Near Earth Objects, our Celestial Neighbors: Opportunity and Risk
Proceedings IAU Symposium No. 236, 2006
A. Milani, G.B. Valsecchi & D. Vokrouhlický, eds.

doi:10.1017/S1743921307003572

Admissible regions for too short arcs: nodal distances and elongations

Stéphane Valk and Anne Lemaitre

Department of Mathematics, University of Namur, Belgium
email: stephane.valk@fundp.ac.be, anne.lemaitre@fundp.ac.be

Abstract. This study is based on the definition of the admissible region introduced by Milani *et al.* (2004); in the search for potential Earth impactors, this theory allows to take into account the partial data of the TSA (*Too Short Arcs*) from which it is impossible to deduce a full orbit. Only a set of 4 variables (two angles and their instantaneous time derivatives), called an *attributable*, is known; a few suitable boundary conditions allow to restrict the motions to a specific bounded 2-dimensional region. In this work, a new inner boundary of this region is introduced, based on the geocentric hyperbolic motion of the immediate impactors; the nodal distances (crossings of the virtual asteroidal orbits with the Earth's orbit) are drawn for two different test attributables, associated with a determination of circular and linear orbits. This could reduce the search for impactors (by propagation of the orbits) to a one-dimensional set. A few comments about elongations and complementary curves complete this paper.

Keywords. orbit determination; space debris; impact risk; MOID

1. Introduction

When a non identified object is observed, the first reaction of the scientific community is to try to determine its orbit. Unfortunately, for the data collected on very short periods of time, the arc of observation is not large enough to give any estimation of the curvature; the determination of the orbit is impossible, using traditional methods of orbital determination, such as Gauss method. If we intend to build a complete catalog of such objects, the conclusion is easy: this object is rejected, and the observers hope to be luckier a few months or years later, to re-observe the same body, on a larger timescale. However, in many cases, the right ascension, the declination and their instantaneous time derivatives are measured.

For the last few years, associations like Space Guard or the specialists of the Near-Earth Asteroids (Minor Planet Center† or NEODYS group‡) have a completely different point of view concerning these unexpected observed objects. The main question is not only the improvement of their orbit, but also the potential hazard that they represent for the Earth: could this unknown body becomes dangerous for us, in a delay of one or two hundreds years?

Virtually, this too short arc (TSA) corresponds to an infinity of orbits. We assume, because it is true in many cases, that its right ascension α and declination δ are know, as well as their time derivatives; on the opposite, there is no data concerning either their distance to the Earth or the time derivative of this distance. Consequently, on a set of six variables $(\alpha, \delta, \dot{\alpha}, \dot{\delta}, r, \dot{r})$, the first four are determined with a specific accuracy, while

† http://cfa-www.harvard.edu/cfa/ps/mpc.html
‡ http://newton.dm.unipi.it/cgi-bin/neodys/neoibo

the last two are completely arbitrary. This means that the object lies in a 2 dimensional subspace of a general 6 dimensional space.

This idea was introduced by Milani *et al.* (2004) and pushed further on in Milani *et al.* (2005a) and Milani *et al.* (2005b). This incomplete set of data (2 angles and 2 time derivatives) is called an "*attributable*" by the authors mentioned above and this denomination is conserved here. Thanks to reasonable hypotheses (the fact that the object belongs to the Solar System, or that it is not a satellite of the Earth), Milani *et al.* (2004) proved that this region, in the plane $(r, \dot{r})$ could be closed and formed of one or two connected sets. Curves of constant values of the osculating keplerian elements can be drawn on this region.

Unfortunately, if a second observation is not available, the admissible region is still very large; one of the challenges is to follow the propagation of this admissible region, by means of linear and non linear techniques, in order to compare its evolution with a potential new arc.

Our purpose here is to concentrate on some aspects of the initial admissible region. Firstly, we recalculate one of its boundaries, for the short distances to the Earth, introducing the hyperbolic shape of the orbit instead of its linear approximation ; secondly, we introduce, on the admissible zone, and in complement of the keplerian elements information, the nodal distances, corresponding to the intersections of the Earth and potential Earth's impactors orbits. We present different situations, where the singularity in inclination is inside or outside the admissible region, following the chosen attributable. These curves could be very interesting in the context of propagation of the motions, reducing the dimension of the admissible region (dimension one instead of two). Thirdly we introduce the concept of elongations on the graphics.

