

## LETTER

# Interplay between intermittency and dissipation in collisionless plasma turbulence

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(Received 1 December 2018; revised 10 April 2019; accepted 11 April 2019)

We study the damping of collisionless Alfvénic turbulence in a strongly magnetised plasma by two mechanisms: stochastic heating (whose efficiency depends on the local turbulence amplitude  $\delta z_\lambda$ ) and linear Landau damping (whose efficiency is independent of  $\delta z_\lambda$ ), describing in detail how they affect and are affected by intermittency. The overall efficiency of linear Landau damping is not affected by intermittency in critically balanced turbulence, while stochastic heating is much more efficient in the presence of intermittent turbulence. Moreover, stochastic heating leads to a drop in the scale-dependent kurtosis over a narrow range of scales around the ion gyroscale.

**Key words:** astrophysical plasmas, plasma dynamics, space plasma physics

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## 1. Introduction

The question of how collisionless plasma turbulence dissipates via kinetic processes has received a great deal of recent interest (Parashar *et al.* 2015). The heating mechanism(s) that effect this dissipation have dramatic consequences for the basic thermodynamic state of the plasma, controlling the ion-to-electron temperature ratio as well as affecting the temperature anisotropy of the plasma with respect to the local magnetic-field direction. Attempts at solving this problem often fall into one of two camps: (i) studies that invoke the ‘quasilinear premise’ (Klein *et al.* 2012; Howes, Klein & TenBarge 2014) and propose that turbulent fluctuations damp at the same rate (e.g. the linear Landau damping rate (Landau 1946)) as linear plasma waves with similar polarisation properties (Howes *et al.* 2006, 2008; Schekochihin *et al.* 2009; Howes, TenBarge & Dorland 2011; TenBarge & Howes 2013; TenBarge, Howes & Dorland 2013; Told *et al.* 2015; Howes, McCubbin & Klein 2018), or,

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alternatively, (ii) studies that focus on intermittency and the associated ‘coherent structures’ (Matthaeus & Lamkin 1986; Burlaga 1991; Horbury & Balogh 1997; Sorriso-Valvo *et al.* 1999; Bruno *et al.* 2007; Cho & Lazarian 2009; Greco *et al.* 2009; Parashar *et al.* 2009; Salem *et al.* 2009; Servidio *et al.* 2009; Greco *et al.* 2010; Osman *et al.* 2011; Servidio *et al.* 2011; Greco *et al.* 2012; Osman *et al.* 2012a,b; Perri *et al.* 2012; Karimabadi *et al.* 2013; Wu *et al.* 2013; Osman *et al.* 2014; Chasapis *et al.* 2015; Bañón Navarro *et al.* 2016; Lion, Alexandrova & Zaslavsky 2016; Perrone *et al.* 2016; Wan *et al.* 2016), arguing that these structures dissipate in a fundamentally different way than linear waves. In this Letter, we straddle these two camps by developing a novel modelling framework for the damping of intermittent turbulence. We use this to predict, for the first time, the quantitative dependence (or independence) of different heating mechanisms on the level of intermittency, and the effect (or lack of effect) of these heating mechanisms on the intermittency itself, with several surprising results. These results suggest a simple new observational test, based on the scale-dependent kurtosis near the ion gyroscale, that will allow us to distinguish between different heating mechanisms in collisionless plasma turbulence, for example in the solar wind.

We first show that, in intermittent, critically balanced turbulence, intermittency has no effect upon the total turbulent heating rate resulting from linear Landau damping, and that linear Landau damping has no effect on the level of intermittency. These results only apply when the turbulence is critically balanced: in both weak turbulence and (unphysical) isotropic turbulence, the linear Landau heating rate does depend on the intermittency. Thus, the ‘linear’ nature of Landau damping does not by itself make (as might naïvely be expected) its associated turbulent heating rate independent of intermittency. We then contrast this with the mechanism of stochastic heating (McChesney *et al.* 1987; Chen, Lin & White 2001; White, Chen & Lin 2002; Chaston *et al.* 2004; Voitenko & Goossens 2004; Fiksel *et al.* 2009; Chandran *et al.* 2010; Chandran 2010; Chandran *et al.* 2011; Bourouaine & Chandran 2013; Vech, Klein & Kasper 2017); when the turbulence amplitude at the ion gyroscale  $\rho = v_{\text{th}}/\Omega_i$  (where  $v_{\text{th}} = \sqrt{2T_i/m_i}$  is the ion thermal speed and  $\Omega_i = ZeB/m_i$  is the ion gyrofrequency) becomes sufficiently large, ion orbits become chaotic, and ions may gain energy by interacting with gyroscale turbulent structures with frequencies much less than  $\Omega_i$ . The stochastic damping rate is a highly nonlinear function of turbulent fluctuation amplitude. We show that (i) intermittency dramatically increases the overall stochastic heating rate, and (ii) stochastic heating reduces the scale-dependent kurtosis of the turbulent fluctuations at the scale  $\rho$ . Finally, we show that because of this strong dependence on intermittency, stochastic heating may remain an important dissipation mechanism in astrophysical situations, where naïvely it would be ignored due to the small overall turbulence amplitude at the gyroscale.

