REFERENCE COORDINATE SYSTEMS FOR EARTH DYNAMICS: A PREVIEW

Ivan I. Mueller
Dept. of Geodetic Science, Ohio State University
Columbus, Ohio 43210 USA

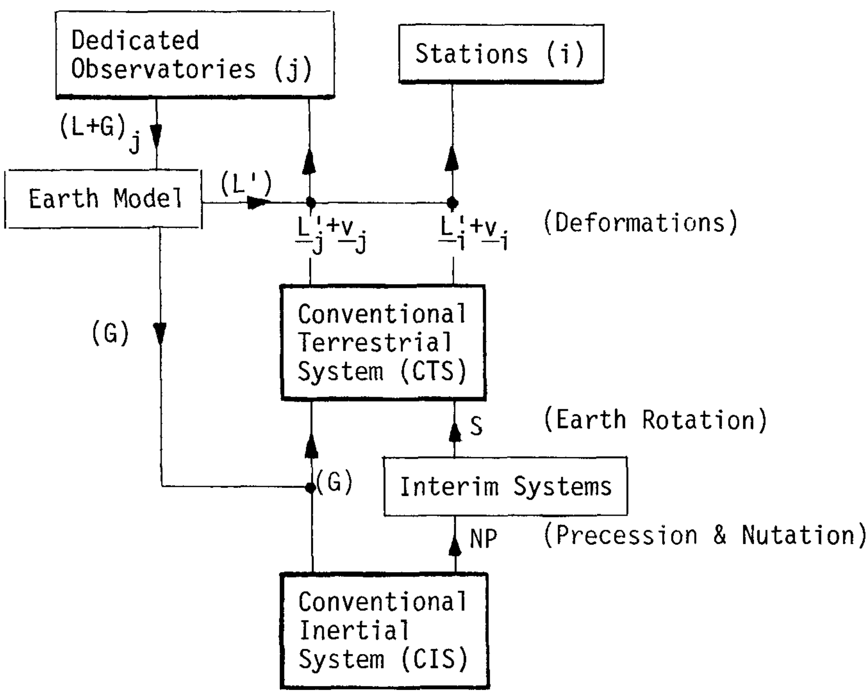
ABSTRACT. A common requirement for all geodynamic investigations is a well-defined coordinate system attached to the earth in some prescribed way, as well as a well-defined inertial coordinate system in which the motions of the terrestrial system can be monitored. This paper deals with the problems encountered when establishing such coordinate systems and the transformations between them. In addition, problems related to the modeling of the deformable earth are discussed.

1. INTRODUCTION

Geodynamics has become the subject of intensive international research during the last decade, involving plate tectonics, both on the intra-plate and inter-plate scale, i.e., the study of crustal movements, and the study of earth rotation and of other dynamic phenomena such as the tides. Interrelated are efforts improving our knowledge of the gravity and magnetic fields of the earth. A common requirement for all these investigations is the necessity of a well-defined coordinate system (or systems) to which all relevant observations can be referred and in which theories or models for the dynamic behavior of the earth can be formulated. In view of the unprecedented progress in the ability of geodetic observational systems to measure crustal movements and the rotation of the earth, as well as in the theory and model development, there is a great need for the definition, practical realization, and international acceptance of suitable coordinate system(s) to facilitate such work. Manifestation of this interest has been the numerous specialized symposia organized during the past decade or so, such as those held in Stresa [Markowitz and Guinot, 1968], Morioka [Melchior and Yumi, 1972; Yumi, 1971], Torun [Kołaczek and Weiffenbach, 1974], Columbus [Mueller, 1975b and 1978], Kiev [Fedorov, Smith and Bender, 1980] and San Fernando [McCarthy and Pilkington, 1979]. There seems to be general agreement that only two basic coordinate systems are needed: a Conventional Inertial System (CIS), which in some "prescribed way" is attached to extragalactic celestial radio sources, to serve as a reference for the motion of a Conventional Terrestrial System (CTS), which moves and

E. M. Gaposchkin and B. Kołaczek (eds.), Reference Coordinate Systems for Earth Dynamics, 1–22.

rotates in some average sense with the earth and is also attached in some "prescribed way" to a number of dedicated observatories operating on the earth's surface. In the latter, the geometry and dynamic behavior of the earth would be described in the relative sense, while in the former the movements of our planetary system (including the earth) and our galaxy could be monitored in the absolute sense. There also seems to be a need for certain interim systems to facilitate theoretical calculations in geodesy, astronomy, and geophysics as well as to aid the possible traditional decomposition of the transformations between the frames of the two basic systems. This scheme is shown in the figure below. The Earth Model block represents the current best knowledge of the geometry and dynamic behavior of the earth, partially deduced from the measurements made at the Dedicated Observatories. This model is continuously improving as more data of increasing accuracy becomes available, and it includes both the local (L) and global (G) phenomena which have theoretical foundations based on physical reality and are mathematically describable. In the final and ideal situation, which may be achieved only after several iterations over an extended period of time, the global part of the model should be identical to the connection between the CIS and CTS frames. Departures (v) from the model (L') observed at the observatories (j) or at other stations (i) are of course most important since they represent new information based on which the model can be improved, after observational random and systematic errors have been taken into proper consideration. The model could eventually include the solid earth as well as the oceans and the atmosphere.



https://doi.org/10.1017/S0252921100081069 Published online by Cambridge University Press

As we will see later, there already seems to be understanding on how the two basic reference systems should be established; certain operational details need to be worked out and an international agreement is necessary. There are, however, a number of more or less open questions which will have to be discussed further. These include the type of interim systems needed and their connections to both CIS and CTS, the type(s) of observatories, their number and distribution, whether all instruments need to be permanently located there or only installed at suitable regular intervals to repeat the measurements; how far the model development should go so as not to become impractical and unmanageable; and how independent observations should be referenced to the CTS, i.e., what kind of services need to be established and by whom.

2. CONVENTIONAL INERTIAL SYSTEMS (CIS) OF REFERENCE

2.1 Basic Considerations

The first law of Newton is as follows: "Every body persists in its state of rest or uniform motion in a straight line unless it is compelled to change that state by forces impressed on it" [Newton, 1686]. It should be obvious that the above *law of inertia* cannot hold in any arbitrary reference frame so that only certain specific reference frames are acceptable. In classical mechanics, reference frames in which the above law is valid are called *inertial frames*. Such "privileged" frames move through space with a constant translational velocity but without rotational motion. Another privileged frame in classical mechanics is the *quasi-inertial*, which also moves without rotational motion, but its origin may have acceleration. Such a frame would be, for example, a non-rotating geocentric Cartesian coordinate system whose origin due to the earth orbit around the sun would move with a non-constant velocity vector. Inertial reference frames thus are either at rest or are in a state of uniform rectilinear motion with respect to *absolute space*, a concept also mentioned by Newton and visualized as being observationally defined by the stars of invariable positions, a dogma in his time.

The refinement of classical mechanics through the theory of relativity requires changes in the above concepts. The theory of special relativity allows for privileged systems, such as the inertial frame but in the *space-time continuum* instead of the absolute space [Moritz, 1967]. Transformation between inertial frames in the theory of special relativity are through the so-called Lorentz transformations, which leave all physical equations, including Newton's laws of motion, and the speed of light invariant. The special theory of relativity holds only in the absence of a gravitational field.