2. The admissible region

Let $\vec{P}_{\mathcal{A}}$ and $\vec{V}_{\mathcal{A}}$ be the heliocentric position and velocity vectors of a celestial body $\mathcal{A}$ at a reference time t. At the same time, the position vector $\vec{P}_{\oplus}$ and the velocity vector $\vec{V}_{\oplus}$ of the Earth are well known.

The heliocentric energy per unit mass of $\mathcal{A}$ is given by

$$E_{\odot} = \frac{1}{2}\|\vec{V}_{\mathcal{A}}\|^2 - k_{\odot}^2 \frac{1}{\|\vec{P}_{\mathcal{A}}\|} \tag{2.1}$$

and its geocentric energy takes the form

$$E_{\oplus} = \frac{1}{2}\|\vec{V}_{\mathcal{A}} - \vec{V}_{\oplus}\|^2 - k_{\oplus}^2 \frac{1}{\|\vec{P}_{\mathcal{A}} - \vec{P}_{\oplus}\|} \tag{2.2}$$

where $m_{\oplus}$ and $m_{\odot}$ are the masses of the Earth and of the Sun respectively. Gauss' constant is defined by: $k_{\odot} = \sqrt{Gm_{\odot}} = 0.01720209895$ and $k_{\oplus}^2 = k_{\odot}^2 \frac{m_{\oplus}}{m_{\odot}}$. The solar mass is taken as the mass unit, the mean semi major axis of the Earth orbit is the distance unit (AU) and the average day is the time unit.

An attributable is defined as a fourth dimension vector

$$\vec{A} = (\alpha, \delta, \dot{\alpha}, \dot{\delta}) \in [-\pi, \pi[\times] - \frac{\pi}{2}, \frac{\pi}{2}[\times \mathbb{R}^2 \tag{2.3}$$

computed at the time t and to which an apparent magnitude M can be associated. We use the classical geocentric equatorial coordinates (α, δ) with α, the right ascension, and δ, the declination. The position vector $\vec{P}_{\mathcal{A}}$ can be expressed as

$$\vec{P}_{\mathcal{A}} = \vec{P}_{\oplus} + r\,\vec{u} \tag{2.4}$$

where r is the geocentric distance of the body $\mathcal{A}$ and $\vec{u}$ is the unit vector in the direction of the observation

$$\vec{u} = (\cos\alpha\cos\delta, \sin\alpha\cos\delta, \sin\delta). \tag{2.5}$$

The first time derivative of equation (2.4) gives the velocity vector

$$\vec{V}_{\mathcal{A}} = \vec{V}_{\oplus} + \dot{r}\,\vec{u} + r\,\dot{\alpha}\,\vec{u}_{\alpha} + r\,\dot{\delta}\,\vec{u}_{\delta}, \tag{2.6}$$

where

$$\vec{u}_{\alpha} = (-\sin\alpha\cos\delta, \cos\alpha\cos\delta, 0) \tag{2.7}$$

$$\vec{u}_{\delta} = (-\cos\alpha\sin\delta, -\sin\alpha\sin\delta, \cos\delta) \tag{2.8}$$

The geocentric position and velocity vectors can be computed as functions of r and $\dot{r}$

$$\|\vec{P}_{\mathcal{A}} - \vec{P}_{\oplus}\|^2 = r^2 \tag{2.9}$$

$$\|\vec{V}_{\mathcal{A}} - \vec{V}_{\oplus}\|^2 = \dot{r}^2 + r^2\,\dot{\alpha}^2\ \cos^2\delta + r^2\,\dot{\delta}^2 = \dot{r}^2 + r^2\eta^2 \tag{2.10}$$

where

$$\eta = \sqrt{\dot{\alpha}^2\cos^2\delta + \dot{\delta}^2} \tag{2.11}$$

is the proper motion. The energies are given by

$$E_{\oplus} = \frac{1}{2}\left[\dot{r}^2 + r^2\,\eta^2 - \frac{2k_{\oplus}^2}{r}\right] \tag{2.12}$$

and

$$E_{\odot} = \frac{1}{2}\left[\dot{r}^2 + c_1\,\dot{r} + W(r) - \frac{2k_{\odot}^2}{\sqrt{S(r)}}\right] \tag{2.13}$$

where the quantities $W(r)$ and $S(r)$ are functions of the geocentric distance r (see Milani *et al.* (2004) for details).

