloud medium. Observational support for the existence of the latter was found by Heiles (1968), who obtained a density of 0.2 cm^{-3} and a velocity dispersion of 6 km s^{-1}, implying an upper limit on T of several thousand degrees. Theoretical work (Hayakawa, Nishimura and Takayanagi 1961; Field 1962; Pikelner 1967; Field, Goldsmith and Habing 1969; Spitzer and Scott 1969) established that such two thermally stable, nearly isothermal phases can exist in pressure balance. The feature of the calculations needed for the establishment of this conclusion is a heating mechanism proportional to the gas density (e.g., cosmic-ray ionization and heating by the produced secondary electrons) and a cooling mechanism proportional to the second power of the gas density (e.g., collisional excitation followed by radiative de-excitation in spectral lines at which the medium is optically thin). Thus, although some of the assumptions (e.g., that the same agent is responsible for both ionization and heating) and conclusions of the above calculations have since been superceded by observations, their main conclusion (namely, the coexistence of two stable phases of interstellar matter in pressure equilibrium) remains valid.

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Matter can "condense" from the intercloud to the cloud phase if the density of the nearly isothermal intercloud gas increases beyond some critical value, thus causing a rise in pressure that cannot be maintained. The critical point marks the onset of a thermal instability (Field 1965) which proceeds almost isobarically; the denser gas cools faster and gets more compressed until the stable cloud phase is reached. This transition relieves the initial excess of pressure so that the ambient pressure is maintained at the critical value (Field et al. 1969). The isobaric nature of the condensation mode implies an upper bound on the fastest-growing wavelengths of a perturbation. It is approximately that distance within which a sound wave can establish pressure equilibrium in a time not exceeding the cooling time ($< 10^6$ yr) of the medium. Since the sound speed is $\lesssim 10$ km s^{-1}, the wavelengths which can grow at a rate near maximum will be $\lesssim 10$ pc. The final size of the resulting condensation is, of course, much smaller than this (< 0.1 pc), and involves only a fraction of a solar mass (see review by Mouschovias 1978, Appendix A). Wavelengths much larger than 10 pc (i) grow nearly isochorically and, therefore, cannot explain the observed cloud densities; (ii) grow at a rate slower than the magnetic Rayleigh-Taylor instability (see below), in which the magnetic field is instrumental, rather than a nuisance, in the formation of large condensations. (Field [1965] has shown that a field strength as small as 1 μgauss prevents the development of the thermal instability, except in a direction parallel to the field.) The above conclusions concerning the inability of thermal instability to form any but dwarf clouds remain valid (Mouschovias 1975b; 1978) even if one considers its development in a cooling medium, which is periodically heated by supernovae (e.g., Schwartz, McCray and Stein 1972).

4.3 Magnetic Rayleigh-Taylor (or, Parker) Instability

A light fluid can support a heavy fluid against a vertical (downward) gravitational field (assumed to be constant for simplicity) if the interface is perfectly horizontal. Deformations of the interface, however, grow, as fingers of heavy fluid protrude downward into the light fluid, thus reducing the energy of the system. Shorter wavelengths along the interface tend to grow faster than longer wavelengths. This is a classical Rayleigh-Taylor instability. It can also develop if the downward gravitational field is replaced by an upward acceleration of the heavy fluid by the lighter one. This instability may be responsible for the observed protrusions of cold neutral matter into adjacent H II regions. The nature of the instability changes if a frozen-in, horizontal magnetic field plays the role of the light fluid in supporting the gas against the gravitational field. The light fluid (the field) and the heavy fluid (the gas) now coexist in the same region of space, and analogies with the nonmagnetic case break down. The nature of the magnetic Rayleigh-Taylor instability has been worked out by Parker (1966) in the context of the interstellar medium. Is it a viable mechanism for the formation of interstellar clouds? In other words, are the unstable wavelengths of the proper size (a few hundred

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parsecs), and are the corresponding e-folding times short enough ($\lesssim 3 \times 10^7$ yr)? In addition, what final densities are achieved?

The pressure scale height of warm ($T \approx 6000°K$) interstellar gas in the (constant, for simplicity) vertical galactic gravitational field ($g \approx 3 \times 10^{-9}$ cm s^{-2}) is $C^2/g \approx 42$ pc; where $C = (kT/\mu m_H)^{1/2}$ is the isothermal speed of sound; k the Boltzmann constant; m_H the mass of a hydrogen atom; and μ the mean mass per particle in units of m_H ($\mu = 1.27$ to account for $n_{He}/n_H = 0.1$). This is smaller than the observed scale height by at least a factor of 3 --possibly, 4. Pressure due to the nearly horizontal frozen-in magnetic field and due to cosmic rays, which are tied to the field, provide additional support to the gas against the galactic gravitational field. Under the simplifying assumption, introduced first by Parker, that these pressures are proportional to the gas pressure (i.e., $\alpha \equiv B^2/8\pi P =$ const., and $\beta \equiv P_{CR}/P =$ const.), the scale height of the gas now becomes $H = (1 + \alpha + \beta) C^2/g$. With $\alpha \approx \beta \approx 1$, a value consistent with observations, one finds that $H \approx 126$ pc --not an unreasonable value. Parker showed, through a linear stability analysis, that this one-dimensional equilibrium state is unstable with respect to deformations of the field lines. The vertical gravitational field acquires a component along a deformed (non-horizontal) field line, thus causing gas to slide along the field line from a raised into a lowered portion. The unloading of gas from the raised portion leaves magnetic and cosmic-ray pressure gradients unbalanced in that region, thereby causing further inflation of the already raised portion of a field line. The component of gravity along the now more vertical field line is larger, with the result that gas can be unloaded more efficiently into the "valley" of the field line. The process will stop only when field lines have inflated enough for their tension to balance the expansive magnetic and cosmic-ray pressure gradients (Mouschovias 1974, 1975a). In this picture, the matter which accumulates in valleys of the field lines represents interstellar clouds. Or, does it?

For the instability to develop, the horizontal (along the field lines) and vertical wavelengths of a perturbation must simultaneously satisfy the following respective inequalities:

$$\lambda_y > \Lambda_y \equiv 4\pi H\{\alpha\gamma/[2(1+\alpha+\beta-\gamma)(1+\alpha+\beta)-\alpha\gamma]\}^{1/2}, \quad (5a)$$

and

$$\lambda_z > \Lambda_z(\lambda_y) \equiv \Lambda_y/[1 - (\Lambda_y/\lambda_y)^2]^{1/2} . \quad (5b)$$

For the interstellar gas $\gamma \equiv d\log P/d\log\rho \approx 1$. If $\lambda_y < \Lambda_y$, the radius of curvature of a typical, deformed field line is small, hence the tension is large and it straightens the field line out --a stable regime. If $\lambda_z < \Lambda_z(\lambda_y)$, the system is stable even though λ_y may exceed its critical value Λ_y. The physical reason lies in the fact that the volume available for the field lines to expand in, and thereby decrease

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the magnetic energy of the system, is limited. The increase in the field strength in the valleys and the pile-up of field lines near the first undeformed field line, which forms a natural "lid" to the system below, represent an increase in magnetic energy which suppresses the instability (see Mouschovias 1975b for a detailed discussion). For a fixed $\lambda_y > \Lambda_y$, the growth rate of the perturbation increases monotonically as λ_z ($> \Lambda_z$) increases. For a fixed $\lambda_z > \Lambda_z$, the growth rate first increases and then decreases as λ_y increases. Equation (5a) shows that the horizontal critical wavelength for $\alpha \approx \beta \approx \gamma \approx 1$ is $\Lambda_y = 1.2\ \pi H \approx 477$ pc. The maximum growth rate occurs at $\lambda_y = 1.8\ \Lambda_y = 2.2\ \pi H \approx 868$ pc and $\lambda_z = \infty$, and its inverse (the e-folding time) is given by

$$\tau_{min} = 1.1\ H/C \qquad (6a)$$

$$= 2.2 \times 10^7\ (T/6000°K)^{1/2}/(g/3 \times 10^{-9}\ \text{cm s}^{-2}) \quad \text{yr}. \qquad (6b)$$

This growth time is short enough to be relevant for cloud formation behind a spiral density shock wave. In fact, it may be smaller than the value given in equations (6a,b) because, in a strict sense, the quantity H is the scale height in the initial state; not its value today. It has been shown by exact determination of final equilibrium states for the Parker instability that, in the valleys of field lines, $H_{final} \approx 1.7\ H_{initial}$ (Mouschovias 1974, Fig. 2b). This implies that $\tau_{min} \approx 1.3 \times 10^7$ yr and $\lambda_y(\tau_{min}) \approx 511$ pc. A further decrease in τ_{min} can take place due to the fact that the instability is externally driven by a spiral density shock wave (Mouschovias 1975b, p. 73). There is yet another reason for which τ_{min} can decrease further. Giz and Shu (1980) took into consideration the actual variation of g with z, and found that the value of g which enters equation (6b) is larger than the one given above by a factor of 3. The amount of matter involved in a cylinder (along a spiral arm) of length 511 pc and diameter 250 pc (the approximate thickness of a galactic shock, as well as the width of the galactic disk in which most of the gas is found) is $8.6 \times 10^5\ M_\odot$. Thus the Parker instability is most suitable for the formation of large-scale condensations (or, cloud complexes), rather than individual interstellar clouds (Mouschovias 1974; Mouschovias, Shu and Woodwood 1974). The implosion by shock waves of individual clouds within these complexes can give rise to OB associations and giant H II regions, all aligned along spiral arms "like beads on a string" and separated by regular intervals of 500 - 1000 pc, in agreement with observations both in our galaxy and in external galaxies (Westerhout 1963; Kerr 1963; Morgan 1970; Hodge 1969). The nonlinear development of the magnetic Rayleigh-Taylor instability and the final equilibrium states which we have calculated also explain the large-scale intimate association between the interstellar gas and field, as revealed by the combined observations of Mathewson and Ford (1970) and Heiles and Jenkins (1976) discussed in §2.2.