In the theory of general relativity, Einstein defined the inertial frames as "freely falling coordinate systems" in accordance with the local gravitational field which arises from all matter of the universe. Thus the inertial frames lose their privileged status. Concerning the existence of inertial frames in the extended portions of the space-time

https://doi.org/10.1017/S0252921100081069 Published online by Cambridge University Press

continuum, Einstein [1956] states that

> "there are finite regions, where, with respect to a suitably chosen space of reference, material particles move freely without acceleration, and in which the laws of special relativity hold with remarkable accuracy."

In other words, one can state [Weinberg, 1972] that

> "At every space-time point in an arbitrary gravitational field, it is possible to choose a locally inertial coordinate system such that, within sufficiently small region of the point in question, the laws of nature take the same form as in unaccelerated Cartesian Coordinate system in the absence of gravitation."

(i.e., as in the theory of special relativity). Our sphere of interest, the area of the solar system, where the center of mass of the earth-moon system is "falling" in an elliptic orbit around the sun, in a relatively weak gravitational field, seems to qualify as such a "small region." Thus we may assume that inertial or quasi-inertial frames of reference exist, and any violation of principles when using classical mechanics can be taken into account with small corrections appropriately applied to the observations and by an appropriate "coordinate" time reference. The effects of special relativity for a system moving with the earth around the sun are in the order of 10^{-8}, while those of general relativity are 10^{-9} [Moritz, 1979]. Since 10^{-8} on the earth's surface corresponds to about 6 cm, corrections at least for special relativity effects are needed when striving for such accuracies. Other than this, the problem, in the conceptual sense, need not be considered further.

2.2 Inertial Systems in Practice

2.21 Extragalactic Radio Source System. This system is attached to radio sources which generally either are quasi-stellar objects (quasars) or galactic nuclei. Very long baseline interferometers rotating with the earth determine the declinations of these sources with respect to the instantaneous rotation axis of the earth, as well as their right ascension differences with respect to a selected source (3C273, NRAO 140, Persei (Algol), etc.). In addition, the observations also determine changes in the earth rotation vector with respect to a selected initial state, the baseline itself, and certain instrumental (clock) corrections. The frame of the Radio Source-CIS can be defined by the adopted true or mean coordinates of appropriately selected sources referred to some standard epoch. The mean coordinates naturally will depend on the model of the transformation from the true frame of date to the adopted mean standard. If, however, the reduction procedure is correct (see more on this later), there are no known reasons for non-radial relative motions of the sources, i.e., for the rotation of the frame. Thus, such a frame could be considered inertial or at least quasi-inertial. The equatorial system of coordinates may be retained for convenience, but the frame could be attached to the sources in any other arbitrary way should this be necessary.

As far as the accuracy of the Radio Source-CIS is concerned, the question has meaning only in the sense of the formal precisions of the

https://doi.org/10.1017/S0252921100081069 Published online by Cambridge University Press

source positions in the catalogue. At the Torun meeting, this number was 0".1 [Moran, 1974]; now it is at most 0".01 [Purcell et al., 1980]. It is hoped that within a few years the precision should reach 0".001 (5×10^{-9}). The problem on this level is that the densification of such a catalogue will be very difficult, since only a relatively few well-defined point-like radio sources have been observed. Others have structures such that identification of the center of the radiation with such accuracy may not be possible. This situation may change when the astrometric satellites (see below) are launched.

2.22 Stellar System. This system will be attached to stars in the FK5 catalogue, i.e., the adopted right ascensions and declinations of the FK5 stars will define the equator and the equinox and thus the frame of the Stellar-CIS. The FK5, to be effective in 1984, will be the fifth fundamental catalogue in a series which began with the FC in 1879 [Fricke and Gliese, 1978]. In the fundamental catalogues the equator is determined from zenith distance (or distance difference) observations of the stars themselves, but the equinox determination also necessitates measurements of the sun or other members of the planetary system. It was always tacitly assumed that coordinate systems attached to the fundamental catalogues were quasi-inertial. However, as more and more observations became available for proper motions and on the various members of the planetary systems, certain small rotations were discovered, which require changes in the positions of the fundamental equator and equinox, in the proper motions and in the precessional constant (all intricately interwoven) when one fundamental catalogue replaces the other. This slow and painstaking process should lead to a quasi-inertial system eventually. We hope that the FK5 will be such a system.

When the FK4 was compiled, a small definitive correction to the declination of FK3 was applied, but there seemed to be no need to change the position of the equinox or the precessional constant [Fricke, 1974]. The FK5 will be a considerably different and improved catalogue. The main changes with respect to the FK4, regarding the issue of the coordinate systems, are as follows [Fricke, 1979a]: (1) New value of general precession in longitude adopted by the IAU in 1976 will be used (more on this later). (2) The centennial proper motions in right ascension will be increased by $0^s.086$/century (this number is provisional) to eliminate the motion of the FK4 equinox with respect to the dynamical equinox (the FK4 right ascensions are decreasing with time due to an error in the FK4 proper motions, see below). (3) Rotation of the FK4 equinox at 1950 by the amount of $0^s.040$ (also a provisional value)so that the FK5 and the dynamic equinoxes will be identical (the FK4 right ascensions at 1950 are too small). (4) Elimination of inhomogeneities of the FK4 system by means of absolute and quasi-absolute observations. (5) Determination of individual correction to positions and proper motions of FK4 stars. (6) Addition of new fundamental stars to extend the visual magnitude from 7.5 to about 9.2. More than 1500 new stars are to be added.

It should be mentioned that the above improvements are possible because of the availability and/or reanalysis of observations of the sun

(1900-1970), of lunar occultations (1820-1977), of Mars (1941-1971), of minor planets (1850-1977), and the JPL DE-108 Ephemeris based on optical or radar observations of the sun, planets and some space probes (Mariner 9, Viking). All in all the number of these observations exceeds 350,000. In addition, more than 150 catalogues of star observations have become available since the completion of the FK4 [Fricke, 1979b].

One should also take note here of the FK5sup catalogue, which will contain the FK5 coordinates of a few extragalactic radio sources with radio and optical positions and thus provide the connections between the Stellar-CIS and the Radio Source-CIS, though with somewhat limited accuracy ($\sim 0''.1$). Improvement of this particular problem is expected from the Space Telescope [Van Altena, 1978] which could increase the number of radio stars, observable by VLBI, in the FK5 to about 50. Such missions (e.g., Hipparcos) could also contribute to the determination of the fundamental equator and equinox with increased accuracies, by observations of the minor planets. This, of course, would mean improved ties with the planetary-CIS (discussed below) which nowadays is based on the observations mentioned in connection with the establishment of the FK5 equator and equinox. The astrometric satellite Hipparcos is described to be able to measure relative positions of some 100,000 stars to a precision of $0''.0015$ and annual proper motions to $0''.002$ over a lifetime of 2.5 years [Barbieri and Bernacca, 1979]. A second mission ten years later could improve this figure by a factor of 5. This compares well indeed with the precision of ground based observations of $0''.04$ at best, requiring something like 50 years to obtain proper motions of comparable precision ($0''.002$).