To determine the admissible regions, let us recall the conditions chosen by Milani *et al.* (2004):

(a) $\mathcal{A}$ is not a satellite of the Earth:

$$\mathcal{D}_1 = \{(r,\dot{r}) : E_{\oplus} \geqslant 0\}$$

(b) $\mathcal{A}$ is not influenced by the Earth's gravity field (of radius R_{SI}):

$$\mathcal{D}_2 = \{(r,\dot{r}) : r \geqslant R_{SI}\}$$

(c) $\mathcal{A}$ is on an elliptic orbit around the Sun:

$$\mathcal{D}_3 = (\{r,\dot{r}) : E_{\odot} \leqslant 0\}$$

(d) $\mathcal{A}$ is obviously outside the Earth's globe (of radius $R_{\oplus}$):

$$\mathcal{D}_4 = \{(r,\dot{r}) : r \geqslant R_{\oplus}\}$$

The admissible region is defined as

$$\mathcal{D} = \{\mathcal{D}_1 \cup \mathcal{D}_2\} \cap \mathcal{D}_3 \cap \mathcal{D}_4 \tag{2.14}$$

A schematic example of such a region is given in Figure 1.

https://doi.org/10.1017/S1743921307003572 Published online by Cambridge University Press

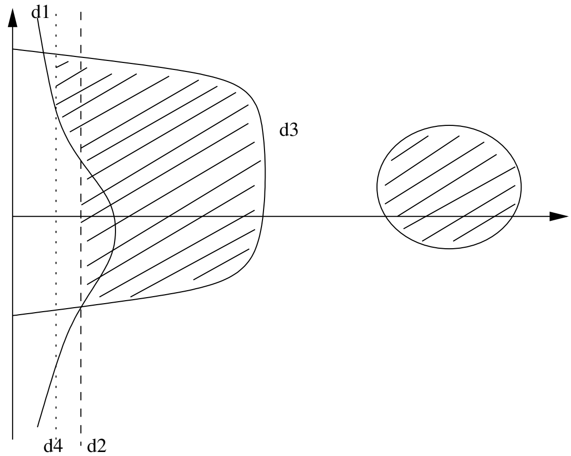

Figure 1. The topology of an admissible region with two connected components in the plane $(r, \dot{r})$; the curves d_i, for $i = 1, 2, 3, 4$ are the boundaries of the regions $\mathcal{D}_i$.

3. Immediate impact trajectory and inner boundary

Our first purpose is to exclude, from the admissible region $\mathcal{D}$, all the objects which are on a collision course with the Earth within a short time span; as mentioned by Milani *et al.* (2004), this gives an additional boundary for the left part of the admissible region, between d_4 and d_2 where the curves d_i are the boundaries of the regions $\mathcal{D}_i$. It is based on the assumption that the trajectory of the body $\mathcal{A}$ is rectilinear and can be written as

$$\frac{\eta\, r^2}{|\dot{r}|} \geqslant R_\oplus \tag{3.1}$$

where $R_\oplus = 4.26352 \times 10^{-5}$ AU. However, when the geocentric speed $\dot{r}$ is low, the hypothesis of linearity is not suitable anymore and the boundary description can be easy improved by using a two body (keplerian) formalism: the object is assumed to move within the sphere of influence of the Earth where its orbit is only controlled by the Earth's gravity field. In this context, our new condition can be expressed as

$$q_\oplus(r, \dot{r}) - R_\oplus > 0 \tag{3.2}$$

where $q_\oplus$ is the perigee corresponding to the geocentric orbit of the near-Earth object $\mathcal{A}$. This latest expression has no simple analytic form as a function of $(r, \dot{r})$, nevertheless it can be computed numerically. As it is obvious that $q_\oplus = a_\oplus\,(1 - e_\oplus)$, we compute the semi-major axis $a_\oplus$ (negative for hyperbolic orbits) and the eccentricity $e_\oplus$ by

$$a_\oplus = -\frac{k_\oplus^2}{2\,E_\oplus} \qquad\qquad e_\oplus = \sqrt{1 - \frac{C_\oplus^2}{a_\oplus\, k_\oplus^2}}$$

where

$$C_\oplus = \|\vec{C}_\oplus\| = \|(\vec{P}_\mathcal{A} - \vec{P}_\oplus) \times (\vec{V}_\mathcal{A} - \vec{V}_\oplus)\| = r^2\,\eta$$

is the norm of the angular momentum $\vec{C}_\oplus$ of the geocentric orbit. The linear and keplerian boundaries are represented in Figure 2 in a descriptive way and in the plane $(r, \dot{r})$ on Figure 3. The two conditions are very similar, even if, as expected, for low radial geocentric speed $\dot{r}$, the differences are significant. Nevertheless, as already pointed out by Milani *et al.* (2004), our new condition is only useful for the discovery of very small objects or of objects with a very small apparent magnitude. In other words, this condition

