Dynamics of Solar and Stellar Convection Zones and Atmospheres
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Resonance and stellar dynamos

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Abstract. Planetary influence on a stellar convective shell can result in a periodic modulation of stellar dynamo drivers. Similar modulation can arise in stellar binary systems. Using the Parker low-mode dynamo model we investigate the properties of nonlinear parametric resonance. This model is a system of four ordinary differential equations and, in the first approximation, describes the processes of generation and oscillation of large-scale magnetic fields in stellar systems. In the absence of nonlinear suppression effects, the problem, by analogy with a system of harmonic oscillations, allows an asymptotic selection of multiple resonant frequencies. Despite the fact that at first glance at these frequencies it is reasonable to expect an increase in the amplitude, the behavior of the system can be just the opposite. All this stuff deserves a systematic analysis of swing excitation in the dynamo sistems in comparison with classical swing excitation in the framework of the Mathieu equation.

Keywords. Solar activity cycle, stellar cycles, stellar dynamo

1. Introduction

Solar activity cycle is a (quasi-)periodic phenomenon. Activity cycles similiar to the solar one are known for many stars similar to the Sun. The activity cycles are believed to be supported by a process in the stellar convective zone known as stellar dynamo. On the other hand, stellar convective zones may be affected by various periodic processes, say, tidal forces arising by gravitational forces from planetary systems and/or members of the binary system. The idea of swing excitation participating in the stellar activity cycles looks natural. The idea was addressed in some papers for quite a long time (see, e.g. Gilman, Dikpati 2011, Moss, Sokoloff 2013). Indeed, numerical modelling provides particular dynamo models which demonstrate behavior similar to the swing excitation (e.g. Moss, Piskunov, Sokoloff 2002) however there are examples of more complicated behavior (e.g. Moss, Sokoloff 2017) rather than just a parametric resonance.The situation becomes dramatic because Jupiter orbital period (about 11 yrs) is closed to the nominal solar cycle length (about 11 yrs) and the idea that this is something more rather a coincidence is discussed in astronomical community (e.g. Klevs, Stefani, Jouve 2023) while search of similar cases in exoplanetary systems with known activity of central star (Obridko, Katsova, Sokoloff 2022) do not support this expectation. It is why a systematic comparison and confrontation of dynamo parametric resonance and classical swing excitation undertaken recently by Kitchatinov, Nepomnyashchikh 2015 and Serenkova, Sokoloff, Yushkov 2023 from different viewpoints becomes important. Here we concentrate on the last approach.

2. Comparing dynamo resonance and classical swing excitation

Comparing dynamo resonance and classical swing excitation we have to take into account that the last one is considered in terms of ordinary differential equations (Mathieu equation), i.e. for system with finite degrees of freedom, while standard solar and stellar dynamo models contain partially differential equations, i.e. consider systems with infinite degrees of freedom (field equations). Of course, any particular kind of swing excitation can be developed to include field equations, however more practical is to reduce a dynamo model to a dynamical system. This can be done by presenting corresponding partial differential equations by Fourier series with further truncation to keep as few variables (degrees of freedom) to get a cyclic behaviour. Nefedov, Sokoloff 2010 performed such truncation for the classical Parker migratory dynamo with an algebraic α-quenching to get a four-dimensional dynamical system which we exploit below. In contrast, the second-order Mathieu equation corresponds to a second order dynamical system. There is no problem to perform such truncation for, say, much more realistic contemporary flux-transport dynamo models, however the exploited over-simplified model is sufficient to isolate substantial difference between dynamo resonance and classical swing excitation.

2.1. *Mathieu resonance*

The classical parametric resonance is described by the Mathieu equation (e.g. McLachlan 1947):

$$\ddot{f} + \omega_0^2(1 + \sigma \sin(\omega t))f = 0. \tag{1}$$

Resonant solutions for the Mathieu equation are sought in the form of a harmonic function with small exponential growth:

$$f(t) = f_1 \exp(st + i(\omega_0 + \varepsilon/2)t) + f_1^* \exp(st - i(\omega_0 + \varepsilon/2)t). \tag{2}$$

Exponential growth is observed in a small corridor near the doubled frequency of the system. In the nonlinear case of parametric resonance for the system, with an increase in the amplitude, the parameter ω_0^2 decreases, due to which the system gradually decouples, and this, in turn, leads to stabilization of exponential growth and quasistationary output. Stationary is understood in the sense that the solution oscillates, but the energy of the oscillations remains constant. The parameter ω_0^2 reduction is set as follows:

$$\omega_{eff}^2 = \frac{\omega_0^2}{1 + \langle f^2 \rangle}. \tag{3}$$

2.2. *Dynamo resonance*

Let us consider the case of dynamo resonance on the example of the Parker system. The key simplification proposed by Parker in the transition to a spherical coordinate system consists in dividing the axially symmetric magnetic field into poloidal and toroidal components (A and B respectively):

$$\begin{aligned} \dot{A} &= R_\alpha B + A_{\theta\theta} - \mu^2 A, \\ \dot{B} &= R_\omega (A \sin(\theta))_\theta + B_{\theta\theta} - \mu^2 B. \end{aligned} \tag{4}$$