## 2. Intermittency model

We restrict our analysis to intermittent Alfvén-wave turbulence and damping mechanisms that are effective at  $k_{\perp}\rho \lesssim 1$ . We assume that the velocity and magnetic-field fluctuations (in velocity units) are much smaller than the background magnetic field, and that the fluctuations are highly anisotropic with respect to the direction of the background magnetic field, i.e. their parallel wavevectors are much smaller than their perpendicular wavevectors,  $k_{\parallel} \ll k_{\perp}$ . This allows us to model the turbulence with the equations of reduced magnetohydrodynamics (RMHD) (Kadomtsev & Pogutse 1973; Strauss 1976; Montgomery 1982), compactly written in terms of

Elsasser (1950) variables  $z_{\perp}^{\pm} = \mathbf{u}_{\perp} \pm \mathbf{b}_{\perp}$ , where  $\mathbf{u}_{\perp}$  and  $\mathbf{b}_{\perp}$  are the perpendicular velocity and magnetic-field (in velocity units) fluctuations respectively. There are a number of different intermittency models (Müller & Biskamp 2000; Chandran, Schekochihin & Mallet 2015) available; here, we will use the MS17 (Mallet & Schekochihin 2017) model, but our results do not depend in detail on this choice.<sup>1</sup> The Elsasser fluctuation amplitude of a structure with perpendicular scale  $\lambda$  is a random variable,

$$\delta z_{\lambda} = \delta z_{L_{\perp}} \Delta^q, \tag{2.1}$$

where  $\delta z_{L_{\perp}}$  is the outer scale amplitude, the constant  $\Delta = 1/\sqrt{2}$  and  $q$  is a Poisson random variable<sup>2</sup> with mean  $\mu = -\log(\lambda/L_{\perp})$ ,  $L_{\perp}$  being the outer scale. This distribution has ‘heavy tails’, becoming heavier at smaller scales  $\lambda$ , a classic hallmark of intermittency (Frisch 1995). This may be usefully quantified by the scale-dependent kurtosis,

$$\kappa_{\lambda} \equiv \frac{\langle \delta z_{\lambda}^4 \rangle}{\langle \delta z_{\lambda}^2 \rangle^2} = \kappa_{L_{\perp}} \left( \frac{\lambda}{L_{\perp}} \right)^{-1/4}. \tag{2.2}$$

The nonlinear and linear time scales of each structure are

$$\tau_{nl\lambda} \sim \frac{\lambda}{\delta z_{\lambda} \sin \theta_{\lambda}}, \quad \tau_{A\lambda} \sim \frac{l_{\parallel\lambda}}{v_A} \tag{2.3}$$

respectively, where  $v_A = B_0/\sqrt{4\pi n_i m_i}$  is the Alfvén speed,  $\theta_{\lambda}$  is the ‘alignment angle’ (Boldyrev 2006) and

$$\sin \theta_{\lambda} \sim \left( \frac{\lambda}{L_{\perp}} \right)^{1/2} \frac{\delta z_{L_{\perp}}}{\delta z_{\lambda}}. \tag{2.4}$$

This model incorporates refined critical balance (Mallet, Schekochihin & Chandran 2015): the linear and nonlinear time scales in each structure are comparable,  $\chi_{\lambda} \equiv \tau_{A\lambda}/\tau_{nl\lambda} \sim 1$ .<sup>3</sup> Thus, either time may be used as the cascade time scale  $\tau_{c\lambda}$ . The cascade power within the local subvolume of a particular structure is

$$\epsilon_{\lambda} \sim \frac{\delta z_{\lambda}^2}{\tau_{c\lambda}}. \tag{2.5}$$

Note that  $\langle \epsilon_{\lambda} \rangle = \delta z_{L_{\perp}}^3/L_{\perp} \equiv \epsilon$ , the injected power, for  $\lambda$  in the inertial range.

### 3. Damping model

In this work, we will assume that the damping mechanisms we study irreversibly dissipate energy that is removed from the Alfvénic cascade.<sup>4</sup> We can then relate the

<sup>1</sup>Provided that the critical balance conjecture (Goldreich & Sridhar 1995) is incorporated in an appropriate way – this will be explained further in §4.

<sup>2</sup>This statement is a slight simplification: in fact, it is a weighted combination of Poissons, which nevertheless exhibits the same statistics.

<sup>3</sup>This principle is obeyed both in numerical simulations of RMHD turbulence (Mallet *et al.* 2015), and in the solar-wind turbulence (Chen 2016).

<sup>4</sup>Technically, this irreversibility arises due to collisions (Schekochihin *et al.* 2009; Zocco & Schekochihin 2011; Loureiro, Schekochihin & Zocco 2013; Bañón Navarro *et al.* 2016; Pezzi, Valentini & Veltri 2016; Servidio *et al.* 2017). There is certainly no guarantee that the collisional heating occurs in the same spatial location as the damping: for example, Bañón Navarro *et al.* (2016) found that in gyrokinetic turbulence, collisional heating was not localised to current sheets, because it takes some time for the velocity-space features generated near the current sheet to phase-mix to small velocity-space scales, and during this time, the plasma containing those velocity-space features flows away from the current sheet (Schekochihin *et al.* 2009). Where exactly the final collisional entropy production happens does not affect the results in this paper.