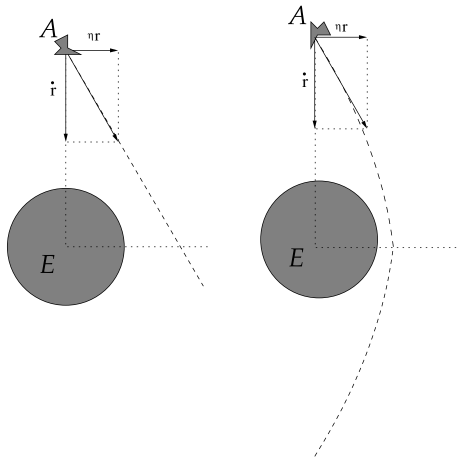

Figure 2. Immediate impact trajectory in the linear and keplerian cases (schematic representation).

allows to discriminate between the population of asteroids and that of future shooting stars.

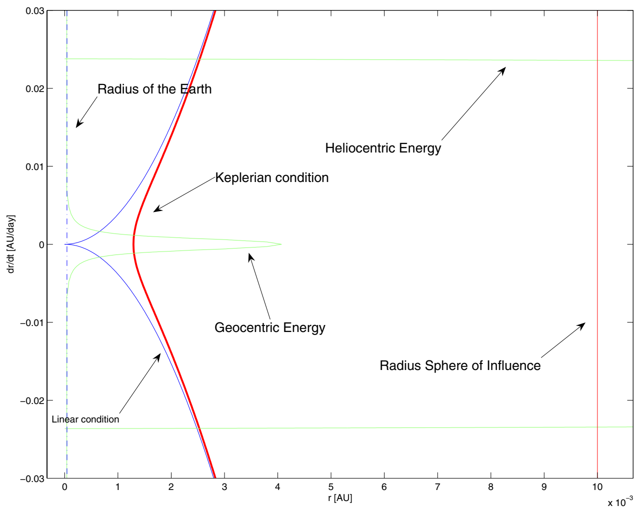

Figure 3. Immediate impact trajectory in the linear and keplerian cases; represented in the $(r, \dot{r})$ plane where r is given in astronomical unit.

A particular region appears clearly in Figure 3, inside the curve "geocentric energy" and to the right of the curve "keplerian condition": this is a region of bodies orbiting around the Earth. Of course, most of the artificial satellites have very precise orbits and do not require such a study. However, it is not always the case for Space Debris where orbital uncertainty is a common fact especially in the case of uncatalogued geostationary Space Debris. A detailed analysis of this confined area, associated to a suitable model of propagation, would give an interesting tool to follow this population and to measure the risk for future missions.

https://doi.org/10.1017/S1743921307003572 Published online by Cambridge University Press

4. Nodal distances

Our second purpose is to define the subset of this admissible region associated with the objects which impact the Earth. The necessary conditions for an impact between the Earth (considered on a circular heliocentric orbit) and an hypothetic object can be easily formulated: the object and the Earth should be exactly at one of the nodes of the orbit at the same time

$$\Omega - \lambda_E = \frac{\pi}{2} \mp \frac{\pi}{2} \tag{4.1}$$

$$\omega + f = \frac{\pi}{2} \mp \frac{\pi}{2} \tag{4.2}$$

where λ_E is the longitude of the Earth, on its geocentric circular orbit, measured in the ecliptic plane, Ω, ω and f are respectively the longitude of the ascending node, the argument of the pericenter and the true anomaly of the heliocentric orbit of the body $\mathcal{A}$, in an ecliptic reference frame. The upper sign corresponds to an impact at the ascending node, and the lower sign to an impact at the descending node. For a collision at the ascending node, the so called *ascending nodal distance* must vanish

$$d_+ = \frac{a(1-e^2)}{1+e\cos\omega} - a_E = 0 \tag{4.3}$$

where a_E is the Earth's semi-major axis, a this of the body $\mathcal{A}$ and e its eccentricity. We have a very similar condition for a collision at the descending node, for the *descending nodal distance*