In the direction $\vec{g} \times \vec{B}$ (the "third direction"), wavelengths ranging from a very small fraction of the vertical scale height H to

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many times H can grow with almost identical growth rates (Parker 1967). If some mechanism (other than a galactic shock) selects wavelengths $\lambda_x \sim 10$ pc in this direction, then the mass involved would be only slightly larger than 10^4 $M_\odot$. This begins to approach masses of individual clouds. Whether in fact individual clouds can form by the Parker instability will be decided only when nonlinear three-dimensional calculations are carried out. Although there is a wealth of ideas on how phase transitions and conversion of atomic to molecular hydrogen in the valleys of field lines can convert the non-gravitating clumps of gas into dense molecular cloud complexes (e.g., Field 1969; Mouschovias 1975b, 1978; Blitz and Shu 1980), no quantitative calculation has been produced yet. Initial perturbations of the field lines which have an odd symmetry about the Galactic plane are more likely to initiate the necessary phase transitions. Such perturbations allow field lines originally coinciding with the Galactic plane to deform, and they therefore can lead to a gas density (and pressure) in the Galactic plane significantly higher (a necessary condition for phase transitions) than its value in the initial (unstable) equilibrium state. Perturbations with even symmetry about the Galactic plane can lead to such phase transitions only under special circumstances (Mouschovias 1975b, pp. 55-57).

5. CONTRACTION, EQUILIBRIUM, AND COLLAPSE OF SELF-GRAVITATING CLOUDS

5.1 Criteria for Collapse

Irrespective of the nature of the mechanism responsible for cloud formation, the contraction, the available equilibrium states (if any), and the collapse of a cloud are significantly affected by a frozen-in magnetic field of initial strength comparable with the observed values (see §2). (Contraction ceases when an equilibrium state is reached. Collapse refers to indefinite contraction past the last available equilibrium state, at least as long as the cloud can be described by the same set of equations which was applicable when collapse began.) Since we are interested here in the diffuse stages of cloud collapse and star formation, $n < 10^9$ cm^{-3}, isothermality is a good approximation. Let us first recall that an isothermal, spherical, nonmagnetic cloud, which is bounded by an external pressure P_{ext} can collapse if its central density ρ_c exceeds the surface density ρ_S by at least a factor 14.3 (Bonnor 1956; Ebert 1955, 1957). The mass and radius at this critical ("Bonnor-Ebert") equilibrium state are given by $M_{BE} = 1.2\ C^4/(G^3P_{ext})^{1/2}$ and $R_{BE} = 0.41\ GM/C^2$, respectively. Thus, an isothermal, nonmagnetic cloud will have no accessible equilibrium states if its mass exceeds $M_{BE} = 2.0 \times 10^3\ (T/80°K)^2\ (\mu/1.27)^{-2}\ (P_{ext}/1600k)^{-1/2}\ M_\odot$. For molecular clouds, $\mu = 2.33$ and $T \approx 40°K$, yielding $M_{BE} \approx 1.5 \times 10^2\ M_\odot$ for the same P_{ext} as above. If thermal pressure were the only means of support of molecular clouds against self-gravity, all of them would be collapsing.

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If an interstellar cloud were to form out of a medium of density ρ_i and magnetic field B_i through spherical isotropic contraction, conservation of mass ($\rho \propto R^{-3}$) and flux ($B \propto R^{-2}$) would imply that $B \propto \rho^{2/3}$. Such a model was employed for a study of the support that magnetic forces can provide against self-gravity (Mestel 1966). A spherically-symmetric density distribution, $\rho(r) = \rho_i + \rho_c \exp[-(r/r_o)^2]$, was assumed; where ρ_c (if $>> \rho_i$) is the central density, and r_o is a radius beyond which ρ decreases rapidly to its background value, ρ_i. Thermal pressure was ignored. Then the magnetic field implied by this density distribution was calculated. Near the center of the cloud ($r << r_o$) the field is nearly uniform and equal to $B_i (\rho/\rho_i)^{2/3}$. In an intermediate region [$1 << r/r_o << (\rho_c/\rho_i)^{1/3}$] the field is almost radial. At larger radii, $r/r_o >> (\rho_c/\rho_i)^{1/3}$, the field becomes uniform and equal to B_i. The nearly radial field,which is solely the result of the imposed spherical contraction, causes large "pinching" forces at the equator --so much so that magnetic forces much exceed gravitational forces. Mestel argues that, if this configuration is achieved through rapid, violent contraction of the cloud, flux dissipation, reconnection and detachment of field lines will take place in the equatorial plane. He points out, however, that preferential flow of matter along field lines might prevent such configuration from being reached. The significant result, and main objective, of this calculation is a criterion for the lateral collapse of the cloud: If the total mass-to-flux ratio exceeds the critical value $(M/\Phi_B)_{crit} = 0.152\ G^{-1/2}$, the gravitational forces exceed the magnetic forces at the equator so that further contraction will ensue. (The quantity G is the universal gravitational constant.)

Criteria for the collapse of magnetic clouds which turn out to be similar with the one found by Mestel can be obtained from the Virial Theorem (Chandrasekhar and Fermi 1953; Mestel 1965; Strittmatter 1966; Spitzer 1968). The advantage of the Virial Theorem lies in the fact that it, being an integral relation, washes out the complex details of the structure of the system. That is also its greatest disadvantage because no conclusion on the internal structure of the system can be arrived at. At times, the Virial Theorem can also lead to misleading results (see discussion by Mestel 1965; Mouschovias 1975b). One would like to obtain a reliable collapse criterion for magnetic clouds analogous to the Bonnor-Ebert condition for nonmagnetic clouds; i.e., by studying exact equilibrium states.

D. A. Parker (1973, 1974) obtained equilibrium states for self-gravitating, magnetic clouds bounded by external pressure. These are true equilibria in that forces are in exact detailed balance, rather than only in an average sense. However, because of his neglect of the constraint of flux-freezing he had to adopt an ad hoc assumption concerning the form of an arbitrary function of the magnetic flux in order to close the system of the magnetohydrostatic equations. This procedure does not allow one to study a sequence of states that can evolve from one to another through continuous deformations of the field lines,

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or to quantify the effectiveness with which the magnetic field can prevent the gravitational collapse of a cloud.

Solution of the self-consistent equilibrium problem for isothermal, pressure-bounded, self-gravitating, magnetic clouds yielded a number of critical states for gravitational collapse under a wide variety of initial values for thermal, gravitational and magnetic energies (Mouschovias 1976a,b). Virial-theorem expressions (Spitzer 1968) were then used as interpolation/extrapolation formulae to determine, after fixing the virial constants so as to obtain agreement with the exact results, critical states for any arbitrary set of initial physical parameters (Mouschovias and Spitzer 1976). The critical mass-to-flux ratio is $(M/\Phi_B)_{crit} = 0.126\ G^{-1/2}$, which can be written in terms of the magnetic field B and the number density of protons n_0 in the cloud in the convenient form

$$M_{crit} = 5.04 \times 10^5 \frac{(B/3\ \mu G)^3}{(n_0/1\ cm^{-3})^2} \qquad M_\odot\ ; \tag{7a}$$

$$= 5.04 \times 10^5\ n_0^{-(2-3\kappa)} \qquad M_\odot\ . \tag{7b}$$

To obtain equation (7b), the relation $B \propto \rho^\kappa$ was used. The exact equilibrium calculations showed that $1/3 \lesssim \kappa \lesssim 1/2$ in the core (see eq. [4]). The critical mass given by equation (7a) is only about half the virial-theorem value. A similar reduction was obtained by Strittmatter (1966), but only for cold, infinitely thin clouds. The maximum external pressure which can be applied to a cloud of given mass and flux without resulting in collapse is equal to 0.60 times the value predicted by the virial theorem for a uniform, magnetic cloud. The main reason for the reduction in the critical values for mass and external pressure at a given flux is the development of a central concentration in the exact equilibrium states which produces stronger gravitational forces. Equation (7a) can be interpreted as yielding that value of B which can stabilize a cloud of given mass and number density no matter how arbitrarily large the external pressure is. For example, a molecular cloud of mass $10^4\ M_\odot$ and density $n_0 = 2 \times 10^4\ cm^{-3}$, is stabilized by a field strength of 600 μG, at any external pressure. For a finite external pressure, a value of B smaller than the one given by equation (7a) is necessary for stabilization against gravitational collapse.