heating rate  $Q_\lambda$  to the damping rate  $\gamma_\lambda$  via

$$Q_\lambda \sim \gamma_\lambda \delta z_\lambda^2. \quad (3.1)$$

To motivate our model, we begin with the non-intermittent cascade model of Howes *et al.* (Batchelor 1953; Howes *et al.* 2008, 2011), which in steady state far from the forcing wavenumber leads to

$$\frac{\epsilon_{k_\perp}}{\epsilon_{k_0}} = \exp \left( - \int_{k_0}^{k_\perp} 2\gamma_{k_\perp} \tau_{ck_\perp} \frac{dk_\perp}{k_\perp} \right), \quad (3.2)$$

where  $\epsilon_{k_\perp} = (\delta z_{k_\perp})^2 / \tau_{ck_\perp}$  is the cascade power at perpendicular wavenumber  $k_\perp$ ,  $\delta z_{k_\perp}$  is the turbulence amplitude at  $k_\perp$  and  $\gamma_{k_\perp}$  is the damping rate at  $k_\perp$ . In order to investigate different damping mechanisms analytically, we make the simplifying assumption that the damping is localised to one particular reference scale  $\rho$ , i.e.  $\gamma_{k_\perp} \tau_{ck_\perp} = \gamma_\rho \tau_{c\rho} \delta[\log(k_\perp \rho)]$ , where  $\delta[\dots]$  denotes the Dirac delta distribution. In practice, various potentially important forms of damping are localised around the ion gyroscale: for example, stochastic heating, and ion Landau damping at high  $\beta_i$  (Howes *et al.* 2006). The cascade power ( $\epsilon_{\rho-}$ ) and turbulence amplitude ( $\delta z_{\rho-}$ ) at  $k_\perp \rho = 1 + d$  may then be written in terms of their counterparts  $\epsilon_{\rho+}$  and  $\delta z_{\rho+}$  at  $k_\perp \rho = 1 - d$  (where  $d \ll 1$ ),

$$\epsilon_{\rho-} = \epsilon_{\rho+} \exp(-2\gamma_\rho \tau_{c\rho}), \quad (3.3)$$

$$\delta z_{\rho-} = \delta z_{\rho+} \exp\left(-\frac{2}{3}\gamma_\rho \tau_{c\rho}\right), \quad (3.4)$$

where to obtain (3.4) we use (2.3), assuming that damping affects the amplitude but not the dynamic alignment.<sup>5</sup>

To generalise this, note that if a turbulent structure has perpendicular scale  $\lambda$  and amplitude  $\delta z_\lambda$ , its fluctuation power  $\delta z_{k_\perp} \sim \delta z_\lambda$  peaks at  $k_\perp \sim 1/\lambda$ . We further assume (Kolmogorov 1962) that the local values of random variables in a structure set its dynamical time scales  $\tau_{c\lambda}$ ,  $\gamma_\lambda^{-1}$ , and promote all the variables in (3.3)–(3.4) to configuration-space random variables. We call  $\gamma_\rho \tau_{c\rho}$  the damping factor.

We would like to stress that, of course, collisionless damping mechanisms do not appear in RMHD, which models the (undamped) Alfvénic fluctuations at  $k_\perp \rho_i \ll 1$  (irrespective of collisionality). However, the intermittency at  $k_\perp \rho_i \sim 1$ , where collisionless damping appears in more complete models, is almost entirely produced by the turbulence in the (assumed to be long,  $L_\perp/\rho_i \gg 1$ ) inertial range in which RMHD is a good approximation. Thus, we model the intermittency using RMHD, and then add the dissipation in the simple way described above at the scale at which the RMHD approximation begins to break down.

We have made the rather drastic simplification that the damping only occurs over an infinitesimal scale interval. No real damping mechanism is truly this localised in scale. To go beyond this approximation, one would have to simultaneously integrate over scale not only the damping part of the process (as in Howes *et al.* 2008) but also the random part of the evolution describing the random log-Poisson evolution of the intermittent probability distribution of amplitudes; replacing the algebraic exponents

<sup>5</sup>Physically, this formula seems reasonable: exponential damping of a structure's amplitude  $\delta z_{\rho+}$  at rate  $2\gamma_\rho/3$  over its lifetime  $\tau_{c\rho}$  leads to precisely (3.4). The RMHD time scales (2.3) do not technically apply beyond the ion gyroscale; we assume the true dynamical time scales are continuous in  $\lambda$  (in the absence of damping), allowing us to use (2.3) to derive (3.4).

of (3.3) and (3.4) with functional integrals. This makes the model analytically intractable. Moreover, across any particular individual scale, the incremental damping of the fluctuations is well described by (3.3) and (3.4), which means that many of our results will not be qualitatively altered by making this approximation.

Finally, it is worth mentioning that other time scales could potentially enter the problem (Matthaeus *et al.* 2014); for example, waves could be excited via instability of the particle velocity distribution function. Indeed, Klein *et al.* 2018 have found that the majority of solar-wind plasma is unstable, although only approximately 10% appears to be strongly unstable in that the growth time is shorter than their estimate of the cascade time. We have ignored this possibility in our analysis here, and assume that the underlying velocity distribution function is stable.