$$d_- = \frac{a(1-e^2)}{1-e\cos\omega} - a_E = 0 \tag{4.4}$$

For any set of values of $(r, \dot{r})$ in the admissible zone, we compute numerically the orbital elements using the usual transformations

$$a = \frac{k_\odot^2}{2\,E_\odot} \qquad e = \sqrt{1 - \frac{C^2}{a\,k_\odot^2}} \qquad \cos I = \left(\frac{C_z}{C}\right)$$

where $\vec{C} = (C_x, C_y, C_z)$ is the angular momentum of the body on its heliocentric orbit and C is its norm. $E_\odot$ is defined by the Equation (2.13). A smart way for calculating ω is to use a scalar product between the line of nodes and the Laplace vector defined by

$$\vec{q} = \vec{V}_\mathcal{A} \times \vec{C} - k_\odot^2 \frac{\vec{P}_\mathcal{A}}{\|\vec{P}_\mathcal{A}\|} \tag{4.5}$$

We present two very different cases; in the first one (corresponding to the attributable $\alpha = 2.018$, $\delta = 0.204$, $\dot{\alpha} = -0.00623$ and $\dot{\delta} = 0.000302$), the level curve $I = 90°$ divides the admissible region into two parts; the level curves seem to converge towards a point located outside the admissible region. In the second one (corresponding to the attributable $\alpha = 2.018$, $\delta = -1.204$, $\dot{\alpha} = -0.0623$ and $\dot{\delta} = 0.00302$), the *convergence* point of the inclination curves is inside the admissible region. This point corresponds to a singularity: the orbit is so elliptic than it becomes a straight line; it means that the inclination is not defined anymore, the orbital plane being reduced to a line.

For both cases, the level curves of the ascending (solid) and descending (bold) nodal distances are plotted, giving a clear idea about the location of virtual impactors in the admissible zone (Figure 4). In the first case, on the left part of Figure 4, the ascending and descending nodal distances curves have no intersection, except for the case $r = 0$ and $\dot{r} = 0$ (i.e. the orbit of the Earth) which is obviously common to both conditions. On the opposite, on the right part of Figure 4, they cross several times, inside the admissible region, for different types of non circular and non coplanar orbits.

The location of the nodal distances in the admissible region is crucial to determine the potential hazard of this attributable. Indeed, it is easy to sample the curves $d_+ = 0$ or $d_- = 0$; to each point of this subset corresponds a set of six orbital elements, i.e. an orbit and an instantaneous position on this orbit. By propagating the motions of the body (on a keplerian orbit, for the simplest case) and of the Earth, we can rapidly check whether a close encounter is scheduled or not for the next few tens of years. By close encounter, we mean that the body enters the sphere of influence of the Earth. A that moment, another analysis has to be developed, using specific variables and formulae (see for example Öpik (1976) and Valsecchi *et al.* (2003)) to make the final model of approach and detect a significant probability of impact.

In a less restrictive use, this new information (the nodal distances location) may also be of great interest to improve the choice of the metric function to enhance some important subsets of the admissible region for future propagation (Milani *et al.* (2004)).

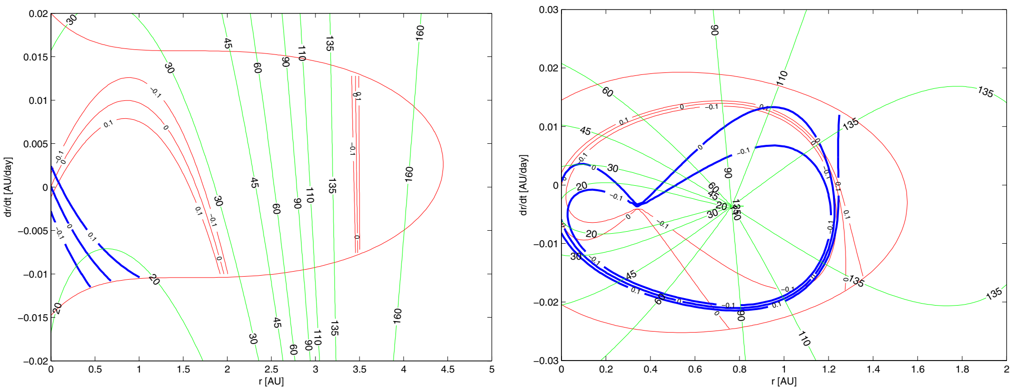

Figure 4. Level curves of the inclination (I = 20, 30, 45, 60, 90, 110, 135, 160 degrees), of the ascending ($d_+ = -0.01, 0, 0.01$ [solid]) and descending ($d_- = -0.01, 0, 0.01$ [bold]) nodal distances for the attributable ($\alpha = 2.018$, $\delta = 0.204$, $\dot{\alpha} = -0.00623$, $\dot{\delta} = 0.000302$) on the left and ($\alpha = 2.018$, $\delta = -1.204$, $\dot{\alpha} = -0.0623$, $\dot{\delta} = 0.00302$) on the right. The unit is the astronomical unit.