5.2 Fragmentation

A necessary condition for fragmentation to occur in a contracting or collapsing cloud is a reduction of M_{crit} upon contraction. Once formed, a fragment will maintain its identity only if it can contract or collapse more rapidly than the background. An incisive discussion of the role of magnetic fields in the fragmentation process has been given by Mestel (1965, 1977). Yet, no detailed quantitative calculation has been undertaken on the subject. The basic effect of the field can be

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illustrated by using equations (7a,b). Spherical isotropic contraction/collapse ($\kappa = 2/3$) leaves M_{crit} unchanged; hence, it prohibits fragmentation. Our calculations have determined, however, that the contraction is both nonhomologous and nonisotropic, with $1/3 \lesssim \kappa \lesssim 1/2$ in a cloud's core. The exponent κ increases to values >> 1 at the magnetic poles of the cloud and decreases to zero, and even to negative values, in the equatorial plane toward the equator; we have argued that similar results should be obtained during collapse, even for a cooling, rather than an isothermal, cloud (Mouschovias 1976b; 1978). It follows then from equation (7b) that, as the density increases, M_{crit} decreases most rapidly in the equatorial plane. Consequently, fragments should form there first. Solar-mass blobs can separate out at densities characteristic of molecular clouds. This conclusion is in agreement with observations which show that the mean density of open clusters is comparable with that of molecular clouds. We had also argued that the extreme nonhomology introduced by the magnetic field has the consequence that low-mass stars may form first, and perhaps only, in the cores of dense clouds. Circumstantial observational evidence for that conclusion has been found recently by Vrba, Coyne and Tapia (1980).

The reason for which fragmentation has been regarded as a necessary element of any theory of star formation lies in the observation that young stars seem to form predominantly in groups. Reasonable as it may seem, it should be borne in mind that it is merely a hypothesis. One could conceive of an alternative, the "pre-existing cloudlet hypothesis". Cloudlets ($\lesssim 1\ M_\odot$) may form via a thermal instability behind a spiral density shock wave and then gathered in valleys of the field lines by the Parker instability (see §4.3). Within the available 3×10^7 yr, several of these cloudlets can coalesce (nonviolently) to give blobs of a few tens of solar masses. In the cores of cloud complexes, efficient conversion of atomic to molecular hydrogen can take place, and these cloudlets can also be shielded from ionizing cosmic-rays, and thus can reduce their magnetic flux via ambipolar diffusion (see §7). Since the Bonnor-Ebert critical mass for dark-cloud parameters is less than 9 $M_\odot$, these cloudlets may collapse, especially if imploded by shocks, and form individual stars in a small region of space. In this scenario no fragmentation is necessary, and the initial (stellar) mass function is largely determined by the mechanism responsible for the formation of cloudlets. If this scenario has anything to do with reality, the initial mass function is likely to be very different in different clusters (Mouschovias and Paleologou 1980a).

5.3 Collapse

A recent calculation (Scott and Black 1980) has followed numerically the collapse of a magnetic cloud with initial parameters near those specifying critical states determined earlier (Mouschovias 1976a, b). The strength of the calculation lies in the fact that it is fully time-dependent, and has therefore followed the increase in central density to three orders of magnitude further than our sequences of

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equilibrium states could go. It is not an accident, however, that the main conclusions of Scott and Black agree quantitatively with our earlier conclusions. We had taken great pains to explain the relevance of equilibrium calculations to star formation (Mouschovias 1976a, §Ic). We had shown analytically that the tension of the field lines will halt the collapse of the outlying portion of a cloud and will form extended envelopes. (This has been verified by the numerical collapse calculation.) As a consequence, the cloud flattens along field lines relatively rapidly, until pressure gradients balance the gravitational forces along field lines. Subsequently the cloud contracts only as rapidly as magnetic forces will allow it to contract laterally, with α_c (the ratio of magnetic and thermal pressure in the core) maintained near unity; hence, for an isothermal cloud, $B_c^2 \propto \rho_c$ or $\kappa = 1/2$ (Mouschovias 1976b, p. 151; 1978; 1979a). Scott and Black emphasize that the result $\kappa = 1/2$ in a flattened core is general and independent of initial conditions. This cannot be valid. For example, consider a cold self-gravitating cloud, threaded by a very strong magnetic field. Once released from its initial spherical, uniform state considered by Scott and Black, it will free-fall along field lines to an infinitesimally thin sheet. During such collapse, $\kappa \gtrsim 0$ (because the density increases without a corresponding increase of the magnetic field), even though a flattened core forms. In the case of a collapsing cloud with a non-negligible thermal pressure, the phase $\kappa \gtrsim 1/2$ is expected to be reached in about one (nonmagnetic) free fall time, because that is roughly the time required for pressure gradients to become comparable with gravitational forces due to flattening along field lines.

Both, our earlier analytical argument and the collapse calculations have shown that the tension of field lines near the equator prevents significant contraction from taking place, and thus field lines deform relatively little there. The large "pinching" forces found in Mestel's (1966) non-self-consistent model do not appear, and magnetic reconnection and detachment of the cloud's field from the background during the early phases of cloud collapse is unlikely to take place. Magnetic reconnection inside clouds, if it occurs at all, will play a role at a very advanced collapse stage, long after ambipolar diffusion has set in.

Thus far we have ignored rotation, which a cloud will inevitably acquire, no matter what its formation mechanism is, by virtue of its being in a rotating system, the Galaxy. In the following section we review the effect of frozen-in magnetic fields on rotation and vice versa.

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6. THE ANGULAR MOMENTUM PROBLEM AND MAGNETIC BRAKING

A blob of interstellar matter of mass $\sim 1\ M_{\odot}$ at the mean density of the interstellar medium $\sim 1\ cm^{-3}$ has an angular momentum ($\vec{J}$) a few times 10^{55} g cm^2 s^{-1} by virtue of its participation in the general galactic rotation. On the other hand, a wide binary star system with members of mass $\sim 1\ M_{\odot}$ each, and a period of 100 yr possesses an angular momentum only a few times 10^{53} g cm^2 s^{-1}. Hence, if binary stars are to form through the collapse and fragmentation of interstellar clouds, a mechanism must exist which can transfer angular momentum efficiently from a collapsing cloud or fragment to the surrounding medium. This is "the angular momentum problem" for binary stars. It is more severe for single stars since a typical star possesses only an angular momentum of order 10^{49} g cm^2 s^{-1}.

The few ideas and calculations which aim at resolving the angular momentum problem (e.g., by putting the angular momentum of the parent cloud into the orbital motion of cluster stars) run either into observational difficulties (e.g., why aren't clusters flat?), or into theoretical problems, or both (see reviews by Mouschovias 1978, §IIc; 1979a, §I). The fastest rotating cloud is the globule B 163 SW, with an angular velocity somewhat less than 10^{-13} rad s^{-1} (Martin and Barrett 1978), and the fastest rotating massive cloud is the Mon R2 (Kutner and Tucker 1975), with $\omega \approx 6 \times 10^{-14}$ rad s^{-1} (see also review by Field 1978). If angular momentum is conserved during the contraction of a cloud, the angular velocity should increase with density as $\omega \approx 10^{-15}$ $n^{2/3}$ rad s^{-1}. Thus, typical dark and molecular clouds with densities in the range $10^4 - 10^6$ cm^{-3} should exhibit angular velocities in the range $4.6 \times 10^{-13} - 10^{-11}$ rad s^{-1}. Clearly, this is not the case. Whatever the nature of the mechanism which resolves the angular momentum problem during star formation, it must operate efficiently during the relatively diffuse stages of cloud formation and collapse.

In a paper concerned with "ways in which the cloud can lose its magnetic energy", Mestel and Spitzer (1956) state the following: "If turbulence is negligible ... the angular momentum present leads to disk formation, and the subsequent evolution of the disk is slow enough for the field and plasma to diffuse outwards, inspite of the increased densities. The objection to this is that the strong frozen-in magnetic field will probably remove angular momentum too rapidly; the time of travel of a hydromagnetic wave across the cloud is of the same order as the time of free-fall, and so it is not obvious that a rotating disk will form" (emphasis added). The underlined, qualitative suggestion is the most promising mechanism proposed as yet for the resolution of the angular momentum problem. It is now referred to as the process of "magnetic braking" of a cloud's rotation.

Figure 1 illustrates the principle of magnetic braking. We consider for simplicity a disk-shaped cloud coinciding with the plane z = 0 and threaded by an initially uniform, frozen-in magnetic field

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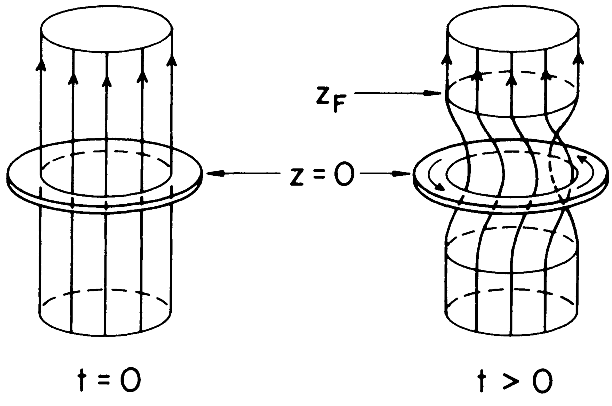

Figure 1. Illustration of the principle of magnetic braking.

