CORRESPONDENCE.

To the Editor of the *Transactions of the Faculty of Actuaries.*

SIR,

King and Reid's Paper.

In a recent Paper, *J.I.A.*, lxvi. p. 419, f.n., I have pointed out that the indeterminate equation which is the basis of Messrs. King and Reid's interesting experiments, *T.F.A.*, xv. p. 111, not only has the infinity of real roots which they considered but also has—provided $\beta^2 > \alpha\gamma$—an infinity of conjugate complex roots of the form $A \pm Bi$ leading to a trigonometric form for μ_x.* It may be of interest to prove this statement and show how pairs of roots may be found. The basic equation may be written

$$\beta n^{15} - \alpha c^{15} n^{15} = \gamma - \beta c^{15} \quad . \quad . \quad . \quad . \quad (1)$$

Put $c^{15} = A + Bi$, $n^{15} = A - Bi$, whence $c^{15}n^{15} = A^2 + B^2$. Substituting in the equation and removing a term βBi from both sides, we get

$$A^2 - 2\frac{\beta}{\alpha}A + B^2 + \frac{\gamma}{\alpha} = 0 \quad . \quad . \quad . \quad . \quad (2)$$

This will give two real values of A, corresponding to a properly selected real value of B, if

$$\frac{\beta^2}{\alpha^2} - \left(B^2 + \frac{\gamma}{\alpha}\right) \overline{\gtrless} 0;$$

$$\textit{i.e.} \text{ if} \quad \beta^2 - \alpha^2 B^2 - \alpha\gamma \overline{\gtrless} 0,$$

$$\text{if} \quad \beta^2 - \alpha\gamma \overline{\gtrless} \alpha^2 B^2.$$

This relation is satisfied

(i) If $\beta^2 - \alpha\gamma = 0$, $B = 0$, in which case the two roots are real and equal, viz. $c^{15} = n^{15} = A = \dfrac{\beta}{\alpha}$;

(ii) If $\beta^2 - \alpha\gamma > 0$, and B has any selected value such that $B^2 \overline{\lessgtr} (\beta^2 - \alpha\gamma)/\alpha^2$. To any such selected value there will correspond two different real values of A (of the form $\beta/\alpha \pm K^{\frac{1}{2}}$, where K is between 0 and $\beta^2 - \alpha\gamma$) leading to two different curves of μ_x for each value of B. On the other hand, if we start by selecting such a value of A, the two corresponding values of B will differ only in sign, corresponding merely to an interchange of c and n and giving only a single curve of μ_x for each value of A.

* See Messrs. King and Reid's further note in this issue.—Ed. *T.F.A.*

Having found c^{15} and n^{15} in the form $A \pm Bi$, we can find c and n as follows. We can put $A \pm Bi$ into the form (cf. *T.F.A.*, ix. p. 237)

$$(A^2+B^2)^{\frac{1}{2}}(\cos\phi \pm i\sin\phi), \text{ where } \tan\phi = B/A,$$

whence by de Moivre's Theorem c and n are

$$(A^2+B^2)^{\frac{1}{30}}\left(\cos\frac{\phi}{15} \pm i\sin\frac{\phi}{15}\right)$$

and the corresponding form for μ_x is, changing the notation and now writing c for $(A^2+B^2)^{\frac{1}{30}}$ and A for the constant part of μ,

$$\mu_x = A + c^x\left(N\cos\frac{x\phi}{15} + M\sin\frac{x\phi}{15}\right),$$

which can be put into the form

$$\mu_x = A + Cc^x\cos\left(\frac{x\phi}{15} + \theta\right).$$

In this way Messrs. King and Reid's experimental methods may be extended to the trigonometric form.

Your obedient servant,

G. J. Lidstone.

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