5. Circular and linear orbits

Let us draw the contour levels of the eccentricity in the admissible zone, for the two selected attributables (Figure 5). There are two apparent centers of circular orbits: the first one coincides with the Earth itself ($r = 0$ and $\dot{r} = 0$) which is assumed to be on a circular orbit; the second one is more interesting and can be characterized as a solution of the two following equations

$$\dot{r} = -\frac{A\, r}{r + B} \tag{5.1}$$

$$p_7 + 2\left[p_2\, \dot{r} + p_4\, r\, \dot{\alpha} + p_6\, r\, \dot{\delta}\right] + \dot{r}^2 + r^2\, \dot{\alpha}^2\, \cos^2\delta + r^2 \dot{\delta}^2 = \frac{k^2}{\sqrt{p_0 + 2\, p_1\, r + r^2}} \tag{5.2}$$

where

$$A = p_2 + p_3\, \dot{\alpha} + p_5\, \dot{\delta}$$
$$B = p_1$$

https://doi.org/10.1017/S1743921307003572 Published online by Cambridge University Press

as well as

$$
\begin{array}{lll}
p_0 = \langle \vec{P}_\oplus, \vec{P}_\oplus \rangle & p_7 = \langle \vec{V}_\oplus, \vec{V}_\oplus \rangle & \\
p_1 = \langle \vec{P}_\oplus, \vec{u} \rangle & p_3 = \langle \vec{P}_\oplus, \vec{u}_\alpha \rangle & p_5 = \langle \vec{P}_\oplus, \vec{u}_\delta \rangle \\
p_2 = \langle \vec{V}_\oplus, \vec{u} \rangle & p_4 = \langle \vec{V}_\oplus, \vec{u}_\alpha \rangle & p_6 = \langle \vec{V}_\oplus, \vec{u}_\delta \rangle
\end{array}
$$

These equations were obtained by combining two conditions characterizing circular orbits. First, the position vector $\vec{P}_{\mathcal{A}}$ must be perpendicular to the velocity vector $\vec{V}_{\mathcal{A}}$

$$\langle \vec{P}_{\mathcal{A}}, \vec{V}_{\mathcal{A}} \rangle = 0 \tag{5.3}$$

secondly, the orbital heliocentric velocity must correspond to

$$\| \vec{V}_{\mathcal{A}} \|^2 = \frac{k_\odot^2}{a}$$

that is

$$\| \vec{V}_{\mathcal{A}} \|^2 \, \| \vec{P}_{\mathcal{A}} \| = k_\odot^2 \tag{5.4}$$

Let us notice that all the values of the eccentricities lie between 0 and 1, the values outside the admissible region correspond to hyperbolic orbits. The curve $a = 1$ (more visible on the right diagram) corresponds to the positions of Earth's Trojans. We plot