(left illustration). At time $t = 0$ the cloud is imparted an arbitrary uniform angular velocity about its axis of symmetry. At a small time later, the situation is as shown in the illustration to the right in Figure 1. The frozen-in field lines (solid lines with arrows) have been twisted by the motion of the cloud. Matter in the external medium adjacent to the cloud surfaces is pulled by the field lines (assuming flux-freezing in the external medium as well) in the direction of motion of the cloud; i.e., it is imparted an angular momentum about the original axis of rotation. Exactly the same argument applied now to this rotating external matter and a layer adjacent to it, further away from the cloud, shows that a rotational disturbance propagates away from the cloud, both above and below, along the z-axis. At any one time there is a "front", shown at position z_F in the figure, beyond which matter is still undisturbed. The speed of propagation of the disturbance (which is referred to as a "torsional Alfvén wave") is the Alfvén speed in the external medium, $v_{A,ext} = B/(4\pi\rho_{ext})^{1/2}$, where ρ_{ext} is the external density. Since the external medium obviously gains angular momentum, which could only come from the original angular momentum of the cloud, the net effect is to slow down the cloud's rotation. We estimate the characteristic time within which a significant amount of angular momentum is transferred from the cloud to the external medium as follows.

A significant angular momentum transfer will take place when the wave fronts have propagated far enough from the cloud to affect a moment of inertia of the external medium comparable with that of the cloud. For a cloud (disk or cylinder) of density ρ_{cl} and half-height Z, this condition is fulfilled when the wave fronts reach a distance $Z_I = Z\ [1 + (\rho_{cl}/\rho_{ext})]$ from the equitorial plane. The time required

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for this to happen (and, hence, the characteristic time for magnetic braking of an aligned disk or cylinder rotator) is

$$\tau_{||} = \int_{Z}^{Z_I} dz/v_{A,ext} = (\rho_{cl}/\rho_{ext})(Z/v_{A,ext}) \; . \qquad (8)$$

It turns out that this is an exact expression for the e-folding time of magnetic braking of an aligned, rigidly rotating disk or cylinder and, if short-lived transients are ignored, an excellent approximation for a differentially rotating disk or cylinder as well, provided that ρ_{cl}/ρ_{ext} is not close to unity (Mouschovias and Paleologou 1980b). (A multiplicative factor 8/15 appears on the right-hand side of equation (8) if one considers a sphere of radius Z or an oblate spheroid of semi-minor axis Z.)

If the field lines are initially radial and perpendicular to the symmetry axis of the disk or cylinder (a "perpendicular rotator"), equality of moments of inertia is achieved when the waves (now propagating perpendicular to the axis) reach a distance from the axis of rotation given by $R_I = R\,[1 + (\rho_{cl}/\rho_{ext})]^{1/4}$, where R is the radius of the cloud. The Alfvén velocity in the external medium is now a function of radial distance r from the axis; namely, $v_A(r) = R\, v_A(R)/r$, where $v_A(R)$ is the Alfvén speed just outside the cloud surface. It therefore follows that the e-folding time for magnetic braking is

$$\tau_{\perp} = \int_{R}^{R_I} dr/v_A(r) = \frac{1}{2}\left[\left(1 + \frac{\rho_{cl}}{\rho_{ext}}\right)^{1/2} - 1\right]\frac{R}{v_A(R)} \; . \qquad (9)$$

This time scale differs from the initial time scale of the exact solution (Mouschovias and Paleologou 1979) by less than a factor of 2. However, it does not reveal the oscillating nature of the approach to corotation of the cloud (or fragment) with the background --which has the important consequence of predicting retrograde rotation in some cloud fragments, and in stellar and planetary systems as a natural consequence of magnetic braking of a perpendicular rotator (see Mouschovias and Paleologou 1980a, §3). The globule B 163 SW, referred to above, exhibits such retrograde rotation. Clark (1980) claims the detection of retrograde rotation in two dense fragments.

The groundwork for calculations on magnetic braking was laid by Ebert, Hoerner and Temesváry (1960), who showed that the disturbances in the external medium obey a wave equation. Kulsrud (1971) obtained expressions, accurate to second order, for the rotational deceleration of stars with dipolar fields. These expressions were adopted by Nakano and Tademaru (1972) to study the loss of angular momentum by interstellar clouds, and also by Fleck (1976) for a similar study. Since the observational evidence in §2.2 and the theoretical arguments concerning magnetic reconnection given in §5.3 indicate that a cloud's magnetic field does not detach from that of the background, the applicability of the results of these studies to interstellar clouds is questionable. The first detailed study of magnetic braking was carried

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out by Gillis, Mestel and Paris (1974), who considered an aligned spherical rotator, with a field-line geometry similar to that of Mestel's (1966) cloud model described in §5.1 above. They assumed that the cloud's angular velocity was constant at all times, and they concluded that magnetic braking was never so efficient as to keep the cloud corotating with the background. A series of analytical but approximate calculations for aligned spherical and oblate rotators (self-gravity was included in the latter) found efficient magnetic braking, which can keep most H I clouds in synchronous galactocentric orbits, and which predicts low angular velocities for molecular clouds compared with expectations based on angular momentum conservation (Mouschovias 1977a, b; 1978; 1979a). A density of about 2.4×10^3 cm^{-3} was predicted above which clouds ought to begin to exhibit angular velocities appreciably higher than 10^{-15} rad s^{-1}, due to the fact that the time scales of magnetic braking and ambipolar diffusion (see §7) become comparable.

An exact calculation for a perpendicular rotator, properly accounting for the time-dependence of the cloud's angular velocity in a manner consistent with the instantaneous magnetic torques exerted on the surface, found a much higher efficiency for magnetic braking than that found for aligned rotators (Mouschovias and Paleologou 1979). The physical origin of this higher efficiency lies in the fact that the rotational waves now set in motion matter at larger and larger distances from the axis of rotation. Hence, a moment of inertia in the external medium comparable with that of the cloud is swept at a much earlier time than in the aligned rotator case --compare $\tau_{\parallel}$ and $\tau_{\perp}$ in equations (8) and (9) for the same $\rho_{cl}/\rho_{ext} >> 1$, $Z \approx R$, and $v_{A,ext} \sim v_A(R)$. It was found that enough angular momentum could be lost in less than about 10^6 yr for binaries to form. The efficiency of magnetic braking increases upon contraction because $\rho_{cl} \propto R^{-2}$ and $v_A(R) \propto R^{-a}$, where $1 \leq a \leq 2$, so that equation (9) yields $\tau_{\perp} \propto R^a$ for $\rho_{cl}/\rho_{ext} >> 1$. On the contrary, for an aligned rotator we have that $\rho_{cl} \propto R^{-2}Z^{-1}$, and $v_{A,ext} \propto R^{-2}$ if ρ_{ext} remains constant and continuity of the field across the cloud surface is assumed; therefore, equation (8) yields that $\tau_{\parallel}$ = const., independent of the stage of contraction. Binaries can form in this case in a time less than 1.4×10^7 yr from the onset of cloud contraction from a density of 1 cm^{-3}.

Gillis, Mestel and Paris (1979) subsequently relaxed their assumption of constant angular velocity of the cloud at all times, and they found an efficiency for magnetic braking similar with that of the aligned rotator described above. Both they and Mouschovias and Paleologou (1980a, b) considered the propagation of nonlinear torsional Alfvén waves within the cloud as well, and followed their numerous partial internal reflections on the cloud surfaces. These waves cause large shearing motions within the cloud which, however, are short lived. Mouschovias and Paleologou compared their solution with an exact solution for a rigid rotator. A rigid rotator can better and better approximate a cloud as contraction goes on. This is so because the Alfvén crossing time decreases upon contraction, thus tending to better

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fulfill the assumption implicit in a rigid-rotator model, namely, that a torque exerted on the cloud surface is communicated instantaneously to any interior point. Mestel and Paris (1979) also employed the virial theorem to study the magnetic braking of a contracting cloud while allowing the cloud's magnetic flux to vary in some arbitrarily prescribed manner, in an effort to simulate the effect of ambipolar diffusion on magnetic braking. They find that clouds with $M > M_{crit}$ contract on a time scale set by magnetic braking, rather than on the free fall time scale. Clouds with $M < M_{crit}$, on the other hand, can contract only if they lose their magnetic flux.

In summary, magnetic braking can resolve the bulk of the angular momentum problem during the diffuse stages of cloud contraction. Clouds should begin to exhibit appreciable rotation above the galactic background for densities in excess of 2.4×10^3 cm^{-3}, due to the fact that ambipolar diffusion, to which we now turn, begins to set in with a time scale comparable with that of magnetic braking for an aligned rotator. Much higher densities will need to be reached for clouds with $\vec{J} \perp \vec{B}$ before they exhibit appreciable rotation above that of their background.