The migration waves generated in such a system (they are often called dynamo waves) are most sensitive to the dynamo number D, i.e. the product of dimensionless parameters responsible for differential rotation (R_ω) and helicity (R_α). The number μ determines the radial part of the diffusion and is proportional to the ratio of the radius to the thickness of the convective shell. Hypothetically, some periodic influence on the system (for example, binary stars on each other or exoplanets on their star) can lead to periodic changes in R_α or R_ω, which, in turn, can cause parametric resonance. To analyze such a situation, the following change is made to the system:

$$R_\omega \to R_\omega(1+\sigma\sin(\omega t)), \tag{5}$$

here, σ is the amplitude of the parametric effect. A more convenient approach for mathematical interpretation of the solution is the so-called low-mode approximation, which has already been used in some works (e.g. Kalinin, Sokoloff 2018, Tarbeeva, Semikoz, Sokoloff 2011). In the case of expansion by four principal modes of the Fourier series, it is possible to obtain a system of Mathieu-type equations:

$$\begin{aligned} \ddot{f}_1 - (R_\alpha R_\omega/2)(1+\sigma\sin(\omega t))(f_1 - f_2) = 0,\\ \ddot{f}_2 - (R_\alpha R_\omega/2)(1+\sigma\sin(\omega t))(f_1 + f_2) = 0. \end{aligned} \tag{6}$$

In this case, the solution has the form of four complex-conjugate exponentials, since now the frequencies have a complex additive responsible for generating outside of a narrow corridor about twice the frequency (see for details Serenkova et al.):

$$\begin{aligned} f_{1,2} = \bar{f}_{1,2}\exp((\gamma_0+s+i\beta)t + i(\omega_0+\varepsilon/2)t) + \bar{f}^*_{1,2}\exp((\gamma_0+s-i\beta)t - i(\omega_0+\varepsilon/2)t)+\\ + \bar{\bar{f}}_{1,2}\exp((\gamma_0+s-i\beta)t + i(\omega_0+\varepsilon/2)t) + \bar{\bar{f}}^*_{1,2}\exp((\gamma_0+s+i\beta)t - i(\omega_0+\varepsilon/2)t). \end{aligned} \tag{7}$$

In the case, when the field is non-linearly stabilized, the growth of the magnetic field leads to a decrease in hydrodynamic helicity, the oscillations have a constant amplitude, and with resonance it can become larger than in the absence of exposure. To ensure stabilization, we will nonlinearly suppress the parameter $R\alpha$:

$$R_\alpha \to \frac{R_\alpha}{1+\max|B^2|}. \tag{8}$$

Some differences between the dynamo resonances and classical swing excitation are clear from advance. Indeed, dynamo systems include self-excitation, i.e. initial exponential growth, and a nonlinear saturation. Natural oscillators with swing excitation may contain similar features however they are not included in the classical Mathieu equation. It is however far to be clear which difference and to what extent is responsible for specific properties of dynamo resonances known from available numerics. In order to clarify this point we mimic the classical derivation of the Mathieu equation to get a similar equation for the dynamical system under consideration. Below we refer to this equation as a generalised Mathieu equation. Because parametric resonance can be interesting for another physical problems with self-excitation and its further nonlinear suppression and in order to mimic somehow more complicated dynamo models we consider numerical coefficients in the generalised Mathieu equation as free parameters and play with numbers nearby their nominal quantities obtained for the Parker migratory dynamo.

3. Specific features of dynamo resonances

The generalised Mathieu equation is quite similar to the classical one however is more bulky. It admits numerical as well to some extent analytical investigation (see for details Serenkova, Sokoloff, Yushkov 2023). Performing the analysis we arrive at the following conclusions.

First of all, periodic modulation of dynamo equations may result in two types of effects, i.e. effects associated with a specific frequency as well as effects presented for a wide range of frequencies. We describe both effects separately.

3.1. *Wide-range effect*

Stellar dynamo excitation is a threshold effect, i.e. intensity of dynamo drivers presented in dynamo equations as dimensionless dynamo numer D should exceed a marginal value D_0 to get self-excitation. Periodic modulation can slightly diminish D_0. In other words, periodic modulation can transform a slightly subcritical dynamo into a slightly supercritical one (Sokoloff, Serenkova, Yushkov 2022). It looks quite improbable that the dynamo number for a particular stellar dynamo is so close to the corresponding marginal value however if it happens the resulting effect on the dynamo supported magnetic field occurs to be crucial.

This effect does not require a fine tuning of the modulation frequency. Because standard swings do not presume self-excitation without modulation, the effect is impossible for this case. We do not insist that the effect have to be referred to as a resonance however it looks interesting by itself.

3.2. *True resonances*

The generalized Mathieu equation contains growing solutions associated with a particular value of the perturbation frequency. In order to separate them from the wide-range effect we refer to them as true resonances. Depending on coefficients in the dynamical system, i.e. on details of dynamo system, we can obtain a resonance peak at a particular modulation frequency which may coincide with the eigenfrequency of the system or do not coincide with it or do not obtain a peak at any frequency. In some cases resonance effects diminish the amplitude of the stellar cycle rather than enlarge it. In other words, a resonant absorption instead of resonant excitation is possible. In any case, resonance effects are not very strong, they modify the stellar cycle amplitude of about 10% at most.

4. Conclusions

We conclude that varity of the effects associated with periodic modulation of dynamo drivers is much more wide rather that one for the standard swing excitation. From the other hand, the effects are not very strong in comparison with the initial dynamo self-excitation and hardly can be very important in many stellar systems. From the other hand the effects can be important for some quite rare stellar systems.

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