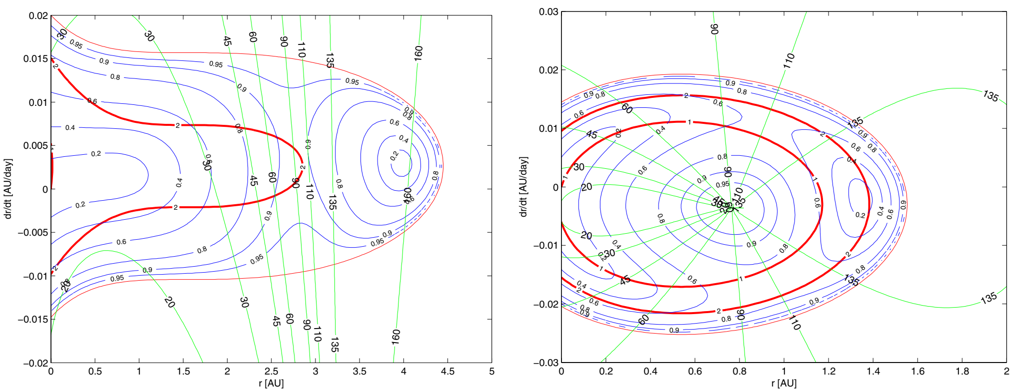

Figure 5. Values of the eccentricity ($e = 0.2, 0.4, 0.6, 0.8, 0.9$) and of the inclination ($I = 20, 30, 45, 60, 90, 100, 135, 160$ degrees) for the two test attributables: ($\alpha = 2.018$, $\delta = 0.204$, $\dot{\alpha} = -0.00623$, $\dot{\delta} = 0.000302$) on the left and ($\alpha = 2.018$, $\delta = -1.204$, $\dot{\alpha} = -0.0623$, $\dot{\delta} = 0.00302$) on the right. Two level curves of the semi-major axis are also drawn corresponding to $a = 1$ AU and $a = 2$ AU [bold].

the curves corresponding to conditions (5.3) and (5.4) in Figure 6. The condition (5.3) describes the objects which are exactly at perihelion or aphelion dividing the admissible region into two distinct parts. The intersections of the curves (5.3) and (5.4) give the circular orbits in the admissible region. Beside the obvious case ($\dot{r} = 0, r = 0$), two potential circular orbits appear in the case of the first attributable and only one for the second one. The supplementary solution hidden in (Figure 5) appears clearly as shown in (Figure 6, left). Let us remark that the virtual impactors detected by vanishing the nodal distances (except in the trivial case) correspond to non-circular orbits, which is obvious, the Earth moving on a circular orbit itself.

https://doi.org/10.1017/S1743921307003572 Published online by Cambridge University Press

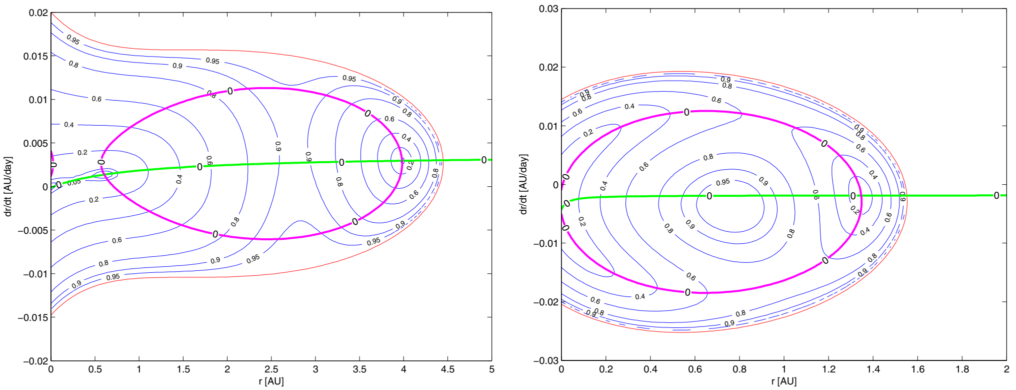

Figure 6. Condition (5.3) and (5.4) for the two test attributables: ($\alpha = 2.018$, $\delta = 0.204$, $\dot{\alpha} = -0.00623$, $\dot{\delta} = 0.000302$) on the left and ($\alpha = 2.018$, $\delta = -1.204$, $\dot{\alpha} = -0.0623$, $\dot{\delta} = 0.00302$) on the right.

6. Elongations and related angular distance

Let us remind that the attributable consists of two angles and their time derivatives; usually they are connected to α, the right ascension and δ, the declination, the geocentric equatorial coordinates, but they could also be replaced by λ and β, the ecliptic longitude and latitude of the object, or deduced from each other thanks to the relations.

$$\begin{aligned}
\cos\beta\cos\lambda &= \cos\delta\cos\alpha \\
\cos\beta\sin\lambda &= \sin\epsilon\sin\delta + \cos\epsilon\cos\delta\sin\alpha \\
\sin\beta &= \cos\epsilon\sin\delta - \sin\epsilon\cos\delta\sin\alpha
\end{aligned}$$

where ϵ is the obliquity, i.e the angle between the ecliptic and the equatorial plane. A quantity directly linked to the attributable is the *elongation*, denoted by ϕ, the angular distance between the Sun and the body $\mathcal{A}$ as viewed from the Earth. The elongation is given by the expression

$$\cos\phi = -x_E\cos\lambda\cos\beta - y_E\sin\lambda\cos\beta \tag{6.1}$$

where $(x_E, y_E, 0)$ is the heliocentric position of the Earth on its circular ecliptic orbit. For the first attributable, the elongation is $\phi = 166.87°$. This value suggests that the observations have been performed close to the opposition ($\phi = 180°$). The elongation value corresponding to the second attributable is $\phi = 91.48°$. In this particularly case, the observations would have been acquired near quadrature.