7. THE MAGNETIC FLUX "PROBLEM" AND AMBIPOLAR DIFFUSION

If flux-freezing remained valid all the way up to main-sequence densities, spherical isotropic collapse would result in typical stellar fields $B_* \gtrsim n^{2/3}$ μgauss $\sim 10^{10}$ Gauss, which are much too strong compared to observed values. Flattening along field lines reduces the exponent κ to $1/3 \lesssim \kappa \lesssim 1/2$ in a cloud's core (Mouschovias 1976b), so that a nonspherical collapse yields $B_* \gtrsim 3 \times 10^2 - 3 \times 10^6$ Gauss, if the field remains frozen in the matter. The surface field of the Sun is near the lower limit of the above range. Consequently, observations of relatively weak stellar fields do not constitute compeling evidence for breakdown of flux-freezing during some stage of cloud collapse and star formation. Mestel (1977) also makes this point. Ambipolar diffusion, nevertheless, which was introduced in §3, can lead to a breakdown in flux-freezing at a relatively early stage in cloud collapse ($n \gtrsim 10^4 - 2 \times 10^6$ cm^{-3}) and thus explain the entire range of periods of binary stars from 10 hr to 100 yr (Mouschovias 1977a).

There are only three significantly different solutions for ambipolar diffusion thus far. Spitzer (1968 or 1978) obtained a steady-state solution for the drift velocity of the plasma relative to the neutrals for an infinite cylinder of uniform density supported laterally against self-gravity by a magnetic field parallel to the axis of symmetry. (A steady-state refers to a solution found under the assumption that locally the magnetic force on the ions is balanced by the gravitational force on the neutrals.) The implied time scale for ambipolar diffusion in an H I cloud is

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$$\tau_D = 5 \times 10^{13}\ x \qquad \text{yr} , \qquad (10)$$

where the quantity $x \equiv n_i/n_H$ is the degree of ionization in the cloud. By definition, τ_D is the time required for the plasma to drift relative to the neutrals a distance r from the axis of symmetry with a drift speed equal to its value at r. This expression is therefore expected to underestimate τ_D significantly in a cloud's core (Mouschovias and Paleologou 1981).

Mouschovias (1979b) found a steady-state solution for ambipolar diffusion in a pressure-bounded cylindrical cloud which contracted to an equilibrium state from an initially uniform configuration. The characteristic time scale is now a function of distance from the axis of symmetry and, for a typical molecular cloud, is given by

$$\tau_B(r) = 1.16 \times 10^{13}\ \zeta(r)\ x(r) \qquad \text{yr} . \qquad (11)$$

It is the time required for the plasma to drift relative to the neutrals a distance equal to the local magnetic scale height, and is thereby a measure of the efficiency of ambipolar diffusion as a function of position within a cloud. The function $\zeta(r)$ decreases monotonically from its value of ∞ on the axis of symmetry to 4×10^{-4} on the cloud boundary; the variation is very rapid for small r and gradual near the boundary (see his Table 1, column 4). Since the degree of ionization $x(r) \equiv n_i(r)/n_{H_2}(r)$ increases with r, $\tau_B(r)$ is likely to have a minimum within the cloud, near the cloud core. Stars may thus form first in a ring near the core, but not at the center or on the axis of symmetry --if, of course, such center or axis of symmetry exists in a real cloud.

Nakano (1979) devised what may be the closest approximation to an attack on the time-dependent process of ambipolar diffusion without actually solving the time-dependent fluid equations. He employed the formulation and method of solution developed by Mouschovias (1976a) for the study of sequences of equilibrium configurations of pressure-bounded, self-gravitating, magnetic interstellar clouds. A new feature was introduced which consisted of allowing Mouschovias' mass-to-flux ratio $dm(\Phi)/d\Phi$, characterizing each flux tube of a cloud, to vary slightly from one equilibrium configuration to the next in accordance with a prescription dependent on the local strength of the magnetic force. He found that equilibrium configurations are possible until the cloud's core loses a significant fraction of its magnetic flux. At that stage rapid contraction seems to ensue, but the numerical scheme stops there because it is suitable only for quasi-static evolution.

A number of other papers essentially apply Spitzer's solution to different situations. Nakano (1973; 1976; 1977) employs it for approximate studies of fragmentation after the decoupling of the field from the matter. He (1978) also studied the same problem in the compressed layer between an ionization front and a shock front, which had

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been suggested as the location of star formation (Elmegreen and Lada 1977). Scalo (1977) estimated the heating of a cloud due to released magnetic energy during ambipolar diffusion and concluded that a value of $\kappa \gtrsim 2/3$ would result in too much heating. Elmegreen (1979) considered the effect of charged grains on steady-state ambipolar diffusion, and concluded that flux-freezing remains valid even when the degree of ionization is much smaller than 10^{-8}. Nakano and Umebayashi (1980) find, however, that grains become important only for neutral densities $n_n \gtrsim 10^9$ cm^{-3} because only then is the density of grains (n_g) comparable with that of the ions (n_i) and much larger than the density of electrons (n_e). For $n_n \lesssim 10^9$ cm^{-3}, they find that $n_i \approx n_e >> n_g$, and the grains are not always attached to the magnetic field. A linear stability analysis of the magnetohydrodynamic equations concluded what was already known from the steady-state solutions; namely, the critical mass and the collapse time scale decrease once the field decouples from the matter, at low degrees of ionization (Langer 1978). Using Spitzer's solution we had shown that ambipolar diffusion can decouple the field from the matter at relatively low densities (10^4 - 2×10^6 cm^{-3}) in a significantly short time; thus, grain effects are not expected to become important (Mouschovias 1977a). Assuming that angular momentum is essentially conserved once ambipolar diffusion became effective, we showed that there is a one-to-one correspondence between the gas density at decoupling and the residual angular momentum in a collapsing blob which will ultimately appear as orbital angular momentum of a binary (or multiple) star system. The above range of densities for decoupling is exactly what is required to account for the entire range of periods of binary stars from 10 hr to 100 yr.

It is commonly thought that ambipolar diffusion necessarily reduces the magnetic flux and magnetic energy of a cloud. We have argued, however, that although that is possible, it is by no means the essential feature of ambipolar diffusion (Mouschovias 1978; 1979b). The essential feature of ambipolar diffusion is a redistribution of mass in at least some of the interior flux tubes of a cloud. The relatively high degree of ionization in the envelopes of self-gravitating clouds maintains the field frozen in the matter there. Ambipolar diffusion sets in when ionizing high-energy ($\gtrsim$ 100 MeV) cosmic rays are screened out of a dense core, and allows the neutrals to contract, under the influence of gravity, more rapidly than the ions, which are acted upon by the full strength of the retarding magnetic forces. Thus, the total flux of the cloud does not necessarily change, and its magnetic energy can even increase while ambipolar diffusion is in progress. It is gravitational energy, not magnetic, which is converted first into kinetic energy of neutrals and then into heat via neutral-ion collisions. The additional important consequence of this picture is that collapsing protostars may retain a larger magnetic energy than previously realized, and magnetic braking may continue past the point of initiation of ambipolar diffusion to allow the formation of single stars through only one stage of fragmentation in the parent cloud.

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The first time-dependent solution for ambipolar diffusion has been obtained recently (Mouschovias and Paleologou 1981). It refers to the case in which ambipolar diffusion both redistributes mass in the flux tubes of the system as well as reduces the total flux and magnetic energy of a cloud. It applies to a layer of gas compressed relatively rapidly (e.g., by a strong shock), with a magnetic field parallel to the surfaces of the slab. The slab-cloud is in pressure balance with a hot and tenuous external medium, whose field is negligible compared to that of the cloud. The neutrals are assumed to be at rest. The drift velocity of the plasma at the cloud boundary is shown as a function of time in Figure 2 (scale on left side of the figure). Each curve is labeled by the value of the neutral density in units of cm^{-3}. It increases rapidly and reaches a maximum (filled circles) within a time τ_* equal to 130 - 300 times the ion neutral collision time (within about 6×10^{-4} - 3 yr, for neutral density in the respective range

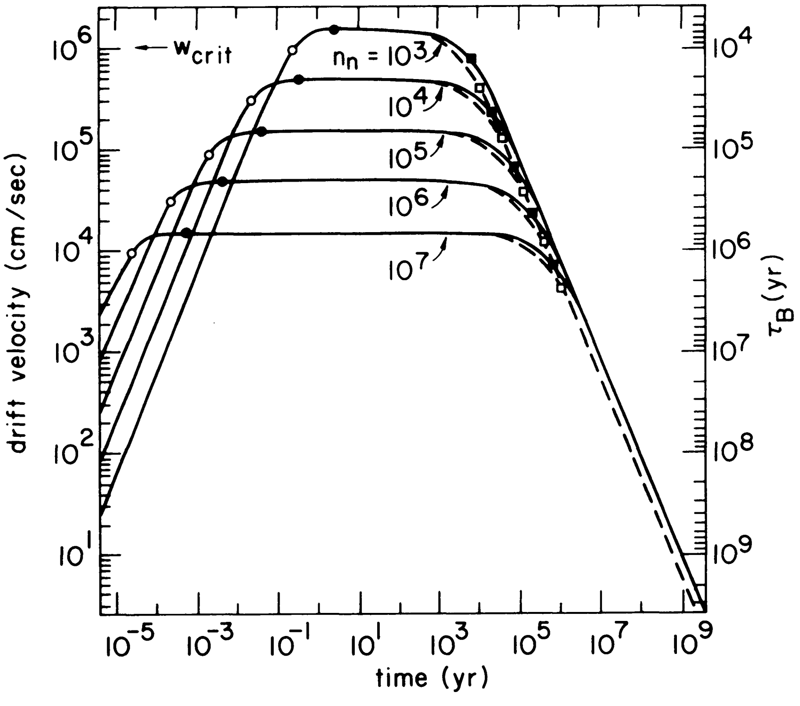

Figure 2. The drift velocity (scale to the left of the frame) and the characteristic time for time-dependent ambipolar diffusion (scale to the right of the frame) as functions of time for different values of the neutral density.

