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ERRATUM TO: ON THE LINK BETWEEN COGNITIVE DIAGNOSTIC MODELS AND KNOWLEDGE SPACE THEORY

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The status of the so-called witness condition was assessed incorrectly in Propositions 9 and 10 of Heller, Stefanutti, Anselmi, & Robusto 2015. The parts showing its necessity do not hold for an arbitrary competence structure, but only if it is equal to the power set on the set of skills. A weaker condition holds in the general case. The following formulates this condition, provides a correct statement of the two propositions, and lets the reference to the weaker condition replace that to the witness condition in two sentences. The rest of the paper remains unchanged.

Replace the sentence before Proposition 9 by: "It turns out that a weaker property is relevant for arbitrary competence structures. We say a skill function (Q, S, μ) respects the *weak witness condition* with respect to the competence structure $\mathcal{C}$ whenever

$$\text{for every atom } A \in \mathcal{C} \text{ there is some item } q \in Q \text{ such that } T \in \mu(q) \text{ and } T \subseteq A. \qquad \text{(wW-}\mathcal{C}\text{)}$$

The following result shows that the witness condition (W-$\mathcal{C}$) with respect to some competence structure $\mathcal{C}$ is sufficient for the problem function to be one-to-one on $\mathcal{C}$ in case of a conjunctive skill functions. In general, p one-to-one implies (wW-$\mathcal{C}$) for arbitrary $\mathcal{C}$, and it implies (W-2^S) in case of $\mathcal{C} = 2^S$."

The subsequently reformulated Proposition 9 provides the details.

Proposition 9. *Let (Q, S, μ) be a skill function, p the corresponding problem function, and $\mathcal{C}$ a competence structure on S. If p is one-to-one then μ respects (wW-$\mathcal{C}$), and in case of $\mathcal{C} = 2^S$ it respects (W-2^S). If μ is a conjunctive skill function respecting (W-$\mathcal{C}$) then p is one-to-one on $\mathcal{C}$.*

Proof. To show necessity of (wW-$\mathcal{C}$), suppose that μ does not respect (wW-$\mathcal{C}$). This implies that $p(A) = \emptyset$ for some atom A of $\mathcal{C}$, meaning that there are at least two competence states (namely $\emptyset$ and A) that are mapped onto the same performance state $\emptyset$ and thus p is not one-to-one on $\mathcal{C}$. The necessity of (W-2^S) in case of $\mathcal{C} = 2^S$ follows immediately from its equivalence to (wW-2^S).

The proof of sufficiency for conjunctive skill functions proceeds by contradiction. Suppose p is not one-to-one on $\mathcal{C}$. Then there are distinct competence states C_1, C_2 in $\mathcal{C}$ with $p(C_1) = p(C_2)$.

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Without loss of generality there is an atom $A \subseteq C_1$ in $\mathcal{C}$, which is not included in C_2. Now, assume that the conjunctive skill function μ respects (W-$\mathcal{C}$). Then there is a $q \in Q$ such that $A \in \mu(q)$. But since $q \in p(C_2)$ there is a subset $T \subseteq C_2$ with $T \in \mu(q)$ and $T \neq A$, a contradiction. So μ cannot respect (W-$\mathcal{C}$). □

The above corrections also have consequences for Proposition 10, which now states an implication instead of an equivalence, and reads as follows.

Proposition 10. *Let Q be a knowledge domain, S a set of skills, μ a conjunctive skill function, and $\mathcal{C}$ a competence structure on S. If μ respects (W-$\mathcal{C}$) then its induced problem function p is an order-isomorphism from $\mathcal{C}$ to $\mathcal{K}$ (with respect to $\subseteq$).*

Proof. Let μ satisfy (W-$\mathcal{C}$). Then by definition $C_1 \subseteq C_2$ implies $p(C_1) \subseteq p(C_2)$ for all $C_1, C_2 \in \mathcal{C}$ and p is onto $\mathcal{K}$. Moreover, p is one-to-one on $\mathcal{C}$ by Prop. 9. It remains to show that $p(C_1) \subseteq p(C_2)$ implies $C_1 \subseteq C_2$ for all $C_1, C_2 \in \mathcal{C}$. Assume that $p(C_1) \subseteq p(C_2)$ holds for $C_1, C_2 \in \mathcal{C}$, and let $s \in C_1$. Then there is an atom A at s in $\mathcal{C}$ with $A \subseteq C_1$. By (W-$\mathcal{C}$) it follows that there exists an item $q \in Q$ such that $\mu(q) = \{A\}$. By assumption $q \in p(C_2)$, which means that $A \subseteq C_2$ and thus $s \in C_2$. □

These modifications do not affect the rest of the paper, except for the following two sentences. Replacing (W-$\mathcal{C}$) by (wW-$\mathcal{C}$) they should read:

- "If the skill function μ does not respect (wW-$\mathcal{C}$), then there are distinct competence states in $\mathcal{C}$ delineating the same performance state, so that any assessment based on the latter remains ambiguous." (5.2 Toward Restoring Identifiability, beginning of second paragraph)
- "Section 5.2 shows that besides extending the set of items in order to respect the witness condition (W-2^S), which has already been discussed as a potential remedy (e.g., Tatsuoka, 1990; DeCarlo, 2011), its newly introduced generalization (wW-$\mathcal{C}$) allows for putting restrictions on the set of possible competence states." (7. Conclusions, mid of third paragraph)

Reference

Heller, J., Stefanutti, L., Anselmi, P. & Robusto, E. (2015). On the link between cognitive diagnostic models and knowledge space theory. Psychometrika.

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