Let us notice that all the virtual asteroids corresponding to the same attributable have the same elongation. On the contrary the opposite angle θ, between the object $\mathcal{A}$ and the Earth, as viewed from the Sun, for a fixed elongation, is a function of r and its level contours are vertical lines in the admissible region, as shown in (Figure 6). Let us remark that the two selected attributables have quite different proper motions. Indeed, we have $\eta_2 \approx 4\,\eta_1$ with $\eta_1 = 6.1 \times 10^{-3}$ rad/day where η_1 and η_2 are the proper motions of the first and second attributable respectively. On the other hand, the declination δ of the second attributable differs significantly from the first one giving to this last case a more theoretical and singular aspect. As a consequence, the internal structure of the admissible region associated to the second attributable shows several uncommon properties such as the inclination singularity for example.

https://doi.org/10.1017/S1743921307003572 Published online by Cambridge University Press

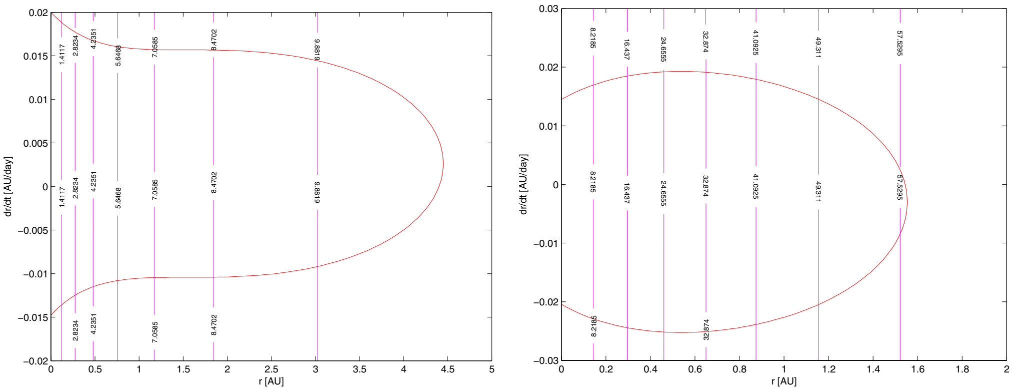

Figure 7. The vertical lines correspond to the values of opposite angular distance θ (in degrees), for the two selected attributables.

7. Conclusion

The topology of the admissible region is clearly dependent on the selected attributable, as shown by our two test attributables, corresponding to two very different observations: at the opposition or at the quadrature.

We have showed that the number and the positions of the potential circular orbits, the location, the shape and the length of the ascending and descending nodal distances, the behaviour of the inclination level curves, are very different from one situation to the other one and contain many informations about any virtual body compatible with the partial set of observations.

In the search for potential Earth's impactors, we propose to complete this preliminary study in two directions: the first idea would be to sample the curves of zero nodal distances, and to propagate this set of points for several years. This procedure reduces in a significant way the size of the admissible region and allows to use specific propagation methods, adapted to close encounters. The second project would be to compute the minimal orbital intersection distance (MOID) to the Earth using Öpik (1976) formalism instead of the nodal distances; however, this computation requires to check the values of Tisserand parameter, for any concerned orbit; the validity of the theory requires Tisserand parameters smaller than 3, which is not always the case, in particular for some of the orbits generated by our two test attributables.

References

Milani, A., Gronchi, J.F., De Michieli Vitturi, M. & Knežević, Z. 2004, *Celest. Mech. Dyn. Astron.* 90, 57

Milani, A. Gronchi, G.F., Knežević, Z., Sansaturio, M.E. & Arratia, O. 2005, *Icarus* 179, 350

Milani, A. & Knežević, Z. 2005, *Celest. Mech. Dyn. Astron.* 92

Öpik, E.J. 1976, *Interplanetary Encounters*, Elsevier, New York

Valsecchi, G.B., Milani, A., Gronchi, G.F. & Chelsey, S.R. 2003, *Astron. Astrophys.* 408, 1179