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10^7 - 10^3 cm^{-3}). For this density range, the maximum drift velocity lies in the range 0.25 - 15 km s^{-1}. The corresponding characteristic time τ_B for ambipolar diffusion is in the range 6.6 × 10^5 - 6.6 × 10^3 yr (scale on the right side of the frame). Beyond the time τ_*, the driving magnetic force is almost exactly balanced by the retarding collisional force between ions and neutrals, and the asymptotic behavior of the drift velocity is t^{-1}. These results are insensitive to the rate at which ionization equilibrium tends to be re-established as plasma escapes the cloud. Solid curves are for very slow and dashed curves for very rapid re-establishment of ionization equilibrium compared with the rate at which ambipolar diffusion progresses.

The magnetic field, normalized to its initial value in the slab, is shown as a function of time in Figure 3. The labeling of the curves and the meaning of solid and dashed curves are as in Figure 2. Up to

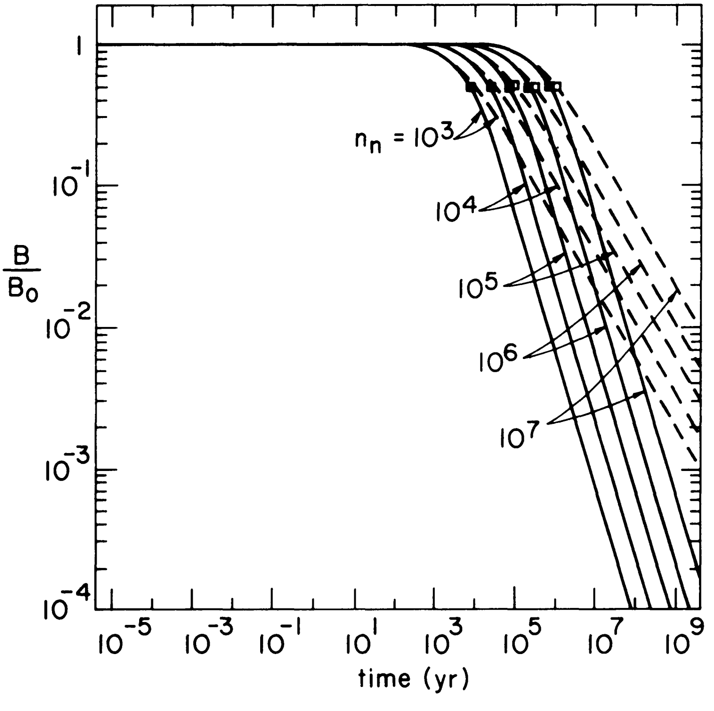

Figure 3. The magnetic field as a function of time for different values of the neutral density.

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a time $\tau_{B,min}$ (indicated by filled and open squares on the two sets of curves), which is equal to the minimum value attained by the characteristic time for ambipolar diffusion, the magnetic field hardly changes. Near $\tau_{B,min}$, however, the field (and the flux) decreases to 1/2 its original value. If the original mass-to-flux ratio of the cloud (or fragment) is near the critical value for gravitational collapse (see eqs. [7a,b]), by the time $\tau_{B,min}$ the cloud (or, fragment) will begin to contract dynamically with a significant fraction of its magnetic flux trapped in (and hence its magnetic energy increasing in time), although ambipolar diffusion is in progress. A quantitative discussion of the collapse phase with ambipolar diffusion will be discussed elsewhere (Paleologou and Mouschovias 1981). The asymptotic behavior of the field is t^{-1} and $t^{-1/2}$ if re-establishment of ionization equilibrium is slow and rapid, respectively. This behavior is relevant only for clouds whose mass-to-flux ratio is well below critical, so that they can wait quiescently for at least 10^4 - 10^6 yr while their flux decreases in time. In the case of clouds with mass-to-flux ratio near critical, the asymptotic behavior of the field has only an academic significance. Dynamical contraction will set in by the time $\tau_{B,min}$.

A widespread misconception exists, that ambipolar diffusion proceeds more rapidly the smaller the degree of ionization is. The results just described show that this is not necessarily so. To illustrate the point further, we consider a slab-shaped cloud (as above) of moderate density so that self-gravity is not dominant. We then ask: (i) What is (are) the dimensionless free parameter(s) whose specification determines uniquely the solution for ambipolar diffusion? (ii) How does the efficiency of ambipolar diffusion depend on this (these) free parameter(s)? It is straightforward to show on physical grounds (see Mouschovias 1980) or rigorously (see Mouschovias and Paleologou 1981) that there is only one free parameter (ν_A) in this problem. It is the ratio of two natural time scales; namely, the time $\tau_{A,i}$ it takes an Alfvén wave (strictly, a magnetosonic wave) to traverse the thickness L of the cloud with a speed equal to the Alfvén speed in the ions, and the ion-neutral collision time τ_{in} --the latter refers to collisions of a single ion in a sea of neutrals. Thus the dimensionless parameter $\nu_A \equiv \tau_{A,i}/\tau_{in} \propto L\, n_i^{1/2}\, n_n / B$ represents the number of collisions in one Alfvén crossing time. The larger ν_A is, the more collisions an ion suffers in one Alfvén crossing time; hence, the larger the collisional drag and the smaller the rate of ambipolar diffusion. The degree of ionization is not a relevant parameter in this case. In fact, it is clear from the expression for ν_A that a given magnetic configuration can drive a given ion density through the neutrals more efficiently the smaller the neutral density (and, hence, the larger the degree of ionization) is.

The situation changes somewhat, in that x_i is a legitimate free parameter, if self-gravity is strong enough to drive neutrals through ions, which are retarded by magnetic forces (Mouschovias 1980). Yet, even in this case, x_i is only one of three free parameters. If it were for x_i alone, ambipolar diffusion would indeed tend to be more efficient as x_i decreases. The other two free parameters, however, $\bar{\nu}_A$ (the number

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of collisions of a neutral particle in one Alfvén crossing time) and ν_{ff} (the number of collisions of a neutral particle in one free fall time) can counter this tendency and can reduce the efficiency of ambipolar diffusion depending on precisely how and due to what physical cause x_i decreases.

8. CONCLUSION

We have reviewed the state of the art of star formation in magnetic interstellar clouds, particularly as it attempts to provide answers to the six basic questions posed in §1. Progress is being made rapidly in understanding the precise role of the magnetic field. What seems certain now, observationally and theoretically, is that the field is important, if not crucial, in star formation, particularly in resolving the thorny angular momentum problem. One should maintain an open mind, however, and renormalize one's thinking if new evidence, observational or theoretical, challenges our present conclusions or offers better alternatives.

REFERENCES

Aannestad, P. A., and Purcell, E. M. 1973, Ann. Rev. Astron. & Astrophys., **11**, 309.

Appenzeller, I. 1968, Astrophys. J., **151**, 907.

Beichman, C. A., and Chaisson, E. J. 1974, Astrophys. J. Letters, **190**, L21.

Bless, R. C. 1968, in Stars and Stellar Systems, Vol. **7**, Nebulae and Interstellar Matter, ed. B. Middlehurst and L. H. Aller (Univ. of Chicago Press, Chicago), pp. 667-684.

Blitz, L., and Shu, F. H. 1980, Astrophys. J., **238**, 148.

Bonnor, W. B. 1956, Mon. Notices Roy. Astron. Soc., **116**, 351.

Chaisson, E. J., and Vrba, F. J. 1978, in Protostars and Planets, ed. T. Gehrels (Univ. of Arizona Press, Tucson), pp. 189-208.

Chandrasekhar, S., and Fermi, E. 1953, Astrophys. J., **118**, 116.

Clark, F. O. 1980, preprint.

Crutcher, R. M., Evans, N. J., Troland, T., and Heiles, C. 1975, Astrophys. J., **198**, 91.

Daniel, R. R., and Stephens, S. A. 1970, Space Sci. Rev., **10**, 599.

Davis, L., Jr. 1958, Astrophys. J., **128**, 508.

Davis, L., Jr., and Berge, G. L. 1968, in Stars and Stellar Systems, Vol. **7**, Nebulae and Interstellar Matter, ed. B. Middlehurst and L. H. Aller (Univ. of Chicago Press, Chicago), pp. 755-770.