As far as the accuracy of the FK5-CIS is concerned, the question again is meaningful only in the sense of how precise the star positions in the FK5 will be. It is hoped that in the worst regions this will not be worse than $0''.02$ in position and $0''.0015$ in the annual proper motion. There should be better regions, of course.

2.23 Dynamical Systems. The dynamics expressed in the equations of motion define a number of non-rotating planes which could be the basis of reference frames. Considering the observable planes that could be the basis of such a Dynamic-CIS, there are the planetary (including the earth-moon barycenter) orbital planes, the equator, the lunar orbital plane, and the orbital planes of certain high flying, thus only slightly perturbed, artificial earth satellites (e.g., Lageos or GPS). Since all of these planes have relative rotations, it is possible to derive a mean plane for a given epoch from an observable apparent plane, or a non-observable invariant plane could be adopted [Duncombe et al., 1974]. At this point, the definition of the origin of the system becomes important also, because relativisitc effects necessitate the distinction between proper and coordinate times. In the radio-source or stellar quasi-inertial systems, the question of origin can be settled through appropriate corrections for aberration and parallax, etc., but here it is also necessary that a uniform and unambiguous time scale referenced to a non-rotating frame of specified origin be established

https://doi.org/10.1017/S0252921100081069 Published online by Cambridge University Press

(coordinate time). The practical implications of a global coordinate time scale is not treated here, but the problem should not be ignored (cf. [Ashby and Allan, 1978]). In more practical (observational) terms one can distinguish between Planetary, Lunar and (artificial) Satellite CIS's, each frame defined, in theory, by two of the above-mentioned planes, and in practice, by the available ephemerides.

In the case of the *planetary systems*, the defining planes are the equator and the ecliptic, their intersection being the line of the equinoxes. In practical terms the frame of the Planetary-CIS is defined by the ephemerides of the centers of masses of the planets, including the barycenter of the earth-moon system. The ephemerides, such as the JPL DE-108 mentioned earlier, are based on observations of the sun, the planets, possibly space probes. Since most modern ephemerides are computed through the numerical integration of the orbital equations of motion, the degree of satisfaction that can be obtained depends only on the completeness of the modeling, including the astronomical constants, the determination of the starting conditions and, of course, on the type, accuracy and distribution of the observed data. In this sense each planetary ephemeris defines its own reference frame. These should agree with each other within the observational accuracies. Connection between the Planetary-CIS's and the Stellar-CIS's is through the determination of the equinox and the equator, as explained earlier.

In the case of the *lunar system*, the main references are the orbital plane of the moon and the equator of the earth. In practice the Lunar-CIS frame is again defined by the lunar ephemeris, which nowadays is most accurately determined from lunar laser observations made from the surface of the earth to reflectors deposited on the lunar surface. For this reason, the adequacy of the definition also depends on how well the lunar rotation (librations) can be computed. Since the most frequently used lunar ephemerides are generally calculated through numerical integration, the above dependence on modeling (especially on the effect of tidal dissipation in the earth), and on initial conditions, apply here also. The identity of the coordinate frame, such defined, may be compared to the other frames to certain accuracies. Lunar occultation of stars, or the earlier Markowitz moon-camera photography, provide a connection to the Stellar-CIS; differential VLBI observations between radio sources deposited on the moon and the extragalactic ones would tie to the Radio Source-CIS. The connection to the Planetary-CIS is through solar eclipse observations, and also through the planetary ephemeris used when calculating the lunar ephemeris. There are also some other looser connections stemming from the orientation of the earth when its non-spherical gravitational effects on the lunar motions are taken into consideration. Present observations reveal a residual rotation (or accelerations) in the order of a few seconds of arc per century squared. This seems to be the present stability (i.e., the accuracy) of this quasi-inertial frame. It is unlikely that without stronger connections to a frame of better stability, this rotation can be eliminated. As it is, the accuracy of this CIS should compare favorably with that defined by the FK5 but only over a period of, say, a decade [Kovalevsky, 1979].

https://doi.org/10.1017/S0252921100081069 Published online by Cambridge University Press

In the case of *satellite systems*, the problem is compounded by additional modeling problems related to the force field in which the satellite moves and by the fact that nowadays there are no direct connections to other frames of reference. Modern satellite tracking techniques (laser, Doppler, etc.) all basically observe ranges or range differences and contain no directional information. The main reference planes, the orbital plane of the satellite and the equator, intersect along the line of nodes, the initial orientation of which therefore must be defined more or less arbitrarily. In the "old days" of satellite geodesy, when satellites were observed photographically in the background of stars, this direction could be determined with respect to the FK4, though not much better than a few tenths of a second of arc. The accumulation of errors in describing the motion of the node with respect to a selected zero point, even for the most suitable high flying and small heavy spherical satellites (Lageos), prevents a Satellite-CIS from being accurate over a long period of time, say beyond several months. In any case, in observational terms such a frame would be defined by the satellite ephemeris made available to the users by organizations which provide for the continuous tracking of the satellite in question. A current example would be the Precise Ephemeris of the U.S. Navy Navigational Satellite (Transit) System. As far as the connections to other systems are concerned, the only accurate possibility seems to be indirectly through the tracking stations. If two observational systems occupy the same station, one observing the satellite, the other, say, the radio sources, either simultaneously or after a short time interval (during which the movement of the station can be modeled), the connection between the satellite and radio source frames can be established. In fact, the now classical disparity between the JPL and SAO frames came to light just through such an arrangement, when the SAO longitudes determined from satellite camera tracking (thus in the FK4 frame) differed by those determined by JPL space probe tracking (in the planetary frame) by an amount (about $0\overset{''}{.}7$ in the early 1970's) consistent with the FK4 equinox motion with respect to the dynamical equinox, mentioned earlier. Only through such continuously maintained connections can the lifetime of a Satellite-CIS be extended, thus its accuracy increased

2.3 Conclusions

From the above discussion, the following conclusions can be drawn:

1. The most accurate, long-term CIS will be the one attached to extragalactic radio sources. It is accessible through VLBI observations Other systems can be accurately connected to it by station collocation or the Space Telescope.

2. The CIS attached to the FK5 is somewhat less accurate. Direct access to it is through optical star observations, which by nature are generally less accurate than VLBI observations. Its main value is in defining the fundamental mean system of coordinates and thereby providing a direction (the FK5 equinox) for the time (UT1) definition, and for the possible orientation of the Radio Source-CIS. The latter

function, however, stems from more of a traditional requirement and not from theoretical needs.

3. Of the Dynamical-CIS's, the accuracy of the planetary system should be equivalent to the FK5. The lunar and satellite systems by themselves are suitable for medium-term to short-term work only. Their stability can be extended by connections to the Radio Source-CIS through accurate and continuous observations at collocated stations. Ties between the radio source and the planetary systems may also be available through the proposed Very Large Array (VLA) observations of minor planets. Solar eclipse observations provide a connection between the lunar and planetary systems.