#### 4. Linear Landau damping

One important and well-studied damping mechanism is linear Landau damping (Howes *et al.* 2006), for which the damping rate may be written (in Fourier space)

$$\gamma_{k_{\perp}} = F_{k_{\perp}} k_{\parallel} v_A, \tag{4.1}$$

where  $F_{k_{\perp}}$  is a function of  $k_{\perp}$  and plasma parameters, but not  $\delta z_{k_{\perp}}$ . Since (refined) critical balance states that (for all structures)  $k_{\parallel} v_A \sim \tau_{ck_{\perp}}^{-1}$ , the damping factor is

$$\gamma_{k_{\perp}} \tau_{ck_{\perp}} = F_{k_{\perp}} \iff \gamma_{\lambda} \tau_{c\lambda} = F_{\lambda} \tag{4.2}$$

where  $F_{\lambda}$  is a function of  $\lambda$  but not of  $\delta z_{\lambda}$ . This result is true for any intermittency model that incorporates refined critical balance, not solely in the MS17 model (Mallet & Schekochihin 2017); it is also the case in the CSM15 model (Chandran *et al.* 2015). (3.4) yields

$$\log \delta z_{\rho-} = \log \delta z_{\rho+} - (2/3)\gamma_{\rho} \tau_{c\rho}. \tag{4.3}$$

Because  $\gamma_{\rho} \tau_{c\rho}$  is independent of  $\delta z_{\rho+}$ , the effect of the damping is to shift the whole distribution of log-amplitudes over by the constant  $(2/3)\gamma_{\rho} \tau_{c\rho}$ ; i.e. the shape of the distribution is not changed. As a corollary, the kurtosis

$$\kappa_{\rho-}^{\text{LD}} = \frac{\langle \delta z_{\rho-}^4 \rangle}{\langle \delta z_{\rho-}^2 \rangle^2} = \frac{\langle \delta z_{\rho+}^4 \rangle e^{-(8/3)F_{\rho}}}{\langle \delta z_{\rho+}^2 \rangle^2 e^{-(8/3)F_{\rho}}} = \kappa_{\rho+} \tag{4.4}$$

is unchanged. Similarly, the average heating rate per unit volume,

$$\langle Q_{\rho}^{\text{LD}} \rangle = \langle \epsilon_{\rho+} - \epsilon_{\rho-} \rangle = (1 - e^{-2F_{\rho}}) \epsilon, \tag{4.5}$$

is not affected by the intermittency at all. However, if one looks at the structures in which the heating is happening, the intermittency is relevant: the heating rate random variable for each structure,

$$Q_{\rho}^{\text{LD}} = \epsilon_{\rho+} - \epsilon_{\rho-} = (1 - e^{-2F_{\rho}}) \epsilon_{\rho+}, \tag{4.6}$$

follows the (intermittent) distribution of the random variable  $\epsilon_{\rho+}$ , and damping is concentrated in the higher-amplitude, intermittent structures. Thus, Landau damping certainly does not lead to homogeneous wave damping – a point also made recently by Howes *et al.* (2018). These results would also apply generically to any damping mechanism for which the damping factor  $\gamma_{\rho} \tau_{c\rho}$  is independent of  $\delta z_{\rho}$ .

It might be naïvely thought that these results (4.4), (4.5) are obvious due to the linear nature of Landau damping. Thinking more carefully, these results only apply if the turbulence is critically balanced in the refined sense. For example, if the turbulence were isotropic ( $l_{\parallel} \sim \lambda$ ) at the gyroscale,  $\gamma_{\rho} \tau_{c\rho} \propto 1/\delta z_{\rho}$ . Likewise, if the turbulence were weak ( $l_{\parallel} \sim \text{const.}$  and  $\tau_c \sim \lambda^2 v_A / l_{\parallel} \delta z_{\rho}^2$ ),  $\gamma_{\rho} \tau_{c\rho} \propto 1/\delta z_{\rho}^2$ . In both these non-critically balanced cases, the Landau damping is less important in higher-amplitude structures, i.e. the heating rate is more homogeneous than the distribution of  $\epsilon_{\rho+}$ . This is yet another argument for why critical balance is a crucial organising principle for magnetised plasma turbulence, and for why one cannot neglect either linear or nonlinear physical phenomena when modelling such turbulence.

### 5. Stochastic heating

The damping rate of gyroscale fluctuations by stochastic heating may be written (Chandran *et al.* 2010)<sup>6</sup>

$$\gamma_{\rho} = \frac{c_1}{2} \frac{\delta z_{\rho}}{\rho} \exp\left(-\frac{c_2 v_{\text{th}}}{\delta z_{\rho}}\right). \quad (5.1)$$

We take  $c_1 = 0.75$  and  $c_2 = 0.34$  (cf. Chandran *et al.* 2010). The exponential suppression depends on the random variable

$$\xi \equiv \frac{\delta z_{\rho}}{v_{\text{th}}} \sim \beta_i^{-1/2} \frac{\delta z_{\rho+}}{v_A}, \quad (5.2)$$

where  $\beta_i = 8\pi n_i T_i / B_0^2$ . Using (2.3) and (2.4), the damping factor is

$$\gamma_{\rho} \tau_{c\rho} = \frac{c_1}{2} \left(\frac{L_{\perp}}{\rho}\right)^{1/2} \left(\frac{\delta z_{\rho+}}{\delta z_{L_{\perp}}}\right) \exp\left(-\frac{c_2 \beta_i^{1/2} v_A}{\delta z_{\rho+}}\right), \quad (5.3)$$

a (highly nonlinear) function of  $\delta z_{\rho+}$ .<sup>7</sup> In a qualitative sense, our results on the efficiency of intermittent stochastic heating and its effect on intermittency also apply generically to all mechanisms for which  $\gamma_{\rho} \tau_{c\rho}$  is an increasing function of  $\delta z_{\rho+}$ .