Davis, L., Jr., and Greenstein, J. L. 1951, Astrophys. J., **114**, 206.

Dyck, H. M., and Lonsdale, C. J. 1979, Astron. J., **84**, 1339.

Earl, J. A. 1961, Phys. Rev. Letters, **6**, 125.

Ebert, R. 1955, Zs. f. Astrophys., **37**, 217.

_____. 1957, Zs. f. Astrophys., **42**, 263.

https://doi.org/10.1017/S0074180900073605 Published online by Cambridge University Press

Ebert, R., von Hoerner, S., and Temesvary, S. 1960, Die Entstehung von Sternen durch Kondensation diffuser Materie (Springer-Verlag, Berlin), p. 184.
Elmegreen, B. G. 1979, Astrophys. J., 232, 729.
Elmegreen, B. G., and Lada, C. J. 1977, Astrophys. J., 214, 725.
Few, R. W. 1979, Mon. Notices Roy. Astron. Soc., 187, 161.
Field, G. B. 1962, in Interstellar Matter in Galaxies, ed. L. Woltjer (W. A. Benjamin, Inc., New York), p. 183.
_____. 1965, Astrophys. J., 142, 531.
_____. 1969, Interstellar Gas Dynamics, ed. H. J. Habing (Reidel, Dordrecht), p. 51.
_____. 1978, in Protostars and Planets, ed. T. Gehrels (Univ. of Arizona Press, Tucson), p. 243.
Field, G. B., Goldsmith, D. W., and Habing, H. J. 1969, Astrophys. J. Letters, 155, L149.
Fleck, R. C. 1976, Mon. Not. Roy. Astron. Soc., 175, 335.
Gardner, F. F., and Davies, R. D. 1966, Australian J. Phys., 19, 441.
Gardner, F. F., Morris, D., and Whiteoak, J. B. 1969, Australian J. Phys., 22, 813.
Gardner, F. F., Whiteoak, J. B., and Morris, D. 1967, Nature, 214, 371.
Gillis, J., Mestel, L., and Paris, R. B. 1974, Astrophys. Space Sci., 27, 167.
_____. 1979, Mon. Not. Roy. Astron. Soc., 187, 311.
Ginzburg, V. L., and Syrovatskii, S. I. 1965, Ann. Rev. Astron. & Astrophys., 3, 297.
Giz, A., and Shu, F. H. 1980 (private communication).
Hall, J. S. 1949, Science, 109, 166.
Hayakawa, S., Nishimura, S., and Takayanagi, K. 1961, Public. Astron. Soc. Japan, 13, 184.
Heiles, C. 1968, Astrophys. J. Suppl., 15, No. 136.
_____. 1976, Ann. Rev. Astron. & Astrophys., 14, 1.
Heiles, C., and Jenkins, E. B. 1976, Astron. & Astrophys., 46, 333.
Hiltner, W. A. 1949, Science, 109, 165.
Hodge, P. W. 1969, Astrophys. J., 156, 847.
Hornby, J. M. 1966, Mon. Notices Roy. Astron. Soc., 133, 213.
Jeans, J. H. 1928, Astronomy and Cosmogony (Cambridge University Press, Cambridge).
Jones, R. V., and Spitzer, L., Jr. 1967, Astrophys. J., 147, 943.
Kerr, F. J. 1963, The Galaxy and the Magellanic Clouds, ed. F. J. Kerr (Reidel, Dordrecht), p. 81.
Kulsrud, R. M. 1971, Astrophys. J., 163, 567.
Kutner, M. L., and Tucker, K. D. 1975, Astrophys. J., 199, 79.
Langer, W. D. 1978, Astrophys. J., 225, 95.
Lo, K. Y., Walker, R. C., Burke, B. F., Moran, J. M., Johnston, K. J., and Ewing, M. S. 1975, Astrophys. J., 202, 650.
Manchester, R. N. 1974, Astrophys. J., 188, 637.
Martin, R. N., and Barrett, A. H. 1978, Astrophys. J. Suppl., 36, 1.
Mathewson, D. S. 1968, Astrophys. J. Letters, 153, L47.
_____. 1969, Proc. Astron. Soc. Australia, 1, 209.
Mathewson, D. S., and Ford, V. L. 1970, Mem. Roy. Astron. Soc., 74, 143.

https://doi.org/10.1017/S0074180900073605 Published online by Cambridge University Press

Mathewson, D. S., and Nicholls, D. C. 1968, Astrophys. J. Letters, 154, L11.
Mestel, L. 1965, Quart. J. Roy. Astron. Soc., 6, 161, 265.
_____. 1966, Mon. Not. Roy. Astron. Soc., 133, 265.
_____. 1977, Star Formation, eds. T. de Jong and A. Maeder (Reidel, Boston), p. 213.
Mestel, L., and Paris, R. B. 1979, Mon. Not. Roy. Astron. Soc., 187, 337.
Mestel, L., and Spitzer, L., Jr. 1956, Mon. Notices Roy. Astron. Soc., 116, 503.
Meyer, P. 1969, Ann. Rev. Astron. & Astrophys., 7, 1.
Meyer, P., and Vogt, R. 1961, Phys. Rev. Letters, 6, 193.
Miller, C. R. 1962, Ph.D. Thesis, California Institute of Technology.
Morgan, W. W. 1970, The Spiral Structure of Our Galaxy, eds. W. Becker, and G. Contopoulos (Reidel, Dordrecht), p. 9.
Morris, D., and Berge, G. L. 1964, Astron. J., 69, 641.
Mouschovias, T. Ch. 1974, Astrophys. J., 192, 37.
_____. 1975a, Astron. & Astrophys., 40, 191.
_____. 1975b, Ph.D. Thesis, University of California, Berkeley.
_____. 1976a, Astrophys. J., 206, 753.
_____. 1976b, Astrophys. J., 207, 141.
_____. 1977a, Astrophys. J., 211, 147.
_____. 1977b, in Star Formation, eds. T. de Jong and A. Maeder (Reidel, Boston), p. 235.
_____. 1978, in Protostars and Planets, ed. T. Gehrels (Univ. of Arizona Press, Tucson), pp. 209-242.
_____. 1979a, Astrophys. J., 228, 159.
_____. 1979b, Astrophys. J., 228, 475.
_____. 1980, Astrophys. J. Letters, submitted.
Mouschovias, T. Ch., and Paleologou, E. V. 1979, Astrophys. J., 230, 204.
_____. 1980a, The Moon and the Planets, 22, 31.
_____. 1980b, Astrophys. J. 237, 877.
_____. 1981, Astrophys. J., submitted.
Mouschovias, T. Ch., Shu, F. H., and Woodward, P. R. 1974, Astron. & Astrophys., 33, 73.
Mouschovias, T. Ch., and Spitzer, L., Jr. 1976, Astrophys. J., 210, 326.
Nakano, T. 1973, Publ. Astron. Soc. Japan, 25, 91.
_____. 1976, Publ. Astron. Soc. Japan, 28, 355.
_____. 1977, Publ. Astron. Soc. Japan, 29, 197.
_____. 1978, Publ. Astron. Soc. Japan, 30, 681.
_____. 1979, Publ. Astron. Soc. Japan, 31, 697.
Nakano, T., and Tademaru, E. 1972, Astrophys. J., 173, 87.
Nakano, T., and Umebayashi, T. 1980, preprint.
Paleologou, E. V., and Mouschovias, T. Ch. 1981, in preparation.
Parker, D. A. 1973, Mon. Notices Roy. Astron. Soc., 163, 41.
_____. 1974, Mon. Notices Roy. Astron. Soc., 168, 331.
Parker, E. N. 1966, Astrophys. J., 145, 811.
_____. 1967, Astrophys. J., 149, 535.
Pikelner, S. 1967, Astr. Zh., 44, 1915.
Roberts, W. W. 1969, Astrophys. J., 158, 123.

https://doi.org/10.1017/S0074180900073605 Published online by Cambridge University Press

Scalo, J. M. 1977, Astrophys. J., 213, 705.
Schwartz, J., McCray, R., and Stein, R. F. 1972, Astrophys. J., 175, 673.
Scott, E. H., and Black, D. C. 1980, Astrophys. J. 239, 166.
Scoville, N. Z., Solomon, P. M., and Sanders, D. B. 1979, in The Large Scale Characteristics of the Galaxy, ed. W. Burton (Reidel, Dordrecht), p. 277.
Spitzer, L., Jr. 1951, Problems of Cosmical Aerodynamics (Central Air Documents Office, Dayton, Ohio), p. 31.
_____. 1962, Physics of Fully Ionized Gases, 2nd ed. (Interscience, New York).
_____. 1968, Diffuse Matter in Space (Interscience, New York).
_____. 1978, Physical Processes in the Interstellar Medium (Wiley-Interscience, New York).
Spitzer, L., Jr., and Scott, E. H. 1969, Astrophys. J., 157, 161.
Strittmatter, P. A. 1966, Mon. Notices Roy. Astron. Soc., 132, 359.
Turner, B. E., and Verschuur, G. L. 1970, Astrophys. J., 162, 341.
Verschuur, G. L. 1971, Astrophys. J., 165, 651.
Vrba, F. J., Coyne, G. V., and Tapia, S. 1980, preprint.
Vrba, F. J., Strom, S. E., and Strom, K. M. 1976, Astron. J., 81, 958.
Westerhout, G. 1963, The Galaxy and the Magellanic Clouds, ed. F. J. Kerr (Reidel, Dordrecht), p. 78.
Woltjer, L. 1965, in Stars and Stellar Systems, Vol. 5, Galactic Structure, eds. A. Blaauw and M. Schmidt (Univ. of Chicago Press, Chicago), pp. 531-587.
Wright, W. E. 1973, Ph.D. Thesis, California Institute of Technology.