It is an unavoidable conclusion that for geodetic and geodynamic applications the most useful CIS is the one attached to the extragalactic radio sources, observable by VLBI, whose orientation is defined through ties to the FK5. The origin of the system can be chosen at will at the center of mass of the earth, of the solar system or elsewhere depending on the application or on operational convenience.

3. CONVENTIONAL TERRESTRIAL SYSTEMS (CTS) OF REFERENCE

As mentioned in the Introduction, the CTS is in some "prescribed way" attached to observatories located on the surface of the earth. The connection between the CTS and CIS frames by tradition (to be preserved) is through the rotations [Mueller, 1969]

$$[\ \underline{CTS}\ ] = SNP\ [\ \underline{CIS}\ ]$$

where P is the matrix of rotation for precession, N for nutation (to be discussed in Section 4), and S for earth rotation (including polar motion). Polar motion thus is defined as the angular separation of the third (Z) axis of the CTS and the axis of the earth for which the nutation (N) is computed (e.g., instantaneous rotation axis, Celestial Ephemeris Pole, Tisserand mean axis of the mantle (see Section 4)).

3.1 Current Situation

At present the internationally accepted Woolard series of nutation (the IAU 1979 series becomes effective only with the 1984 ephemerides) is computed for the instantaneous rotation axis of the rigid earth, and the Z axis of the CTS is the Conventional International Origin (CIO), defined by the adopted astronomic latitudes of the five International Latitude Service (ILS) stations, located approximately on the 39°08' parallel. These are assumed to be motionless relative to each other, and without variations in their respective verticals (plumb lines) relative to the earth. Thus, conceptually, polar motion should be determined from latitude observations only at these ILS stations. This has been done for over 80 years, and the results are the best available *long-term* polar motions, properly, but not very accuractely, determined.

https://doi.org/10.1017/S0252921100081069 Published online by Cambridge University Press

The first axis of the CTS is defined by the assigned astronomic longitudes of time observatories (around 50) participating in the work of the Bureau International de l'Heure (BIH).

Due to the fact that in most geodetic and astronomical applications accurate shorter-term variations of polar motion are needed, which are not available with sufficient accuracy from the ILS observations, polar motion is also determined from latitude and/or time observations at a larger number of observatories participating in the work of the International Polar Motion Service (IPMS), as well as of the BIH. In the resulting calculations the earlier definition of the CIO cannot be maintained. The common denominator being the Woolard series of nutation, observationally the Z axis of the CTS is defined by the coordinates of the pole as published by the IPMS or by the BIH. Thus it is legitimate to speak of IPMS and BIH poles of the CTS (in addition to the CIO). The situation recently has become even more complicated because Doppler and laser satellite tracking, VLBI observations, and lunar laser ranging also can determine *variations* in the earth rotation vector (including polar motion), some of which are incorporated in the BIH computations. Further confusion arises due to the fact that the BIH has two systems: the BIH 1968 and the BIH 1979, the latter due to the incorporation of certain annual and semiannual variations of polar motion determined from the comparisons of astronomical (optical) results with those from Doppler and lunar laser observations [Feissel, 1980].

Though naturally every effort is made to keep the IPMS and BIH poles of the CTS as close as possible to the CIO, the situation cannot be considered satisfactory from the point of view of the geodynamic accuracy requirement of a few parts in 10^9. The current accuracy of the pole position is estimated to be $0\overset{''}{.}01$, and that of the UT1, 1 ms ($\sim 5 \times 10^{-8}$) for five-day averages [Guinot, 1978]. These figures, of course, do not include biases from the definition problems mentioned.

3.2 The Future CTS

There seems to be general agreement that the new CTS frame conceptually be defined similarly to the CIO-BIH system [Bender and Goad, 1979; Guinot, 1979; Kovalevsky, 1979; Mueller, 1975a], i.e., it should be attached to observatories located on the surface of the earth. The main difference in concept is that these can no longer be assumed motionless with respect to each other. Also they must be equipped with advanced geodetic instrumentation like VLBI or lasers, which are no longer referenced to the local plumblines. Thus the new transformation formula may have the form

$$\underline{[OBS]}_j = \underline{L}'_j + \underline{[CTS]}_j + \underline{v}_j = \underline{L}'_j + SNP\ \underline{[CIS]}_j + \underline{v}_j \quad ,$$

where $\underline{L}'_j$ is the vector of the "j" observatory's movement on the deformable earth with respect to the CTS, computed from suitable models (see the figure and Section 4); NP, the nutation and precession matrices computed with the new 1976 IAU constants and the 1979 IAU series of nutation

https://doi.org/10.1017/S0252921100081069 Published online by Cambridge University Press

(provided the latter is not going to be changed; see Section 4); and S, the rotation matrix between the CTS and the true frame for which the nutation is computed. Variations in S can be determined by a future international service (like the BIH) by comparing repeatedly observed observatory coordinates ($[\underline{OBS}]_j$), corrected for the modelable deformations ($-\underline{L}'_j$), and by minimizing the residuals ($\underline{v}_j$) in the least squares sense. This rotation can either be determined from the residuals of the Cartesian coordinates (e.g., [Moritz, 1979]) or, for possible better sensitivity, since the rotation is least sensitive to variations in height, only from those of the horizontal coordinates (geodetic latitude and longitude) (e.g., [Bender and Goad, 1979]). It is unlikely that the rotation will continue to be determined (as presently) from astronomical coordinates, i.e., from the direction of the vertical, for the reason of inadequate observational accuracy.

As far as the origin of the CTS is concerned, it could be centered at the center of mass of the earth, and its motion with respect to the stations can be monitored either through observations to satellites or the moon, or, probably more sensitively, from continuous global gravity observations at properly selected observatories [Mather et al., 1977]. For the former method a translational term may easily be incorporated in the above transformation equation.

Since the above method or some variation thereof provides only changes in S and in the translation, the new CTS needs to be initialized in a way to provide continuity. This could be done through the CIO or the IPMS or BIH poles, and the BIH zero meridian, at the selected initial epoch, uncertainties in their definition mentioned earlier mercifully ignored.

It is probably not useless to point out that if such a system is established, the most important information for the users will be the transformation parameters, but for the scientist new knowledge about the behavior of the earth will come from the analysis of the residuals after the adjustment.

It is hoped that the IAU and IUGG will make practical recommendations on the establishment of such or a very similar Conventional Terrestrial System, including the necessary plans for supporting observatories and services. One of the recommendations ought to be that due to the fact that the ultimate goal is the determination of the total transformation between the CTS and CIS, the future service must publish not only the parameters of the S matrix determined from the repeated comparisons (the situation at present), but also the models and parameters in L' as well as in NP, i.e., the parameters defining the whole *system*.

4. MODELING THE DEFORMABLE EARTH

In this section we will try to highlight the modeling problems

https://doi.org/10.1017/S0252921100081069 Published online by Cambridge University Press

associated with the components of transformation between the CIS and CTS mentioned in Section 3.