To illustrate our results, we use a numerically sampled log-Poisson distribution. We take the outer scale amplitudes  $\delta z_{L_{\perp}}$  to be distributed as the magnitude of a normal random variable with zero mean and standard deviation  $\sigma = 0.1 v_A$ . We multiply  $\delta z_{L_{\perp}}$  by the log-Poisson factor  $\Delta^q$  (2.1), generating  $10^7$  samples of the intermittent distribution  $\delta z_{\rho+}$  just above the gyroscale  $\rho$ . We then apply damping using (3.4) and (5.3) with various different values of  $\beta_i$  and  $L_{\perp}/\rho$ , obtaining the distributions of  $\delta z_{\rho-}$  used in figures 1–3.

### 6. Distribution of fluctuation amplitudes

The shape of the distribution of  $\log(\delta z_{\rho-}/\sigma)$  resulting from stochastic heating is shown for  $L_{\perp}/\rho = 10^4$  (a value similar to that in the solar wind) and various values of  $\beta_i$  (i.e. various different overall damping rates) in Figure 1. As the damping becomes

<sup>6</sup>The damping rate (5.1) is only expected to be valid at low  $\beta_i$ ; a different formula applies at high  $\beta_i$  (Hoppock *et al.* 2018); we do not consider this case.

<sup>7</sup>This is also true using the Chandran *et al.* (2015) intermittency model, although the precise amplitude dependence is different.

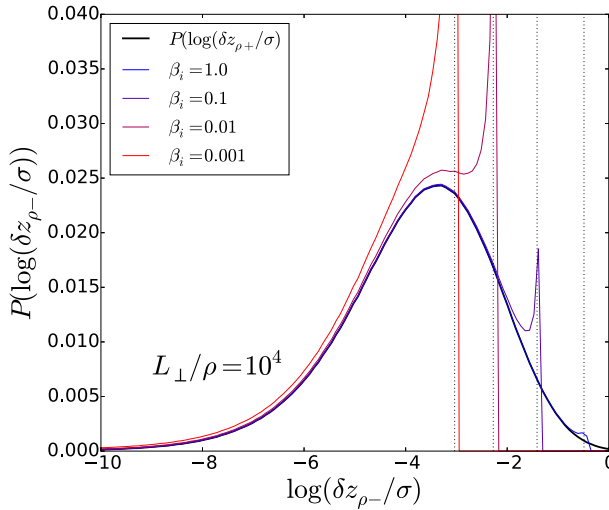


FIGURE 1. The distribution of  $\log(\delta z_{\rho-}/\sigma)$  resulting from (nonlinear) stochastic heating at  $\rho = 10^{-4}L_{\perp}$ , for various values of  $\beta_i = 1, 0.1, 0.01, 0.001$  (blue to red). Vertical dotted lines show  $\delta z_{\max}$  for each  $\beta_i$ . The distribution of  $\log(\delta z_{\rho+}/\sigma)$  is shown in black.

more important (i.e. at lower  $\beta_i$ ), the fluctuations with higher amplitude are heavily damped, causing a relatively sharp upper limit on  $\delta z_{\rho-}$ . This limit is the amplitude  $\delta z_{\max}$  for which

$$\left. \frac{d \log(\delta z_{\rho-})}{d \log(\delta z_{\rho+})} \right|_{\delta z_{\max}} = 0, \tag{6.1}$$

shown in figure 1 as a vertical dotted line for each  $\beta_i$ .

Because of this modification of the shape, the kurtosis (2.2) is heavily affected by the damping. In the inertial range, the kurtosis increases as  $\lambda$  decreases, reaching a value of  $\kappa_{\rho+} = 30$  just above  $\rho = 10^{-4}L_{\perp}$ . As the stochastic heating becomes more important (with decreasing  $\beta_i$ ), the kurtosis just below  $\rho$ ,  $\kappa_{\rho-}^{\text{SH}}$ , decreases significantly – see figure 2(a). Such a decrease in kurtosis is a generic property of nonlinear damping mechanisms for which  $\gamma_{\lambda} \tau_{c\lambda}$  is an increasing function of  $\delta z_{\lambda}$ .

### 7. Heating

Unlike in the linear case, the average stochastic heating rate

$$\langle Q_{\rho}^{\text{SH}} \rangle = \langle \epsilon_{\rho+} - \epsilon_{\rho-} \rangle = \langle (1 - \exp(-2\gamma_{\rho} \tau_{c\rho}) \epsilon_{\rho+}) \rangle \tag{7.1}$$

is affected by the intermittency of the turbulence. This heating rate may be compared with  $\langle \epsilon_{\rho+} \rangle = \langle \epsilon_{L_{\perp}} \rangle = \epsilon$ , and also with the heating rate that would be obtained without intermittency,  $Q_{\text{rms}}^{\text{SH}}$ , using the root-mean-square (r.m.s.) amplitude  $\delta z_{\text{rms}\rho+} \sim \sigma(\rho/L_{\perp})^{1/4}$  in place of the random variable  $\delta z_{\rho+}$ . These intermittent and r.m.s. heating rates, calculated using (5.3) and normalised to  $\epsilon$ , are shown in Figure 2(b), again with  $L_{\perp}/\rho = 10^4$ . The value of  $\beta_i$  at which the damping removes approximately half of the cascade power is significantly higher (by approximately a factor of 20) with intermittency:  $\langle Q_{\rho}^{\text{SH}} \rangle \gtrsim 0.5$  for  $\beta_i \lesssim 0.1$ , while  $Q_{\text{rms}}^{\text{SH}} \gtrsim 0.5$  for  $\beta_i \lesssim 0.005$ .

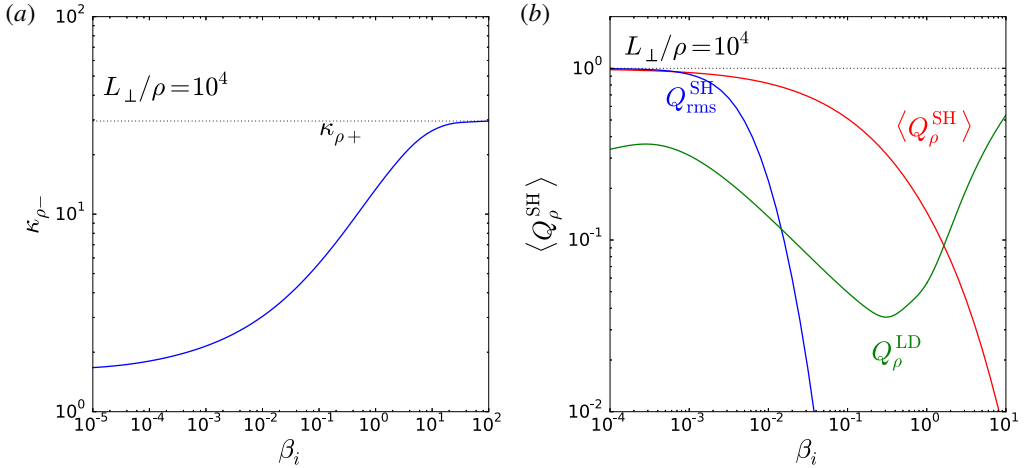


FIGURE 2. (a) In blue, the kurtosis after damping,  $\kappa_{\rho^-}^{\text{SH}}$ , as a function of  $\beta_i$ . The black dotted line is the kurtosis without damping  $\kappa_{\rho^+}$ . (b) The heating rates as functions of  $\beta_i$ :  $\langle Q_{\rho}^{\text{SH}} \rangle$  calculated using the intermittent distribution (red),  $Q_{\text{rms}}^{\text{SH}}$  using the root-mean-square (r.m.s.) turbulent amplitude (blue), and  $Q_{\rho}^{\text{LD}}$ , the linear Landau-damping heating rate, normalised to the cascade power  $\epsilon$ . In both panels,  $L_{\perp}/\rho = 10^4$  and the outer-scale turbulence amplitude distribution is fixed, parametrised by  $\sigma = 0.1v_A$  (see text).

Finally, we calculate the kinetic-Alfvén-wave damping rates  $\gamma_{\rho}^{\text{KAW}}$  and real frequencies  $\omega_{\rho}^{\text{KAW}}$  for  $k_{\perp}\rho = 1$  and  $k_{\parallel}/k_{\perp} = 10^{-3}$ , using the PLUME numerical Vlasov–Maxwell linear dispersion solver (Klein & Howes 2015), thus estimating the average heating rate from linear Landau damping,<sup>8</sup>  $\langle Q_{\rho}^{\text{LD}} \rangle$  (using (4.5) with  $F_{\rho} = \gamma_{\rho}^{\text{KAW}}/\omega_{\rho}^{\text{KAW}}$ ), plotted in figure 2(b). At the gyroscale, intermittent stochastic heating is comparable to linear Landau damping even for  $\beta_i = 1$ .

## 8. Length of the inertial range

The level of intermittency at the gyroscale  $\rho$  depends on the length of the inertial range  $L_{\perp}/\rho$  (cf. 2.2).  $Q_{\text{rms}}^{\text{SH}}$  is a strongly decreasing function of  $L_{\perp}/\rho$ , simply because the r.m.s. amplitude  $\delta z_{\text{rms}\rho^+} \sim \sigma(\rho/L_{\perp})^{1/4}$ . The intermittent stochastic heating rate  $\langle Q_{\rho}^{\text{SH}} \rangle$  has a weaker dependence on  $L_{\perp}/\rho$ , because intermittent, high-amplitude fluctuations in the MS17 model resemble discontinuities with (up to) the outer scale amplitude  $\delta z_{L_{\perp}}$ .

The dependence of the stochastic heating rates for  $\beta_i = 0.1, 1.0$  on  $L_{\perp}/\rho$  are shown in figure 3, along with  $\langle Q_{\rho}^{\text{LD}} \rangle$ . The weak dependence of  $\langle Q_{\rho}^{\text{SH}} \rangle$  on  $L_{\perp}/\rho$  means that, for  $\beta_i = 0.1$ , stochastic heating still removes approximately 10% of the overall cascade power at  $L_{\perp}/\rho \approx 10^{11}$ . Moreover, it remains comparable to  $\langle Q_{\rho}^{\text{LD}} \rangle$  up to  $L_{\perp}/\rho \approx 10^{12}$ . Thus, intermittency may have important astrophysical consequences: even at only moderately low  $\beta_i$ , stochastic heating may (i) convert a large portion of the total cascade power into ion thermal energy at the gyroscale in solar-wind turbulence, where  $L_{\perp}/\rho \approx 10^4$ , and (ii) be non-negligible (and comparable to linear Landau damping) in the warm interstellar medium (ISM), where  $L_{\perp}/\rho \approx 10^{11} - 10^{13}$  (Ferrière 2001; Cox 2005; Beck 2007; Haverkorn *et al.* 2008).

<sup>8</sup>Note that in the low beta regime, Landau damping is dominated by the transfer of energy to electrons.