DISCUSSION

Bodenheimer: How many orders of magnitude in specific angular momentum can be lost through magnetic braking given sufficient time? Can you summarize your results?

Mouschovias: Our results depend neither on the origin nor on the magnitude of the initial angular momentum of the cloud--that's the advantage of solving a problem in dimensionless form. Exactly by how many orders of magnitude the initial angular momentum will be reduced, depends on the density n_{dec} at which ambipolar diffusion will decouple the field from the neutral matter relatively efficiently. I have shown that enough angular momentum is lost to account for the entire range of periods of binary stars from 10 hours to 100 years (through a single fragmentation process). To form the Sun-Jupiter "binary", n_{dec} must be $\sim 10^9$ cm^{-3}.

Bodenheimer: Can you elaborate on your statement that the degree of ionization has nothing to do with the ambipolar diffusion rate?

Mouschovias: My statement, in the form you quoted, it referred to clouds in which gravity is not dominant. In such objects, the dimensionless free parameter is the ratio of the Alfvén crossing time in the ions and the collision time of an ion in a sea of neutrals; this varies as $n_i^{1/2} n_n$.

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Physically, it means that each ion has a harder time diffusing through the neutrals as the neutral density n_n increases. So, the efficiency of ambipolar diffusion decreases as the degree of ionization decreases. In self-gravitating clouds, the degree of ionization is relevant in the "classical" sense but, still, it is only one of three free parameters; its decrease does not necessarily mean more efficient ambipolar diffusion.

Bodenheimer: I do not entirely agree with your statement that the angular momentum problem has already been solved at the molecular cloud stage. The observations I quoted earlier show that J/M is at least 10^{23} in massive clouds. Could you clarify your statement?

Mouschovias: That was not my statement. I said that the bulk of the angular momentum problem has been resolved. In other words, the same mechanism which removed so much angular momentum from the "rapidly" rotating clouds (which, as you mentioned, rotate slowly compared to the angular velocities implied by conservation of angular momentum from an initial density of 1 cm^{-3} and $\omega \sim 10^{-15}$ sec^{-1}), will have an even easier time removing the necessary additional, relatively small amount of angular momentum. To re-iterate, I meant to convey the message that, as you stated in your talk, there is an angular momentum problem for dense clouds. But the angular momentum problem is much more severe if one considers the earlier, more diffuse, stages of such dense clouds. Still, magnetic braking can resolve even this more severe angular momentum problem.

Sugimoto: What is the physical situation which corresponds to $\kappa = 1/3$?

Mouschovias: First, let me recall that $\kappa = 1/2$ corresponds to rapid establishment of near hydrostatic equilibrium between gravity and pressure gradients along field lines. Virtually "instantaneous" re-adjustment along field lines can take place, and the contraction of a cloud then proceeds only as rapidly as magnetic forces allow the cloud to contract perpendicular to the field lines -- while near-hydrostatic equilibrium is maintained along field lines. A value $\kappa < 1/3$ means, of course, a smaller increase of the magnetic field strength for a given increase in the gas density. This could be the result of the development of a central condensation in which the magnetic force is partly determined by field lines that are frozen in the envelope as well as in the external (intercloud) medium. There is no theoretical lower limit on κ; e.g., if $\underline{B}$ is very strong, only motion along field lines will occur, and $\kappa \simeq 0$ for such motion.

Nariai: Is not the energy density of the gravitational field of the galaxy comparable to or larger than the energy density of the magnetic field?

Mouschovias: The galactic gravitational field is indeed important. In fact, it is responsible for the Parker instability, which, when triggered by a galactic shock, may account for the formation of cloud complexes, OB associations, and giant H II regions along spiral arms separated by regular intervals of about one kiloparsec, like "beads on a string" (see Mouschovias, Shu, and Woodward 1974, Astron. & Astorophys., 33, 73).

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Nariai: In the case where most of the magnetic lines of force lie in the plane of rotation, wouldn't you expect transfer of mass among neighboring clouds connected by the magnetic field, which may reduce the timescale of the change in $m(\Phi)$?

Mouschovias: No, I would not. The mean separation of interstellar clouds along the same field lines is much too large (at least several hundred parsecs and maybe larger) and matter velocities much too small ($v \leq v_{Alfvén} \sim 10$ km/sec) for exchange of mass to be relevant. In addition, field lines "buckle" in the space between clouds and extend high above the galactic plane. This geometry makes the kind of mass exchange which you are suggesting very unlikely.

Schatzman: I suggest that you consider mass loss from the cloud, since such mass loss can carry away a large amount of angular momentum.

Mouschovias: Mass loss due to what? And over what time scale? If somebody estimates significant mass loss (e.g., due to compressional hydromagnetic waves), then we'll surely have to consider it. However, I am finding that magnetic braking by itself can resolve the angular momentum problem during the early, diffuse stages of star formation.

Nakano: What configuration did you take for the cloud?

Mouschovias: As shownin Figure 4 below, the magnetic fields vector of a cylindrical (or disk) cloud, rotating about its axis of symmetry, is initially perpendicular to the axis of rotation. Under our assumptions, the results are independent of the length of the cylinder (or, the thickness of the disk).

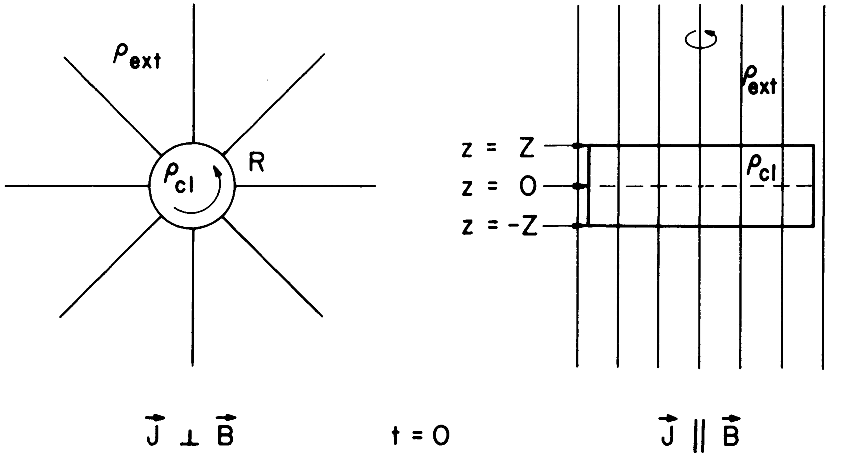

Figure 4. The geometries employed by Mouschovias and Paleologou in their studies of magnetic braking of perpendicular (left) and aligned (right) disk or cylinder rotators. Solid lines are field lines, shown at time t = 0.

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van den Heuvel: Would not field line reconnection rapidly destroy a spiral-shaped magnetic field pattern?

Mouschovias: We have considered the issue of reconnection and concluded that, if it happens at all, it happens far from the cloud -- at least 20 cloud radii away. In fact, we estimated the energy of cosmic rays which may be produced *in situ* in the intercloud medium by such reconnection (see 1979, *Ap. J.*, *230*, 204).

Nariai: Have you solved the equations for *i*, *B*, and *E* as well as the equation of motion, or have you solved only the equation of motion using the frozen condition? I would not say that a problem of this type is solved rigorously unless *i*, *B*, and *E* are given as functions of time.

Mouschovias: For the case $\vec{J} \parallel \vec{B}$, we impose the condition that both components of $\vec{B}$ are continuous across the cloud surface, and indeed we solve the problem rigorously. The current density is always given by $\nabla \times \vec{B} = (4\pi/c)\vec{j}$ and the electric field by $\vec{E} = -(\vec{v}/c) \times \vec{B}$, since the magnetic field is strictly frozen in the matter. We also solved this same problem by relaxing the condition of continuity of the azimuthal component of $\vec{B}$ across the cloud surface (i.e., by considering a rigidly rotating cloud), and showed that the behavior of the cloud (as far as its angular velocity is concerned), except for short-lived transient effects, is virtually identical in the two cases. We also found that the rigid-body approximation becomes better and better as the ratio ρ_{cl}/ρ_{ext} increases (see 1980, *Ap.J.*, *237*, 877). For these reasons, we assumed rigid-body rotation of the cloud in the case $\vec{J} \perp \vec{B}$, which concerns us here. One should always distinguish between assumptions which, if relaxed, alter the qualitative results and assumptions which, if relaxed, only affect somewhat the quantitative nature of the conclusions.

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