

## CORRESPONDENCE.

ON THE NUMBER OF YEARS IN WHICH PREMIUMS  
AMOUNT TO TWICE THE TOTAL SUM PAID.

*To the Editors of the Journal of the Institute of Actuaries.*

DEAR SIRs,—Mr. Touzel's interesting communication which appears in the issue of the *Journal* for November 1925 illustrates a type of problem closely associated with the practical side of our science.

If a solution be sought by operating upon continuous functions some simple and direct results are evolved. Thus suppose, in the first instance, we require to ascertain the relation between  $n$  and  $\delta$  when  $n$  represents the number of years in which a continuous annuity of unity per annum will amount to  $2n$ .

We have 
$$\bar{s}_{\overline{n}|} = \frac{e^{n\delta} - 1}{\delta} = 2n$$

or 
$$\frac{e^{n\delta} - 1}{n\delta} = 2$$

The form of the left hand member of this equation shows at once that under the supposed conditions  $(n\delta)$  will be a constant. By inversely entering Makeham's table of  $\log \frac{e^x - 1}{x}$  (*J.I.A.*, vol. xv, p. 437 *et seq.*) it is found that  $(n\delta)$  equals  $125.64/100$ .

When, however, we come to deal with annual payments, we find that the rate of interest is involved: that is  $(ni)$  is not a constant, but still very nearly so. For  $\bar{s}_n$  is the equivalent of the amount of an annual payment of  $\frac{d}{\delta}$  in advance for  $n$  years, and we therefore require the value of  $(n\delta)$  where

$$\frac{e^{n\delta} - 1}{n\delta} = \frac{d}{\delta}$$

The value of  $(ni)$  will then be obtained from

$$\log (ni) = \log (n\delta) + \log \left( \frac{i}{\delta} \right)$$

For practical values of  $i$  say .03 and .06 it is found that  $(ni)$  equals 125.03/100 and 124.40/100 respectively.

It is seen, therefore, that  $n(100i)$  is subject to a very small range of variation and the adoption of a general and easily remembered value of 125 as suggested by Mr. Touzel is sufficient for the practical purposes in view.

Yours faithfully,

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#### ON THE VALUES OF $P_x$ AT VARIOUS RATES OF INTEREST BY DIFFERENT MORTALITY TABLES.

*To the Editors of the Journal of the Institute of Actuaries.*

DEAR SIRS.—Recently, in checking some monetary tables which I had constructed on the basis of Australian mortality, I was struck with a feature of the function  $P_x$  which appears likely to be of service in certain cases where monetary tables for a given experience are available for a very limited range of rates of interest.

2. I found, when the values of  $P_x$  for any given age at various rates of interest by one mortality table are compared rate by rate with the corresponding values for the same age by another table, that there is roughly a constant difference between these respective values, or more correctly, that these differences, while increasing or decreasing with increases in the rate of interest involved, do so with marked regularity and very slowly.

3. An example will make my meaning clearer. Taking the  $H^M$  (Makeham Graduation) and the  $O^M$  Tables, the corresponding values of  $P_x$  for rates 3 per-cent,  $3\frac{1}{2}$  per-cent and 4 per-cent by each table and the differences between these values are as follows: