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A STUDY OF WELL-BALANCED FINITE VOLUME METHODS AND REFINEMENT INDICATORS FOR THE SHALLOW WATER EQUATIONS

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Water flows can be modelled mathematically and one available model is the shallow water equations. This thesis studies solutions to the shallow water equations analytically and numerically. The study is divided into three parts.

The first part is about well-balanced finite volume methods to solve steady and unsteady state problems. A method is said to be well balanced if it preserves an unperturbed steady state at the discrete level [10]. We implement hydrostatic reconstructions proposed by Audusse *et al.* [1] for the well-balanced methods with respect to the steady state of a lake at rest. Four combinations of quantity reconstructions are tested. Our results indicate an appropriate combination of quantity reconstructions for dealing with steady and unsteady state problems [3].

The second part presents some new analytical solutions to debris avalanche problems [5, 7] and reviews the implicit Carrier–Greenspan periodic solution for flows on a sloping beach [8]. The analytical solutions to debris avalanche problems are derived using characteristics and a variable transformation technique. The analytical solutions are used as benchmarks to test the performance of numerical solutions. For the Carrier–Greenspan periodic solution, we show that the linear approximation of the Carrier–Greenspan periodic solution may result in large errors in some cases. If an explicit approximation of the Carrier–Greenspan periodic solution is needed, higher order approximations should be considered. We propose second order approximations of the Carrier–Greenspan periodic solution and present a way to get higher order approximations.

The third part discusses refinement indicators used in adaptive finite volume methods to detect smooth and nonsmooth regions. In the adaptive finite volume methods, smooth regions are coarsened to reduce the computation costs and nonsmooth regions are refined to get more accurate solutions. We consider

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the numerical entropy production [11] and weak local residuals [2] as refinement indicators. Regarding the numerical entropy production, our work is the first to implement the numerical entropy production as a refinement indicator into adaptive finite volume methods used to solve the shallow water equations. Regarding weak local residuals, we propose formulations to compute weak local residuals on nonuniform meshes. Our numerical experiments show that both the numerical entropy production and weak local residuals are successful as refinement indicators.

Some publications corresponding to this thesis are listed in [3–9].

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