4.1 Precession (P)

At the XVIth General Assembly in Grenoble in 1976, the IAU adopted a new speed of general precession in longitude of 5029".0966 per Julian century at the epoch J2000.0 (JED 2451545.0). This value when referred to the beginning of the Besselian year B1900.0 is 5026".767 per tropical century, which may be compared to the previously adopted (and presently still used) value of 5025".64 per tropical century at B1900.0. The change was calculated by Fricke [1977] from proper motions of stars in the systems GC, FK3, N30, and FK4. From the results, the correction of +1".10 per century to Newcomb's luni-solar precession in longitude was recommended. This value combined with a correction to Newcomb's planetary precession, due to the improved 1976 IAU values of planetary masses, resulted in the above new precessional constant. Expressions to compute the effect of precession from one epoch to another were developed by Lieske et al. [1977]; and the usual equatorial parameters, z, θ, ζ_0, to be used in the precession matrix [Mueller, 1969],

$$P = R_3(-z)\, R_2(\theta)\, R_3(-\zeta_0) \quad ,$$

to and from the epoch J2000 were computed by Lieske [1979]. The above matrix allows the currently best transformation between the CIS (say, the FK5 at J2000.0) and an interim "Mean Equator and Equinox Frame" of some date.

4.2 Nutation (N)

The nutation story is much more complex. First of all, the nutation matrix is [Mueller, 1969]

$$N = R_1(-\varepsilon - \Delta\varepsilon)\, R_3(-\Delta\psi)\, R_1(\varepsilon) \quad ,$$

where ε is the obliquity of the ecliptic, $\Delta\varepsilon$ is the nutation in obliquity, and $\Delta\psi$ the nutation in longitude, computed from a certain theory of nutation. This matrix allows transformation from the aforementioned interim mean frame of date to the (also) interim true frame of the same date. This part is clear and without controversy. The complexities are in the agreement reached (or still to be reached) on the theory of nutation when computing the above parameters. Kinoshita et al. [1979] give an historical review:

> "In astronomical ephemerides, nutation has been computed until now by the formulae which were given by Woolard (1963). The coefficients of the formulae are calculated assuming that the Earth is rigid. However, it has been found in recent analyses of observations ... that some coefficients of actual nutations are in better agreement with values calculated by the non-rigid Earth theory.
>
> "Moreoever, Woolard (1953 gave the nutation of the axis of rotation. Therefore, a small and nearly diurnal variation appears

https://doi.org/10.1017/S0252921100081069 Published online by Cambridge University Press

in the latitude and time observations, which is the so-called dynamical variation of latitude and time, or Oppolzer terms. In the global reduction of latitude and time observations, such as polar motion or time services, the Oppolzer terms have been until now removed from the data at each station (cf. BIH Rapport Annuel 1977, pA-3) or counted out as a part of the non-polar common z and τ-terms (IPMS Annual Report 1974, p. 11). On the other hand, Atkinson (1973) pointed out that if the (forced) nutation of the axis of figure is calculated instead of rotation axis, such a complicated treatment becomes unnecessary.

"Considering these situations, the IAU investigated the treatment of nutations, together with the system of astronomical constants which should be used in new ephemerides, and set up the 'Working Group of IAU Commission 4, on Precession, Planetary Ephemeris, Units, and Time-Scales'. The results by the Working Group are given in the report of Joint Meeting of Commissions 4, 8, and 31, in Grenoble, 1976 (Duncombe et al. 1976). In the report, the proposal by Atkinson is adopted, and the formula for computing the (forced) nutation of figure axis is shown clearly and in detail, by using the equation-numbers given by Woolard (1953). However, the amendments of coefficients taking account of the non-rigidity of the Earth have not been adopted. In regard to this problem, it was noted that there should be a possibility of making further amendments in Kiev Symposium ... .

"At the IAU Symposium No. 78 in Kiev in 1977, the problem with the non-rigid values of nutation was discussed, and a series of new values were recommended which seemed to be based on Molodenskij's non-rigid theory. In the Symposium, however, it was recommended that the axis for which the nutation should be computed was the axis of rotation. This recommendation reversed the resolution given at Grenoble.

"In accordance with the resolution at the Kiev Symposium, an 'IAU Working Group on Nutation under Commission 4' was set up and is investigating these two problems, in order to prepare a fully documented proposal for the next IAU General Assembly in Montreal in 1979. In the second draft of the Working Group circulated on Nov. 16, 1978, the following conclusions are reported: (1) as for the axis to be referred, the Grenoble resolution is still valid, and (2) as for the coefficients of nutation series, the value in which the non-rigidity of the Earth is taken into account should be adopted as a working standard of astronomical observations. In the draft, a table of nutation series is given, and the numerical values in the table are based on the rigid theory by Kinoshita (1977), with use of IAU (1976) System of Astronomical Constants, and are modified by Molodenskij's non-rigid theory (Molodenskij 1961)."

As we understand it, the Kinoshita theory above is for the nutation of the axis of maximum moment of inertia of the "mean shape of the elastic mantle" (briefly, "mean axis of figure of the mantle"). To add to the

https://doi.org/10.1017/S0252921100081069 Published online by Cambridge University Press

history, after the above-quoted Working Group Report was circulated, a new proposal was made by J.M. Wahr and M.L. Smith of CIRES that it would be preferable to adapt the non-rigid earth results of Wahr [1979] for the earth model 1066A developed by Gilbert and Dziewonski [1975]. This model is a rotating, elliptically stratified linearly elastic and oceanless earth with a fluid outer core and a solid inner core. The nutations are computed for the "Tisserand mean figure axis of the surface," which is also a mean mantle fixed axis [Wahr, 1979]. The IAU in Montreal in 1979 considered both proposals and opted for the Kinoshita et al. [1979] series. A few months later in December, 1979, the IUGG in Canberra, in Resolution No. 9 addressed to the IAU, requested reconsideration in favor of the Wahr model. This is where the matter stands now.

It should be pointed out that regardless of the fact that in geodetic or geodynamic applications we are only concerned with the total transformation SNP, it is of scientific importance to understand clearly the definition of the interim true equator and equinox frame of date, more specifically, the exact definition and the desirability (from the observability point of view) of the axis for which the nutation is computed.

In order to simplify the discussion, let us start with the rigid model. The motion of each of the axes, i.e., the axis of figure (F) (maximum moment of inertia), of the angular momentum (H), and the instantaneous rotation axis (I) are described by differential equations. If we want to refer to one of these axes we have to consider the complete solution of the differential equations, i.e., the free solution and the forced solution components. Confusion can arise if one refers to only one solution component (forced or free), but still calls it axis of figure, instantaneous rotation axis, etc. It is mandatory to point out which solution component one refers to. Neglecting to do so has been the reason for the by now classical confusing controversy about the Atkinson papers, though Atkinson [1975, p.381] clearly states:

> "Accordingly, when we speak of computing the nutations for either axis, we mean here computing the forced motion only, excluding the appropriate fraction of the non-computable Chandlerian wobble."

Unfortunately, he, and others as well, then continue to use the term "axis of figure" sometimes in the sense of the axis of maximum moment of inertia and at other times in the sense of the forced motion of the axis of figure.

A remark concerning the "Eulerian pole of rotation" (E_0) as given by Woolard seems in order also. Quoting once again Atkinson [1976]:

> "The wording of the resolution on nutation, and the notes on it, which have been circulated by the Working Group, avoid all explicit mention of the axis of figure, even though they specify that the coefficients which Woolard gives for that axis shall be inserted, and they refer to the "Eulerian pole of rotation" although this cannot ever, in principle, coincide with the celestial pole and really has no more direct connection with the observations than

https://doi.org/10.1017/S0252921100081069 Published online by Cambridge University Press

is shown for it in [his] Fig. 2, i.e., none at all."
The difference between the Eulerian pole of rotation (E_0) and the pole which Atkinson talks about is due to a homogeneous solution component. (E_0) is obtained from the complete solution of (I) by subtracting the periodic diurnal body-fixed motions of (I).

Consequently, the point E_0 has no periodic motion with respect to the crust, but it does have such a motion in space which is exactly the free nutation. Although this spatial motion is conceptually insignificant considering the observation technique (fundamental observations at both culminations), one gets another point, which is called the (true) Celestial Pole (C) in [Leick and Mueller, 1979], by subtracting the forced body-fixed motions of (H) from the complete nutation set of (H). The thus obtained axis (C) has no periodic diurnal spatial motion because the homogeneous solution of the angular momentum (H) is constant (zero). Equivalently, one can say that the nutations of (C) correspond to the forced solution of the axis of figure (rigid case, of course). This is the pole which Atkinson talks about and which is called (mistakenly) the "mean axis of figure." There is no doubt that this is the point to which the astronomical observations as well as lunar laser ranging refer, and the nutation should be adopted for this point. As for terminology, the IAU in 1979 named this (C) pole appropriately the *Celestial Ephemeris Pole* because its motion characteristics, i.e., no periodic diurnal motion relative to crust or space, have always been associated with the concept of the celestial pole. It would be preferred that the word "figure" be dropped entirely for several reasons. First, one intuitively associates the axis of figure with the one for which the moment of inertia is maximum. This is true for the (C) only if the free solution (Chandler) is zero. But this is, generally, not the case. Second, the conceptual definition of (C) can easily be extended to elastic models or models with liquid core (the IAU 1979 case). Moreover, in order to emphasize that the observations take place on the earth surface, it would be useful to denote the actual pole accessible to the fundamental observation techniques by another designation, e.g., (CO), similarly to UTO. The "O" would indicate that the nutations of this pole can in principle be determined only from observations because of the lack of a perfect earth model. Any nutation set based on a model is only an approximation to the nutations of the (CO). In this sense the rigid earth nutations of (I), (H) or (F) are all equivalent. Each of these nutations defines its own pole which has a diurnal motion around the (CO). The purpose of the measuring efforts is to find the corrections to the adopted set of nutations in order to get those of the (CO), the only pole which is observable.

Some have suggested the term "zero excitation figure axis" for what is called above the (CO). The term "zero excitation" would not reduce the confusion. The spatial motion of this axis is computed by adding Atkinson's terms to Woolard's series, but this is equivalent to the *forced* motion of the axis of figure (rigid case). The observed motion of the (CO) relative to the crust only appears as a motion of zero excitation (free motion) at the first sight. Since the conceptual observation time of one position determination is one day, the observed position of the (CO) will always include effects due to oceans, atmospheric mass re-distribution, etc., i.e., the geophysical nutations. These motions are better known as the annual polar motion and the sub-harmonics. Therefore, the zero-excitation pole is not directly observable. On the other hand, the concept of the (CO) can still be used in this case since it is by definition the pole which has no periodic diurnal motions relative to the crust or to space.

There is also the common offset of both the rotation axis and the (CO) caused by the tidal deformation [McClure, 1973]. This is an offset of (I) and (CO) relative to (H) for the perfectly elastic model as compared with the rigid model. We have to remember, again, that the observations refer to the (CO). Therefore, any nutation correction which is derived from observations (based on an adopted set of nutations) will automatically give the corrections to the (CO). Consequently, there is no need for a special consideration of this possible separation, at least not for those harmonic motions whose amplitudes are derived from observations. In fact, the analysis of the observed fortnightly term seems to contradict somewhat the predicted amplitude for the perfectly elastic model.

From the above discussion, it also seems clear that ideas advocating the adoption of nutations for the axis of angular momentum violate the concept of observability. It is true that the direction of (H) in space is the same for the rigid, elastic, or any other reasonable earth model. But this property is not of much interest to the astronomer or geodesist who tries to determine the orientation of the earth. It is *conceptually simpler* to refer to an axis which is observable.

Returning now to the problem of the IAU 1979 adopted set of nutations, there seems to be little difference whether the Kinoshita series is retained or the Wahr set is adopted. Using more and more realistic earth models is certainly appealing. On the other hand, severely model-dependent developments are liable to change as models improve. A more important point is that whichever series is adopted, it should be for the Celestial Ephemeris Pole (C), which (again) has no periodic diurnal motion relative to the *crust* (not the mantle!) or the CIS.

4.3 Earth Rotation (S)

The two components of the S matrix [Mueller, 1969],

$$S = R_2(-x_p)\ R_1(-y_p)\ R_3(\phi) \quad ,$$

are the rotational angle of the first (X) axis of the CTS with respect to the first axis of the interim true equator and equinox frame of date, measured in the equator of the Celestial Ephemeris Pole (or whatever is defined in the N matrix), also known as Apparent Sidereal Time (ϕ), and the polar motion coordinates (x_p, y_p) referred to the same pole and the Z axis of the CTS.

In this connection it should be mentioned that some authors prefer a different "true"frame, which would have no rotation about the Z axis [Guinot, 1979; Murray, 1979; Kinoshita et al., 1979]. It is in such an interim frame where, for example, a nutational theory can be conveniently developed, or satellite orbits calculated [Kozai, 1974]. Such a frame can be obtained from the CIS by a modified NP transformation, where

$$N = R_1(-\Delta\varepsilon\cos M + \Delta\psi\sin\varepsilon\sin M)\ R_2(\Delta\psi\sin\varepsilon\cos M + \Delta\varepsilon\sin M), \text{ and}$$

$$P = R_3(-z + M)\ R_2(\theta)\ R_3(-\zeta_0),$$

where M is the precession in right ascension.

In this case the rotation of CTS about the Z axis (ϕ) is the Apparent Sidereal Time from which the general precession and nutation in right ascension are removed. What is left, thus, is the rotational angle of the X axis of the CTS directly with respect to that of the CIS. Such a definition of the sidereal angle would, of course, necessitate the redefinition of UT1, a possibility for controversy. It should be noted also, that the above transformation is independent of the ecliptic, a preference of many astronomers.

Here there is not very much modeling that can be considered really useful. Of course, the rotation rate of the earth could be modeled as constant and possibly in the UTC scale. This would then mean that observed departures could immediately be referenced to that scale, a current practice. If one really wanted to go overboard, polar motion could also be modeled with the Chandlerian cycle of, say, 428 days and a circular movement of radius 0".15, centered at the Z axis of the CTS. More complex models may be developed (e.g., Markowitz, 1976, 1979], but since there are no valid physical concepts yet for the excitation of the amplitude of the Chandler motion, such modeling would not serve much purpose.

4.4 Deformations (L')

The deformations which reasonably can be modeled at the present state of the art are those due to the tidal phenomena and to tectonic plate movements.

4.41 Tidal Deformations. Tides are generated by the same forces which cause nutation; thus models developed for the latter should be useful for the former. One would think that for earth tides it may not be necessary to use the theories based on the very sophisticated earth models: the amplitude of the phenomena being only around 30 cm, an accuracy of 3% should be adequate for centimeter work. This should be compared, for example, with the accuracy of the Wahr nutation model claimed to be at the 0.3% level. However, the tides and nutations differ in one important respect. The nutations hardly depend upon the elasticity and are affected only slightly by the liquid core (this is one reason why modern theories such as those of Wahr and Kinoshita give only slightly different results). Thus, except perhaps for the largest terms, one can depend upon theory when dealing with nutation. The tides, on the other hand, depend intimately upon the internal properties of the earth, and one must use tidal theories with caution [Newton, 1974]. Additional problems are handling the transformation of the potential into physical displacements and on the calculations of regional (ocean loading) or local tidal deformations.

As far as the transformation of the tidal potential into displacement is concerned, the traditional way to do this is through the Love numbers for the solid effect and through "load" numbers for ocean loading. These numbers, however, are spherical approximations which, for the purely elastic earth, are global constants. For more sophistication, elliptic terms can be added, but they will change the results by 1-2% only. A liquid core model produces resonance effects, which will result in a frequency dependency. The actual numbers representative for a given location can be determined only through in situ observations, such as gravity, tilt, deflections, which are all sensitive to certain Love number combinations and frequencies. Difficulties in this regard include the frequency dependence of the Love number. For example, the Love number h for radial (vertical) displacement can be determined locally from combined gravity and tilt meter observations by the analysis of the O_1 tidal component, but the real radial motion of geodetic interest is influenced by the M_2 and other semidiurnal tidal components.

Tidal loading effects have recently been very successfully computed by Goad [1979] using the 1° square Schwiderski [1978] M2 ocean tide model. Global results show agreement with gravimetrically observed deformation on the 0.5 μgal (5×10^{-10}) level. From this it would seem that with good quality ocean tide models and with proper attention to the frequency dependence, this problem is manageable.

Suitable equations for displacement, gravity change, deflection change, tilt and strain calculations due to tides may be found in [Melchior, 1978; Vanicek, 1980] and in [Wahr, 1979] for the elliptic case.

As a conclusion one can reasonably state that the global and regional station movements due to tides can be estimated today within centimeters. Local effects, however, can be sizable and unpredictable, and therefore they are best determined from in situ observations. Thus most of the tidal effect in fact can and should be removed from the observations.

4.42 Plate Tectonic Mass Transfer. The concept that the earth lithosphere is made up of a relatively small number of plates which are in motion with respect to each other is the central theme of global plate tectonics. The theory implies the transfer of masses as the plates move with velocities determined from geologic evidence (see, e.g., [Solomon and Sleep, 1974; Kaula, 1975; or Minster and Jordan, 1978]). Material rises from the asthenosphere and cools to generate new oceanic lithosphere, and the lithospheric slabs descend to displace asthenospheric material (see, e.g., [Chapple and Tullis, 1977]). A good example of how such a theory can be used to estimate the vertical motions of observatories located on the lithosphere (in terms of changes in geoid undulations) is given in [Larden, 1980], based on specific models constructed in [Mather and Larden, 1978]. The results indicate that changes in the geoid can reach 150 mm/century. Horizontal displacements can be estimated from the plate velocity models mentioned directly with certain possible amendments [Bender, 1974].

4.43 Other Deformations. If one wants to carry the modeling further, it is possible to estimate seasonal deformations due to variations in air mass and groundwater storage, for which global data sets are available [Van Hylckama, 1956; Stolz and Larden, 1979; Larden, 1980]. A more esoteric effect would be the expansion of the earth (e.g., [Dicke, 1969; Newton, 1968]). The rate of possible expansion is estimated to be 10 - 100 mm/century.

One could continue with other modeling possibilities, but there is a real question on the usefulness of modeling phenomena of this level of magnitudes and uncertainties. As a general philosophy, one could accept the criteria that modeling should be attempted only if reliable and global data is available related to the phenomena in question, and if the magnitudes reach the centimeter per year level or so.

One last item which should be brought up is the fact that the issue of referencing observations and/or geodynamic phenomena is not exhausted by the establishment of reference frames of the Cartesian types discussed in this paper. An outstanding issue is still the geoid as a reference surface. Though it is true that three-dimensional advanced geodetic observational techniques do not need the geoid as a reference, there are still others, such as spirit leveling, which are used in the determination of crustal deformations in the local scale. In addition, the geoid is needed to reference gravity observations on a global scale (one should remember that a 1 cm error in the geoid corresponds to a 3 μgal error in the gravity reduction, which is (or soon will be) the accuracy of modern gravimeters). Further, in connection with the use of satellite altimetry for the determination of the departures of sea surface topography from the equipotential geoid (a topic of great oceanographic interest), there is a requirement for a geoid of at least 10 cm accuracy. The determination of such a geoid globally, or even over large areas, is a very difficult problem, which, however, is not the subject of the present paper.

ACKNOWLEDGEMENT. This work was supported by NASA Grant No. NSG 5265 and by Grant No. 4810/79 of the Deutsche Forschungsgemeinschaft (DFG) while the author visited at the Geodetic Institute, Stuttgart University, during June-July, 1980.

REFERENCES

Ashby, N. and Allan, D.W.: 1979, *Radio Science* 14, 649.
Atkinson, R.d'E.: 1973, *Astron. J.* 78, 147.
Atkinson, R.d'E.: 1975, *Monthly Notices Roy. Astron. Soc.* 71, 381.
Atkinson, R.d'E.: 1976, 'On the Earth's Axes of Rotation and Figure',

pres. at XVI General Assembly of IUGG, Grenoble.
Barbieri, C. and Bernacca, P.L.(eds.): 1979, *European Satellite Astrometry*, Ist. di Astronomia, Univ. di Padova, Italy.
Bender, P.L.: 1974, *see* Kołaczek and Weiffenbach (eds.), 85.
Bender, P. and Goad, C.: 1979, *The Use of Artificial Satellites for Geodesy and Geodynamics, Vol. II*, National Tech. Univ. Athens.
Chapple, W.M. and Tullis, T.E.: 1977, *J. Geophys. Res.* 82, 1967.
Dicke, R.H.: 1969, *J. Geophys. Res.* 74, 5895.
Duncombe, R.L., Seidelmann, P.K. and Van Flandern, T.C.: 1974, *see* Kołaczek and Weiffenbach, 223.
Duncombe, R.L., Fricke, W., Seidelmann, P.K. and Wilkins, G.A.: 1976, *Trans. IAU* XVIB, 52.
Einstein, A.: 1956, *The Meaning of Relativity*, Princeton Univ. Press, Princeton, New Jersey.
Fedorov, E.P., Smith, M.L. and Bender, P.L. (eds.): 1980, *IAU Symp.* 79.
Feissel, M.: 1980, *Bull. Geodes.* 54, 81.
Fricke, W.: 1974, *see* Kołaczek and Weiffenbach, 201.
Fricke, W.: 1977, *Veröffentlichungen Astron. Rechen-Inst. Heidelberg* 28, Verl. G. Braun, Karlsruhe.
Fricke, W. and Gliese, W.: 1978, *see* Prochazka and Tucker, 421.
Fricke, W.: 1979a, 'Progress Rept. on Preparation of FK5', pres. at Commission IV, IAU XVII General Assembly, Montreal.
Fricke, W.: 1979b, *see* Barbieri and Bernacca, 175.
Gilbert, F. and Dziewonski, A.M.: 1975, *Phil. Trans. R. Soc. London* A278, 187.
Goad, C.C.: 1979, 'Gravimetric Tidal Loading Computed from Integrated Green's Functions', NOAA Tech. Memorandum NOS NGS 22, NOS/NOAA, Rockville, Md.
Guinot, B.: 1978, *see* Mueller, 1978, 13.
Guinot, B.: 1979, *see* McCarthy and Pilkington, 7.
Kaula, W.M.: 1975, *J. Geophys. Res.* 80, 244.
Kinoshita, H.: 1977, *Celes. Mechan.* 15, 227.
Kinoshita, H., Nakajima, K., Kubo, Y., Nakagawa, I., Sasao, T. and Yokoyama, K.: 1979, *Publ. Int. Lat. Obs. of Mizusawa* XII, 71.
Kołaczek, B. and Weiffenbach, G. (eds.): 1974, *On Reference Coordinate Systems for Earth Dynamics, IAU Colloq. 26*, Smithsonian Astrophys. Obs., Cambridge, Mass.
Kovalevsky, J.: 1979, *see* McCarthy and Pilkington, 151.
Kozai, Y.: 1974, *see* Kołaczek and Weiffenbach, 235.
Larden, D.R.: 1980,'Some Geophysical Effects on Geodetic Levelling Networks', *Proc. 2nd International Symp. on Problems Related to the Redefinition of North American Vertical Geodetic Networks*, Canadian Inst. of Surveying, Ottawa.
Leick, A. and Mueller, I.I.: 1979, *manuscripta geodaetica* 4, 149.
Lieske, J.H., Lederle, T., Fricke, W. and Morando, B.: 1977, *Astron. Astrophys.* 58, 1.
Lieske, J.H.: 1979, *Astron. Astrophys.* 73, 282.
Markowitz, Wm. and Guinot, B. (eds.): 1968, *IAU Symp.* 32, Reidel.
Markowitz, Wm.: 1976, 'Comparison of ILS, IPMS, BIH and Doppler Polar Motions with Theoretical', Rep. to IAU Comm. 19 and 31, IAU General Assembly, Grenoble.

Markowitz, Wm.: 1979, 'Independent Polar Motions, Optical and Doppler; Chandler Uncertainties', Rep. to IAU Comm. 19 and 31, IAU General Assembly, Montreal.
Mather, R.S., Masters, E.G. and Coleman, R.: 1977, *Uniserv G* 26, Univ. of New South Wales, Sidney, Australia.
Mather, R.S. and Larden, D.R.: 1978, *Uniserv G* 29, 11, Univ. of New South Wales, Sidney, Australia.
McCarthy, D.D. and Pilkington, J.D.H. (eds.): 1979, *IAU Symp.* 82, Reidel.
McClure, P.: 1973, 'Diurnal Polar Motion', GSFC Rep. X-592-73-259, Goddard Space Flight Center, Greenbelt, Md.
Melchior, P. and Yumi, S. (eds.): 1972, *IAU Symp.* 48, Reidel.
Melchior, P.: 1978, *The Tides of the Planet Earth*, Pergamon Press, Oxford.
Minster, J.B. and Jordan, T.H.: 1978, *J. Geophys. Res.* 83, 5331.
Molodenskij, M.S.: 1961, *Comm. Obs. R. Belgique, 188 S. Geoph.* 58, 25.
Moran, J.M.: 1974, *see* Kołaczek and Weiffenbach, 269.
Moritz, H.: 1967, *Dept. of Geod. Sci. Rep.* 92, Ohio State Univ., Columbus.
Moritz, H.: 1979, *Dept. of Geod. Sci. Rep.* 294, Ohio State Univ.,Columbus.
Mueller, I.I.: 1969, *Spherical and Practical Astronomy As Applied to Geodesy*, Ungar Publ. Co., New York.
Mueller, I.I.: 1975a, *Geophys. Surveys* 2, 243.
Mueller, I.I. (ed.): 1975b, *Dept. of Geod. Sci. Rep.* 231, Ohio State Univ., Columbus.
Mueller, I.I. (ed.): 1978, *Dept. of Geod. Sci. Rep.* 280, Ohio State Univ., Columbus.
Murray, C.A.: 1979, *see* McCarthy and Pilkington, 165.
Newton, I.: 1686, *Philosophiae Naturalis Principia Mathematica*, Univ. of California Press, 1966.
Newton , R.R.: 1968, *J. Geophys. Res.* 73, 3765.
Newton , R.R.: 1974, *see* Kołaczek and Weiffenbach, 181.
Prochazka, F.V. and Tucker, R.H. (eds.): 1978, *Modern Astrometry, IAU Colloq.* 48, Univ. Obs. Vienna.
Purcell, G.H., Jr., Fanselow, J.L., Thomas, J.B., Cohen, E.J., Rogstad, D.H., Sovers, O.J., Skjerve, L.J. and Spitzmesser, D.J.: 1980, *Radio Interferometry Techniques for Geodesy*, p. 165, NASA Conference Publ. 2115, NASA Scientific & Tech. Information Office, Washington, D.C.
Schwiderski, E.M.: 1978, 'Global Ocean Tides, Part 1: A Detailed Hydrodynamical Interpolation Model', US Naval Surface Weapons Center TR-3866, Dahlgren, Va.
Solomon, S.C. and Sleep, N.H.: 1974, *J. Geophys. Res.* 79. 2557.
Stolz, A. and Larden, D.R.: 1979, *J. Geophys. Res.* 84, 6185.
Van Altena, W.: 1978, *see* Prochazka and Tucker, 561.
Van Hylckama, T.E.A.: 1956, *Climatology* 9, 59.
Vanicek, P.: 1980, 'Tidal Corrections to Geodetic Quantities', NOAA Tech. Rep. NOS 83 NGS 14, NOS/NOAA, Rockville, Md.
Wahr, J.M.: 1979, *The Tidal Motions of a Rotating, Elliptical, Elastic and Oceanless Earth*, PhD diss., Dept. of Physics, Univ. of Colorado, Boulder.
Weinberg, S.: 1972, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, Wiley & Sons, New York.
Woolard, E.W.: 1953, *Astronomical Papers Prepared for the Use of the American Ephemeris and Nautical Almanac*, XV, Part I, US Govt. Printing

Office, Washington, D.C.
Yumi, S. (ed.): 1971, *Extra Collection of Papers Contributed to the IAU Symposium No. 48, "Rotation of the Earth"*, International Latitude Obs., Mizusawa, Japan.