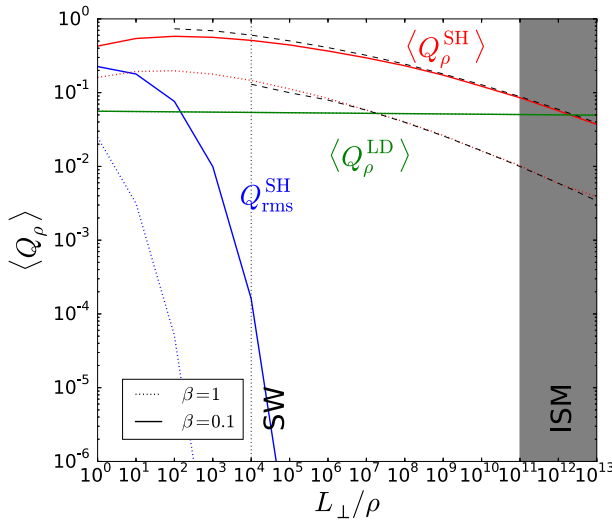


FIGURE 3. Heating rates (normalised to  $\epsilon$ )  $\langle Q_{\rho}^{SH} \rangle$  (red),  $Q_{rms}^{SH}$  (blue), and  $Q_{\rho}^{LD}$  (green), all at  $\beta_i = 0.1$  (solid lines) and  $\beta_i = 1$  (dotted lines) as a function of  $L_{\perp}/\rho$ . The two curves for  $Q_{\rho}^{LD}$  are nearly identical (and thus indistinguishable here – see figure 2(b)). Approximate ranges of  $L_{\perp}/\rho$  in the solar wind and in the warm ISM are labelled as SW (dotted line) and ISM (grey box).  $Q_{\rho}^*$  multiplied by a factor (4 for  $\beta_i = 0.1$  and 6.5 for  $\beta_i = 1$ ) is also shown (black dashed lines).

To explain the shallow dependence of  $\langle Q_{\rho}^{SH} \rangle$  on  $L_{\perp}/\rho$ , we calculate the minimum amplitude  $\delta z_{\rho+}^*$  for which fluctuations are strongly damped. Setting  $\gamma_{\rho} \tau_{c\rho} = 1$  in (5.3), we obtain

$$\log \left( \frac{\delta z_{\rho+}^*}{\delta z_{L_{\perp}}} \right) = W \left[ \frac{c_1 c_2 \beta_i^{1/2}}{2\sigma} \left( \frac{L_{\perp}}{\rho} \right)^{1/2} \right] - \log \left[ \frac{c_1}{2} \left( \frac{L_{\perp}}{\rho} \right)^{1/2} \right], \tag{8.1}$$

where  $W$  is the Lambert  $W$  function. This analytic expression for  $\delta z_{\rho+}^*$  approximates  $\delta z_{max}$  in (6.1). If  $\delta z_{\rho+}^*$  were determined by simply setting the exponent in (5.3) equal to some constant threshold value, then  $\delta z_{\rho+}^*$  would be independent of  $L_{\perp}/\rho$ . However, as  $L_{\perp}/\rho$  increases, the fluctuations are increasingly highly aligned (see (2.4)) at the gyroscale, which increases  $\tau_{c\rho}$  but not  $\gamma^{-1}$ . This introduces the factor  $(L_{\perp}/\rho)^{1/2}$  in (5.3), causing  $\delta z_{\rho+}^*$  to decrease with increasing  $L_{\perp}/\rho$ .

The corresponding heating rate from the damping of the structures with this amplitude is

$$Q_{\rho}^* \sim \frac{(\delta z_{\rho+}^*)^2}{\tau_{c\rho}} P(q^*) \sim \frac{[\log(L_{\perp}/\rho) \Delta^2]^{q^*}}{\sqrt{2\pi} q^* (q^*/e)^{q^*}} \epsilon, \tag{8.2}$$

where  $q^* = \log(\delta z_{\rho+}^*/\delta z_{L_{\perp}})/\log \Delta$  (cf. (2.1)), and we have used Stirling’s formula to approximate the factorial in the Poisson probability mass function.  $Q_{\rho}^*$  is a reasonable analytic estimate for the scaling dependence of  $\langle Q_{\rho}^{SH} \rangle$  on  $L_{\perp}/\rho$  for  $\log(L_{\perp}/\rho) \gg 1$ ; however, it is an underestimate (by a factor approximately independent of  $L_{\perp}/\rho$ ), due to (i)  $\delta z_{\rho+}^*$  being an overestimate of the true cutoff,  $\delta z_{max}$ , (ii) the neglect of the cascade power damped in structures with higher amplitudes  $\delta z_{\rho+}^* < \delta z_{\rho+} < \delta z_{L_{\perp}}$ ,

(iii) the neglect of the width of the outer scale (normal) distribution of fluctuation amplitudes. For each  $\beta_i$ ,  $Q_\rho^*$  multiplied by an empirical correction factor is plotted in figure 3. The analytic expressions for  $Q_\rho^*$  and  $\delta z_{\rho+}^*$  make clear that the slowly decreasing nature of  $\langle Q_\rho^{\text{SH}} \rangle$  with  $L_\perp/\rho$  arises due to a competition between the decreasing volume-filling fraction of structures above any particular amplitude and the decreasing cutoff amplitude  $\delta z_{\text{max}} (\approx \delta z_{\rho+}^*)$ .

