Analytic functions over Banach algebras

R.W. Perry

The purpose of this thesis is twofold:

(a)   to examine the complete analytic function determined by a
      function element over a complex Banach algebra M ; and

(b)   to construct a Riemann hypersurface on which the resulting
      analytic configuration can be represented.

The construction is modelled on an old definition of Giorgi for analytic
matrix functions in the neighbourhood of a diagonalizable matrix (see [3]).
It is motivated by an observation of Gibson and Szekeres ([1]) that, in the
case of 2×2 and 3×3 matrices, the non-diagonalizable function
values can be obtained by a suitable limiting process.

In order to transfer the definition from matrices to general Banach
algebras, it is necessary to assume that M contains a maximal commutative
B^*-subalgebra D , which takes the place of the subalgebra of diagonal
matrices. The generalization leads naturally to meromorphic function
elements and to the definition of points at infinity, which must be
adjoined to M in order that values of meromorphic functions at their
poles may be assigned.

The Riemann hypersurface is constructed from an ordinary (complex)
Riemann surface R and the space $R(N)$ of continuous functions from the
Gelfand structure space N of D into R . The space is provided with a
suitable analytic structure, making essential use of a theorem of Gunning
and Narasimhan [2] on the holomorphic immersion of non-compact open sets of
R into C .

The thesis also examines meromorphic differentials and their integrals

on the hypersurface. It turns out that the residues of differentials can only take certain special values on M, corresponding to conjugates of functions on $R(N)$ which take constant values on the members of a partition of N into a finite number of compact subsets. For instance, if N is connected all residues are scalar multiples of the identity of M.

References

[1]   J.K. Gibson and G. Szekeres, "Analytic matrix functions", *Nieuw Arch. Wisk.* (3) 15 (1967), 233-246.

[2]   R.C. Gunning and Raghavan Narasimhan, "Immersion of open Riemann surfaces", *Math. Ann.* 174 (1967), 103-108.

[3]   R.F. Rinehart, "The equivalence of definitions of a matric function", *Amer. Math. Monthly* 62 (1955), 395-414.