In many astrophysical plasmas,  $\delta z_{L_0} \sim v_A$  at an outer scale  $L_0$  that is beyond the RMHD regime. We can apply our model in such cases on scales  $\lambda$  smaller than an effective outer scale  $L_\perp \ll L_0$ , where  $\delta z_{\text{rms}L_\perp} \approx 0.1v_A$ . For example, if  $\delta z_{\text{rms}\lambda} \propto \lambda^{1/4}$  at  $L_\perp < \lambda < L_0$ , then  $L_\perp = 10^{-4}L_0$ . The stochastic heating rate when the outer scale amplitude  $\delta z_{L_0} \sim v_A$  is then much larger than our numerical example, where  $\delta z_{L_0} \sim 0.1v_A$ , because the gyroscale fluctuation amplitudes are much larger. Our figures thus provide a highly conservative lower limit on the stochastic heating rate in plasmas in which  $\delta z_{L_0} \sim v_A$ .

## 9. Conclusions

In this study, we consider the collisionless damping of critically balanced, intermittent plasma turbulence by two mechanisms. We develop a new general modelling framework for studying the effects of intermittency on dissipation (and *vice versa*), and use it to make specific predictions for the efficiency of two different mechanisms. First, for linear Landau damping (Landau 1946; Howes *et al.* 2006, 2008, 2011),  $\gamma_\lambda \tau_{c\lambda}$  is independent of the turbulent amplitude  $\delta z_\lambda$ . In this case, (i) damping affects neither the shape of the distribution of log-fluctuation amplitudes, nor the kurtosis of the distribution of fluctuation amplitudes, and (ii) the overall efficiency of damping is not enhanced by the presence of intermittency. However, (iii) locally, damping is still concentrated near coherent structures (TenBarge & Howes 2013; Howes *et al.* 2018). Importantly, these results are not an inevitable consequence of the ‘linear’ nature of linear Landau damping: the efficiency would be strongly affected by intermittency if the turbulence did not obey the refined critical balance (Mallet *et al.* 2015).

On the other hand, for stochastic heating (Chandran *et al.* 2010),  $\gamma_\lambda \tau_{c\lambda}$  depends on  $\delta z_\lambda$ , leading to damping that (i) strongly affects the shape of the distribution of log-fluctuation amplitudes and the kurtosis of the distribution of fluctuation amplitudes. In addition, stochastic heating is (ii) much more efficient if one accounts for intermittency and (iii) even more concentrated near coherent structures than heating by linear Landau damping. Our results suggest that, once intermittency is incorporated, stochastic heating may be an important damping mechanism for solar-wind turbulence, and perhaps also for some regimes of interstellar turbulence, even when  $\xi \sim \delta z_{\text{rms}}/v_{\text{th}} \ll 1$  (in which case one would be justified in ignoring stochastic heating if the turbulence were not intermittent).

Our results can be easily extended to other dissipation mechanisms, which may be divided into different classes based on the (in)dependence of  $\gamma_\lambda \tau_{c\lambda}$  on  $\delta z_\lambda$ . This will allow us to quantitatively distinguish between different dissipation mechanisms in observations and simulations of collisionless plasma turbulence. We predict that a nonlinear heating mechanism (for which  $\gamma_\lambda \tau_{c\lambda}$  is an increasing function of  $\delta z_\lambda$ ) decreases the scale-dependent kurtosis just below the dissipation scale. This leads to a simple observational test to establish the presence of a nonlinear mechanism. Indeed, there are numerous observations of decreases in or flattening of the scale-dependent kurtosis at around the ion scale in both numerical and solar-wind turbulence

(Sundkvist *et al.* 2007; Alexandrova *et al.* 2008; Wan *et al.* 2012; Wu *et al.* 2013; Leonardis *et al.* 2016); our model provides a natural explanation for this phenomenon (however, we cannot explain why the scale-dependent kurtosis remains rather constant in the range of scales between the ion and electron gyroradii, as in the results of Wu *et al.* (2013) and Chen *et al.* (2014)). Moreover, there is direct evidence for a nonlinear ion heating mechanism, whose efficiency depends on  $\xi$  (suggestive of stochastic heating, cf. (5.2)), in some numerical simulations (Matthaeus *et al.* 2016; Grošelj *et al.* 2017; Shay *et al.* 2018), while electron heating appears to have  $\gamma_\rho \tau_{cp}$  independent of  $\delta z_\rho$  (suggestive of linear Landau damping; see also Bañón Navarro *et al.* (2016) and Chen, Klein & Howes (2019)). Our new modelling framework provides a useful way to interpret these simulation results.

Our results clarify the role of intermittency in heating by collisionless plasma turbulence: since heating rates for nonlinear mechanisms (e.g. stochastic heating) are dramatically enhanced by intermittency, an understanding of the intermittency is essential for determining relative heating rates of different mechanisms, and thus for explaining the eventual thermodynamic state of a turbulent collisionless plasma.

### Acknowledgements

We thank A. A. Schekochihin and R. Meyrand for useful discussions. A.M. was supported by NSF grant AGS-1624501. K.G.K. was supported by NASA grant NNX16AM23G. C.S.S. was supported by NASA grant NNX16AI59G and NSF SHINE 1622498. Work at Berkeley is also supported by NASA grants NNX14AJ70G and NNX16AP95G. B.D.G. Chandran was supported by NASA grants NNX15AI80, NNX16AG81G, NNN06AA01C and NNX17AI18G, and NSF grant PHY-1500041.

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